1. In the field $\mathbb{F}_{2^5}$ defined by the polynomial $x^5 + x^3 + 1$, compute $x^5, x^6, x^7$ and $x^8$.

2. In the same field as above, let $\alpha = x^3 + x + 1$ and $\beta = x^4 + x$, like in Example 3 on page 5 of the lecture notes. Compute $\alpha^4$ and $(\alpha\beta)^{-1}$.

3. Consider the field $\mathbb{F}_{64}$ defined by the irreducible polynomial $x^6 + x^3 + 1 \in \mathbb{F}_2[x]$. Find the order of $x$.

4. In the field $\mathbb{F}_5$, find the periods of the three sequences

$$\left(2n\right)_{n=1}^{\infty}, \quad \left(n^4\right)_{n=1}^{\infty}, \quad \left(2^n\right)_{n=1}^{\infty}.$$ 

5. Let $A$ be a finite set and let $f : A \to A$ be a function. Let $u_0 \in A$ and let $u_{n+1} = f(u_n)$ for all $n \geq 0$.

(a) Show that the sequence $u_0, u_1, u_2, \ldots$ is ultimately periodic.

(b) Show that the sequence $u_0, u_1, u_2, \ldots$ is periodic if $f$ is a bijection.

6. (a) Prove that $\sum_{\alpha \in \mathbb{F}_q} \alpha = 0$ if $q \neq 2$.

(b) Prove that $\prod_{\alpha \in \mathbb{F}_q^*} \alpha = -1$. 