Intensive Spring Course on Galois Cohomology by Professor Gregory Berhuy*

Basic information

Contents: The course consists of 16-18 hours of teaching (incl. 4-6 hours of introductory lectures and tutoring), a set of exercises, and/or an essay to be hand in by the end of the course.

Credits and grading: 2-3 cu, pass/fail.

Place: Department of Mathematics, University of Turku (classroom XXIV).

Time: Six or seven course days during May 17-25 2010, between 10-16 o'clock. Exact schedule will be announced later.

Registration: By e-mail to Camilla Hollanti: cajoho@utu.fi. The course is free of charge.

Accommodation: Accommodation requests can be sent to Camilla as well. Partial travel and accommodation support available for those in need.

Course description

Preliminaries: Basic algebra (e.g. the advanced course *Algebra* provided by the University of Turku).

Goals: Let us consider the following conjugacy problem: Let $M, M_0 \in \mathrm{M}_n(\mathbf{R})$ be two real matrices. Assume that G is a matrix group (such as $\mathrm{GL}_n, \mathrm{SL}_n, \mathrm{O}_n$, $\mathrm{Sp}_{2n}, \ldots$), and assume that M and M_0 are conjugate by an element of $G(\mathbf{C})$. Are they already conjugate by an element of $G(\mathbf{R})$?

A classical theorem in linear algebra says that it is true for $G = \mathrm{GL}_n$, but simple examples show that it is false for SL_n, even for $n = 2$.

How to explain this difference? This is where Galois cohomology comes into play.

In this course, we will introduce enough material to reinterpret the obstruction to the congujacy problem in terms of Galois cohomology, and to compute explicitely this obstruction. This will be a pretext to introduce general Galois descent problems, to prove the Galois descent lemma and Hilbert's theorem 90.

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