Galois esimerkki:

$$\begin{tikzcd}[column sep=small]
 & \{id, \tau_1, \tau_2, \tau_3\} \arrow[dl, dash] \arrow[d, dash] \arrow[dr, dash] & \\
\{id, \tau_1\} \arrow[dr, dash] & \{id, \tau_2\} \arrow[d, dash] & \{id, \tau_3\} \arrow[dl, dash] \\
 & \{id\} &
\end{tikzcd}
\qquad
\begin{tikzcd}[column sep=small]
 & \mathbb{Q}(\sqrt{3}, \sqrt{5}\,) \arrow[dl, dash] \arrow[d, dash] \arrow[dr, dash] & \\
\mathbb{Q}(\sqrt{3}\,) \arrow[dr, dash] & \mathbb{Q}(\sqrt{5}\,) \arrow[d, dash] & \mathbb{Q}(\sqrt{15}\,) \arrow[dl, dash] \\
 & \mathbb{Q} &
\end{tikzcd}$$

$$G(\mathbb{Q}(\sqrt{3}, \sqrt{5}\,)/\mathbb{Q})$$

Esimerkki 2

- Olkoon $f(x) = x^4 - 2 \in \mathbb{Q}[x]$.
- Sen hajoamiskunta on

$$L = \mathbb{Q}(\sqrt[4]{2}, i).$$

  Nollakohdat ovat $\pm\sqrt[4]{2}$ ja $\pm i\sqrt[4]{2}$.
- Tässä $[\mathbb{Q}(\sqrt[4]{2}\,) : \mathbb{Q}] = 4$ ja $i \notin \mathbb{Q}(\sqrt[4]{2}\,)$, ja $[\mathbb{Q}(\sqrt[4]{2}, i) : \mathbb{Q}(\sqrt[4]{2}\,)] = 2$ ja siten

$$[\mathbb{Q}(\sqrt[4]{2}, i) : \mathbb{Q}] = 8 = \#G.$$

Laajennuksen kuva:

- Laajennuksen kanta on

$$\{1, \sqrt[4]{2}, (\sqrt[4]{2})^2, (\sqrt[4]{2})^3, i, i\sqrt[4]{2}, i(\sqrt[4]{2})^2, i(\sqrt[4]{2})^3\}$$

- Olkoon $\sigma(\sqrt[4]{2}) = i\sqrt[4]{2}$ ja $\sigma(i) = i$.
- τ kompleksikonjugointi: $\tau(i) = -i$.
- Laskemalla todetaan, että
  $G = \{id, \sigma, \sigma^2, \sigma^3, \tau, \sigma\tau, \sigma^2\tau, \sigma^3\tau\}$.
- Tässä $\tau^2 = id$, $\sigma^4 = id$, $\tau\sigma\tau = \sigma^{-1}$.
- G on diedriryhmä D_4.

$$
\begin{tikzcd}
 & & \mathbb{Q}(\sqrt[4]{2}, i) \arrow[dll, dash] \arrow[dl, dash] \arrow[d, dash] \arrow[dr, dash] \arrow[drr, dash] & & \\
\mathbb{Q}(\sqrt[4]{2}) \arrow[dr, dash] & \mathbb{Q}(\sqrt[4]{2}\, i) \arrow[d, dash] & \mathbb{Q}(\sqrt{2}, i) \arrow[dl, dash] \arrow[d, dash] \arrow[dr, dash] & \mathbb{Q}((1+i)\sqrt[4]{2}) \arrow[d, dash] & \mathbb{Q}((1-i)\sqrt[4]{2}) \arrow[dl, dash] \\
 & \mathbb{Q}(\sqrt{2}) \arrow[dr, dash] & \mathbb{Q}(i) \arrow[d, dash] & \mathbb{Q}(\sqrt{2}\, i) \arrow[dl, dash] & \\
 & & \mathbb{Q} & &
\end{tikzcd}
$$

$$
\begin{tikzcd}
 & & D_4 \arrow[dll, dash] \arrow[d, dash] \arrow[dr, dash] & & \\
 & \{id, \sigma^2, \tau, \sigma^2\tau\} \arrow[dl, dash] \arrow[d, dash] \arrow[dr, dash] & \{id, \sigma, \sigma^2, \sigma^3\} \arrow[d, dash] & \{id, \sigma^2, \sigma\tau, \sigma^3\tau\} \arrow[dl, dash] \arrow[d, dash] \arrow[dr, dash] & \\
\{id, \tau\} \arrow[drr, dash] & \{id, \sigma^2\tau\} \arrow[dr, dash] & \{id, \sigma^2\} \arrow[d, dash] & \{id, \sigma\tau\} \arrow[dl, dash] & \{id, \sigma^3\tau\} \arrow[dll, dash] \\
 & & \{id\} & &
\end{tikzcd}
$$