Complexity Issues in Switching of Graphs

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Abstract. In the context of graph transformations we look at the operation of switching, which can be viewed as an elegant method for realizing global transformations of graphs through local transformations of the vertices. We compare the complexity of a number of problems on graphs with the complexity of these problems extended to the set of switches of a graph. Within this framework, we prove a modification of Yannakakis' result and use it to show NP-completeness for the embedding problem. Finally we prove NP-completeness for the 3-colourability problem.

1 Introduction

The operation of switching is an elegant example of a graph transformation, where the global transformation of a graph is achieved by applying local transformations to the vertices. The elegance stems from the fact that the local transformations are group actions, and hence basic techniques of group theory can be applied in developing the theory of switching.

The use of switching presented in this paper is motivated by operations for formalizing specific types of networks of processors, as introduced by Ehrenfeucht and Rozenberg in [3].

For a finite undirected graph $G = (V, E)$ and a subset $\sigma \subseteq V$, the *switch* of G by σ is defined as the graph $G^\sigma = (V, E')$, which is obtained from G by removing all edges between σ and its complement $V - \sigma$ and adding as edges all nonedges between σ and $V - \sigma$. The switching class $[G]$ determined (generated) by G consists of all switches G^σ for subsets $\sigma \subseteq V$.

A switching class is an equivalence class of graphs under switching, see the survey papers by Seidel [8] and Seidel and Taylor [9]. Generalizations of this approach – where the graphs are labeled with elements of a group other than $\mathbf{Z}_2$ – can be found in Gross and Tucker [5], Ehrenfeucht and

Rozenberg [3], and Zaslavsky [12]. Although, in this paper, we will restrict ourselves to graphs, many of the results on complexity hold for these generalizations.

A property $\mathcal{P}$ of graphs can be transformed into an existential property of switching classes as follows:

$$\mathcal{P}_\exists(G) \text{ if and only if there is a graph } H \in [G] \text{ such that } \mathcal{P}(H).$$

We will also refer to $\mathcal{P}_\exists$ as "the problem $\mathcal{P}$ for switching classes".

Let G be a graph on n vertices. It is easy to show that there are 2^{n-1} graphs in $[G]$, and so checking whether there exists a graph $H \in [G]$ satisfying a given property $\mathcal{P}$ requires exponential time, if each graph is to be checked separately.

However, although the hamiltonian cycle problem for graphs (GT37) is NP-complete, the hamiltonian cycle problem for switching classes can be solved in time $O(n^2)$, since one needs only check that a given graph is not complete bipartite of odd order (see [7] or [2]).

After the preliminaries we list a few problems for switching classes that are polynomial for switching classes, yet are NP-complete for ordinary graphs. We continue with giving a procedure for turning algorithms for graph properties into algorithms for the corresponding properties for switching classes.

We generalize to switching classes a result of Yannakakis [10] on graphs (GT21, see [4]), which is then used to prove that the independence problem (GT20) is NP-complete for switching classes. This problem can be polynomially reduced to the embedding problem (given two graphs G and H, does there exist a graph in $[G]$ in which H can be embedded, GT48). Hence, the latter problem is also NP-complete for switching classes. It also turns out that deciding whether a switching class contains a 3-colourable graph is NP-complete (a special case of GT4). The proof of this result nicely illustrates some of the techniques that can be used to tackle this kind of problem.

2 Preliminaries

For a (finite) set V, let $|V|$ be the *cardinality* of V. We shall often identify a subset $A \subseteq V$ with its characteristic function $A : V \to \mathbf{Z}_2$, where $\mathbf{Z}_2 = \{0, 1\}$ is the cyclic group of order two. We use the convention that for $x \in V$, $A(x) = 1$ if and only if $x \in A$. The restriction of a function $f : V \to W$ to a subset $A \subseteq V$ is denoted by $f|_A$.

The set $E(V) = \{\{x, y\} \mid x, y \in V,\ x \neq y\}$ denotes the set of all unordered pairs of distinct elements of V. We write xy or yx for the undirected edge $\{x, y\}$. The graphs of this paper will be finite, undirected and simple, i.e., they contain no loops or multiple edges. We use $E(G)$ and $V(G)$ to denote the set of edges E and the set of vertices V, respectively, and $|V|$ and $|E|$ are called the *order*, respectively, *size* of G. Analogously to sets, a graph $G = (V, E)$ will be identified with the characteristic function $G : E(V) \to \mathbf{Z}_2$ of its set of edges so that $G(xy) = 1$ for $xy \in E$, and $G(xy) = 0$ for $xy \notin E$. Later we shall use both notations, $G = (V, E)$ and $G : E(V) \to \mathbf{Z}_2$, for graphs.

For a graph $G = (V, E)$ and $X \subseteq V$, let $G|_X$ denote the *subgraph* of G *induced* by X. Hence, $G|_X : E(X) \to \mathbf{Z}_2$. For two graphs G and H on V we define $G + H$ to be the graph such that $(G+H)(xy) = G(xy) + H(xy)$ for all $xy \in E(V)$.

A *selector* for G is a subset $\sigma \subseteq V(G)$, or alternatively a function $\sigma : V(G) \to \mathbf{Z}_2$. A *switch* of a graph G by σ is the graph G^σ such that for all $xy \in E(V)$,

$$G^\sigma(xy) = \sigma(x) + G(xy) + \sigma(y) \ .$$

Clearly, this definition of switching is equivalent to the one given at the beginning of the introduction. The set $[G] = \{G^\sigma \mid \sigma \subseteq V\}$ is called the *switching class* of G. We reserve lower case σ for selectors (subsets) used in switching.

A selector σ is *constant* on $X \subseteq V$ if $X \subseteq \sigma$, or $X \cap \sigma = \emptyset$. The name arises from the fact that $G|_X = G^\sigma|_X$.

Some graphs we will encounter in the sequel are K_V, the clique on the set of vertices V, and $\overline{K}_V$, the complement of K_V which is the discrete graph on V; the complete bipartite graph on A and $V - A$ is denoted $K_{A,V-A}$. If the choice of vertices is unimportant we write can K_n, $\overline{K}_n$ and $K_{m,n-m}$ for $n = |V|$ and $m = |A|$.

We now give a few results from the litterature that will be used throughout this paper.

Lemma 1. *The switching class $[\overline{K}_V]$ equals the set of all complete bipartite graphs on V.*

Proof. Given any complete bipartite graph $K_{\{\sigma, V-\sigma\}}$, we can obtain it from $\overline{K}_V$ by switching with respect to σ. Also, every switch of $\overline{K}_V$ can divide its partition at most in two, so it must be a bipartite graph. It is obviously complete bipartite, because $\overline{K}_V$ has no edges. $\square$

From the observation that applying a selector σ to G amounts to computing $G + K_{\sigma, V(G)-\sigma}$ we obtain the following result.

Lemma 2. *It holds that $G \in [H]$ if and only if $G + H \in [\overline{K}_V]$.* $\square$

Lemma 3. *Let $G = (V, E)$ be a graph, $u \in V$ and $A \subseteq V$. If $u \notin A$, then there exists a unique graph $H \in [G]$ such that the neighbours of u in H are the vertices in A.*

Proof. The vertex u has no adjacent vertices in $G_u = G^{N(u)}$, where $N(u)$ is the set of neighbours of u in G. Switching G_u with respect to A connects u to every vertex in A (and no others) yielding H.

To show that H is unique: let H' be such that $N(u) = A$ in H'. Since H and H' are in the same switching class $H + H'$ is a complete bipartite graph (Lemma 2), say $G_{B,V-B}$. Since u has the same neighbours in both, u is isolated in $G_{B,V-B}$. Hence, $G_{B,V-B}$ is a discrete graph and, consequently, $H = H'$. □

We define the *complement* of a switching class as follows:

$$\overline{[G]} = \{\overline{H} \mid H \in [G]\} \ .$$

Lemma 4. *For a graph $G = (V, E)$, $\overline{[G]} = [\overline{G}]$.*

Proof. We show that for a graph $G = (V, E)$ and $\sigma \subseteq V$: $\overline{G^\sigma} = \overline{G}^\sigma$. Indeed, let $x, y \in V$. Then $\overline{G^\sigma}(xy) = 1 - (\sigma(x) + G(xy) + \sigma(y)) = \sigma(x) + (1 - G(xy)) + \sigma(y) = \overline{G}^\sigma(xy)$, because $(1 - a) + (1 - b) = a + b$ for $a, b \in \mathbf{Z}_2$. □

3 Easy problems for switching classes

We now proceed by listing a number of problems that become easy or trivial for switching classes.

Let G be a graph on n vertices. It is easy to show that there are 2^{n-1} graphs in $[G]$, and so checking whether there exists a graph $H \in [G]$ satisfying a given property $\mathcal{P}$ requires exponential time, if each graph is to be checked separately. However, although the hamiltonian cycle problem for graphs (GT37) is NP-complete, the hamiltonianian problem for switching classes can be solved in time $O(n^2)$, since all one needs to check is that a given graph is not complete bipartite of odd order (see [7] or [2]).

Example 1. Let G be the graph of Figure 1(a). The objective is to find a graph in $H \in [G]$ such that H has a hamiltonian path (GT39), in this case $(x_1, \dots, x_6)$. It turns out that this is always possible, something which we shall now illustrate. The technique we use is to change the neighbourhoods of certain vertices using Lemma 3 in such we way that every time it is applied we lengthen the path, without changing the already constructed path.

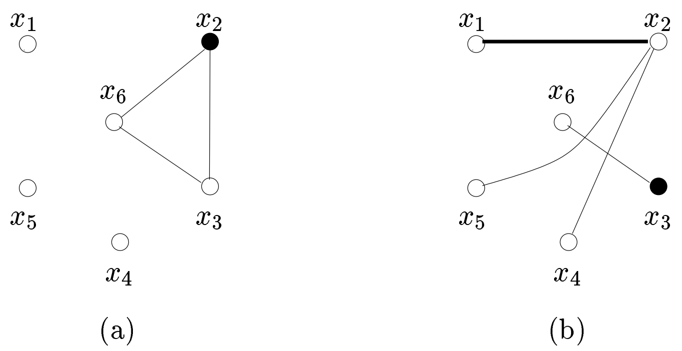


Fig. 1. The graph G and $G_1 \in [G]$ with path (x_1, x_2)

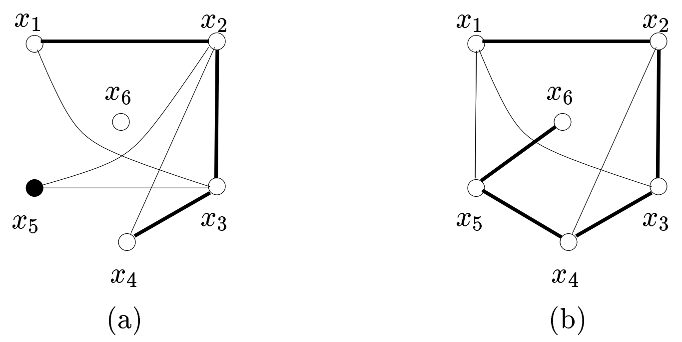


Fig. 2. The graphs G_2 and G_3

Since x_2 is not adjacent to x_1 we apply the selector $\{x_2\}$ (selected vertices are marked in black) and obtain the graph G_1 of Figure 1(b). After applying the selector $\{x_3\}$ to G_3 we add both x_2x_3 and x_3x_4 to the path and obtain G_2 of Figure 2(a) with path (x_1, x_2, x_3, x_4). The last selector we have to apply is $\{x_5\}$ to obtain $H = G_3$ of Figure 2(b), which indeed has a hamiltonian path $(x_1, \dots, x_6)$. $\diamond$

Using the same method we can obtain a spanning tree with maximum degree $\leq k$ (IND1), a spanning tree with at least k leaves (for $2 \leq k \leq n-1$) (IND2), a subgraph that is (noninduced) complete bipartite of order k (for $2 \leq k \leq n$) (a special case of GT21)). Note that these existence problems for graphs are all NP-complete, but as now made clear, trivial for switching classes.

Given any property of graphs decidable in polynomial we can derive an algorithm for deciding the existential property for switching classes.

Theorem 1. *Let $\mathcal{P}$ be a graph property such that $\mathcal{P}(G)$, for G of order n can be decided in $\mathcal{O}(n^m)$ steps for an integer m. Let $\delta(G) \leq d(n)$ where $\delta(G)$ is the minimum degree of G. Then $\mathcal{P}_\exists$ is in $\mathcal{O}(n^{d(n)+1+max(m,2)})$.*

Proof. Let $G = (V, E)$ be the graph, $n = |V(G)|$, $\mathcal{P}$ the property, $d(n)$ the polynomial that bounds the minimum degree of G and m the constant as defined in the theorem. Algorithm 2 checks $\mathcal{P}$ for all graphs such that there is a vertex v of degree at most $d(n)$. It uses Lemma 3.

Algorithm 2.

```
P∃-DegreeBounded(G)
begin
    for all v ∈ V(G) do
        for all subsets V' of V(G) − {v} of size ≤ d(n) do
            σ = (V' − N(v)) ∪ (N(v) − V');
            if P(G^σ) then
                return true;
            else continue;
        od;
    od;
end;
```

The complexity of the algorithm is easily determined. The outer loop is executed n times, the inner loop for each value of v, $\mathcal{O}(n^{d(n)})$ times. The condition of the "if" takes $\max(n^2, n^m)$ and together these yield the complexity as given in the theorem. $\square$

This leads us to the following definition which gives us the predicates such that we need only check a polynomial number of candidate graphs in each switching class.

For $\mathcal{P}$, a predicate on (or property of) graphs, $\mathcal{P}$ is of *bounded minimum degree* k if and only if for all graphs $G = (V, E)$ such that $\mathcal{P}(G)$ holds, there exists $x \in V$ such that the degree of x in G is at most k.

Corollary 1. *If $\mathcal{P}$ is a graph property of bounded minimum degree that is decidable in polynomial time, then $\mathcal{P}_\exists$ is decidable in polynomial time.* $\square$

Example 2. It is well known that planarity of a graph can be checked in time linear in the number of vertices. Because every planar graph has a vertex of degree at most 5, there are only a polynomial number of graphs that can possibly have this property. Since we can find these efficiently, we can check in polynomial time whether $[G]$ contains a planar graph.

Also, every acyclic graph has a vertex of degree at most 1, therefore we get a polynomial algorithm for the acyclicity of switching classes. $\diamond$

By complementation an analogous result can be formulated for predicates that hold for graphs that always have a vertex of degree at least $d(n)$.

4 Yannakakis' result modified for switching classes

Let $\mathcal{P}$ be a property of graphs that is preserved under isomorphisms. We say that $\mathcal{P}$ is

(i) *nontrivial*, if there exists a graph G such that $\mathcal{P}(G)$ does not hold and there are arbitrarily large graphs G such that $\mathcal{P}(G)$ does hold;
(ii) *switch-nontrivial*, if $\mathcal{P}$ is nontrivial and there exists a switching class $[G]$ such that for all $H \in [G]$, $\mathcal{P}(H)$ does not hold;
(iii) *hereditary*, if $\mathcal{P}(G|_A)$ for all $A \subseteq V(G)$ whenever $\mathcal{P}(G)$.

In the following we shall look at nontrivial hereditary properties. If there is a graph for which $\mathcal{P}(G)$ does not hold, then there are arbitrarily large graphs for which $\mathcal{P}$ does not hold, because of hereditarity. Hence, this condition has been omitted from the definition of nontriviality.

Example 3. The following are examples of nontrivial hereditary properties of graphs that are also switch-nontrivial: G is discrete, G is complete, G is bipartite, G is complete bipartite, G is acyclic, G is planar, G has chromatic number $\chi(G) \leq k$ where k is a fixed integer, G is chordal, and G is a comparability graph. $\diamond$

Yannakakis proved in [10] the following general result on NP-hardness and NP-completeness.

Theorem 3. *Let $\mathcal{P}$ be a nontrivial hereditary property of graphs. Then the problem for instances (G, k) with $k \leq |V(G)|$ whether G has an induced subgraph $G|_A$ such that $|A| \geq k$ and $\mathcal{P}(G|_A)$, is NP-hard. Moreover, if $\mathcal{P}$ is in NP, then the corresponding problem is NP-complete.* $\square$

Example 4. If we take $\mathcal{P}$ to be the discreteness property, then Theorem 3 says that given (G, k) the problem to decide whether G has a discrete induced subgraph of order at least k is NP-complete. Note that this problem is exactly one of the standard NP-complete problems: the independence problem (GT20). The result of Yannakakis proves in one sweep that this problem is NP-complete, without having to resort to reduction. $\diamond$

We shall now establish a corresponding result for switching classes. For this, let $\mathcal{P}$ be a switch-nontrivial hereditary property. The property $\mathcal{P}_\exists$ is nontrivial, and $\mathcal{P}_\exists$ is hereditary, since

$$(G|_A)^\sigma = G^\sigma|_A \tag{1}$$

for all $A \subseteq V(G)$ and $\sigma : V(G) \to \mathbf{Z}_2$.

Theorem 4. *Let $\mathcal{P}$ be a switch-nontrivial hereditary property. Then the following problem for instances (G, k) with $k \leq |V(G)|$, is NP-hard: does the switching class $[G]$ contain a graph H that has an induced subgraph $H|_A$ with $|A| \geq k$ and $\mathcal{P}(H|_A)$? If $\mathcal{P} \in NP$ then the corresponding problem is NP-complete.*

Proof. Since $\mathcal{P}_\exists$ is a nontrivial hereditary property, we have by Theorem 3 that the problem for instances (G, k) whether G contains an induced subgraph of order at least k satisfying $\mathcal{P}_\exists$, is NP-hard. This problem is equivalent to the problem stated in the theorem, since by (1), for all subsets $A \subseteq V(G)$, $\mathcal{P}_\exists(G|_A)$ if and only if there exists a selector σ such that $\mathcal{P}((G|_A)^\sigma)$.

If $\mathcal{P}$ is in NP, then we can guess a selector σ and check whether $\mathcal{P}(G^\sigma)$ holds in nondeterministic polynomial time. Hence, $\mathcal{P}_\exists$ is in NP and NP-complete by (the second part of) Theorem 3. $\square$

5 Embedding problems for switching classes

We consider now the embedding problem for switching classes. Recall that a graph H can be embedded into a graph G, denoted $H \hookrightarrow G$, if

H is isomorphic to a subgraph M of G, that is, there exists an injective function $\psi : V(H) \to V(G)$ such that

$$\{\psi(u)\psi(v) \mid uv \in E(H)\} \subseteq E(M)$$

We write $H \hookrightarrow [G]$, if $H \hookrightarrow G^\sigma$ for some selector σ. The embedding problem for graphs is known to be NP-complete, and below we show that it remains NP-complete for switching classes.

For a subset $A \subseteq V(G)$ and a selector $\sigma : V(G) \to \mathbf{Z}_2$ we have by (1) that $[G|_A] = [G]|_A$, where

$$[G]|_A = \{G^\sigma|_A \mid \sigma : V(G) \to \mathbf{Z}_2\}$$

is called the *subclass* of G induced by A.

Hence the switching class $[G]$ contains a graph H which has an independent subset A if and only if the induced subgraph $G|_A$ generates the switching class $[\overline{K}_A]$. As stated earlier in Lemma 1, $[\overline{K}_A]$ equals the set of all complete bipartite graphs on A.

An instance of the independence problem consists of a graph G and an integer $k \leq |V(G)|$, and we ask whether there exists a graph $H \in [G]$ containing an independent set A with k or more vertices. This problem is NP-complete for graphs (that is, the problem whether a graph G contains an independent subset of size $\geq k$) and, by Theorem 4, it remains NP-complete for switching classes.

Theorem 5. *The independence problem is NP-complete for switching classes. In particular, the problem whether a switching class $[G]$ has a subclass $[\overline{K}_m]$ with $m \geq k$, is NP-complete.* □

Recall that a graph $G = (V, E)$ has *clique size* $\geq k$ if there is a set $A \subseteq V$ with $|A| \geq k$ such that $G|_A$ is a clique. By Lemma 4 and Theorem 5 the following corollary holds.

Corollary 2. *For an instance (G, k), where G is a graph and k an integer such that $k \leq |V(G)|$, the problem whether $[G]$ contains a graph with clique size $\geq k$, is NP-complete.* □

From the simple observation that if K_n embeds into a graph G, then it is isomorphic to a subgraph of G, we obtain

Corollary 3. *The embedding problem, $H \hookrightarrow [G]$, for switching classes is NP-complete for the instances (H, G) of graphs.* □

Since we can instantiate H with the clique on V and then use it to solve the clique problem of Corollary 2 using the same value for k, we can conclude the following.

Corollary 4. *For an instance (G, H, k) for graphs G and H on the same domain V of size n and k an integer with $3 \leq k \leq n-1$, the problem whether there is a set $A \subseteq V$ with $|A| \geq k$ such that $H|_A \in [G|_A]$ is NP-complete.* □

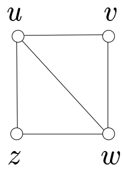


Fig. 3. A graph

Example 5. To illustrate the problem posed in Corollary 4, let $H = K_4$ and let G be the graph of Figure 3.

First of all H cannot be embedded into any graph in $[G]$. So if we take $k = 4$, then the answer to the problem posed in Corollary 4 would be "no". For $k = 3$ however the answer would be "yes", since both graphs contain a triangle. ◇

We write $[H] \hookrightarrow [G]$, if for all $H' \in [H]$ there exists $G' \in [G]$ such that $H' \hookrightarrow G'$. By instantiating H with a clique of size k, the property $[H] \hookrightarrow [G]$ becomes the clique problem of Corollary 2. Hence,

Corollary 5. *For instances (H, G) of graphs the switching class embedding problem $[H] \hookrightarrow [G]$ is NP-complete.* □

• I thought I'd put the following discussion in entirely, since I have the space • Note, however, that the problem to decide whether a given graph H is a subgraph of a graph in $[G]$ is easy. In this case the only difficulty lies in checking whether G can be switched so that H appears within the switch (see [6]). It occurs as a special case of the embedding problem treated here, the case where the embedding is the identity function on $V(H)$. Hence, the NP-completeness arises from the number of possible injections. Note that in Corollary 4 the problem does not lie in the number of injections, because the injection is always the identity function. The NP-completeness here arises from the number of possible subsets A. In this regard the embedding problem as treated above and the problem considered in Corollary 4 are orthogonal generalizations of the membership problem of [6].

5.1 3-colourability for switching classes

We consider in this section the problem of 3-colourability. For a given graph $G = (V, E)$ a function $\alpha : V \to C$ for some set C is a proper colouring of G if for all $uv \in E$, $\alpha(u) \neq \alpha(v)$. The *chromatic number* of G, $\chi(G)$, is the minimum cardinality over the ranges of possible colourings of G. Note that for each $c \in C$. $alpha^{-1}(c)$ is an independent set in G.

The 3-colourability problem for graphs (for a graph G, is $\chi(G) \leq 3$?) is NP-complete. In this section we prove that the 3-colourability problem for graphs can be reduced to the corresponding problem for switching classes, hereby proving that the latter is also NP-complete.

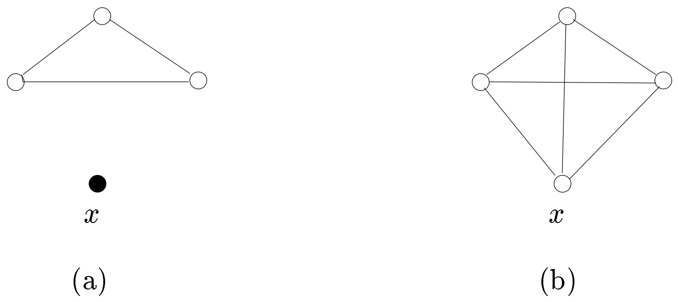


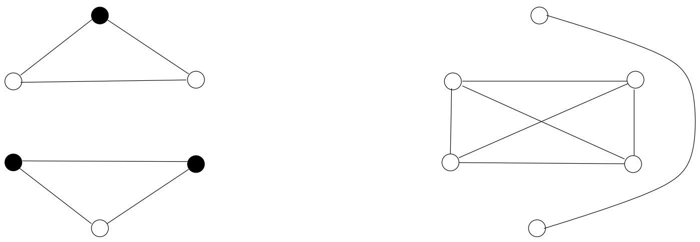
Fig. 4.

Theorem 6. *The problem whether a switching class $[G]$ contains a graph H with chromatic number 3, is NP-complete.*

Proof. Let $G = (V, E)$ be any graph, and let $G_9 = G + 3C_3$ be the graph which is a disjoint union of G and three disjoint triangles. Let A be the set of the nine vertices of the added triangles.

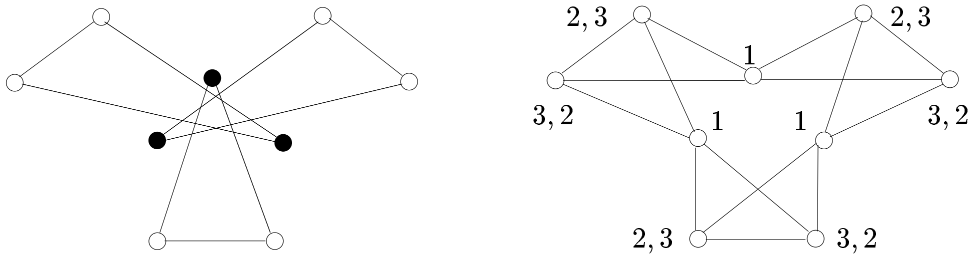
We claim that $\chi(G) \leq 3$ if and only if $[G_9]$ contains a graph H such that $\chi(H) = 3$. Since the transformation $G \mapsto G_9$ is in polynomial time, the claim follows.

It is clear that if $\chi(G) \leq 3$ then $\chi(G_9) = 3$.



(a) (b)

Fig. 5.



(a) (b)

Fig. 6.

Suppose then that there exists a selector σ such that $\chi(G_9^\sigma) = 3$, and let $\alpha : V \cup A \to \{1, 2, 3\}$ be a proper 3-colouring of G_9^σ.

If σ is constant (either 0 or 1) on V, then G is a subgraph of G_9^σ, and, in this case, $\chi(G) \leq 3$.

Assume that σ is not constant on V. Since G_9^σ does not contain K_4 as a subgraph, it follows that σ is not a constant selector on any of the added triangles, see Figure 4(a) and (b). Further, each of these triangles contains equally many selections of 1 (and of 0, of course), since otherwise the subgraph $G_9^\sigma|_A$ would contain K_4 as its subgraph, see Figure 5(a) and (b).

We may assume that each of the added triangles contains exactly one vertex v with $\sigma(v) = 1$ (otherwise consider the complement of σ, $V(G_9) - \sigma$), see Figure 6(a). Let these three vertices constitute the subset $A_1 \subset A$.

In the 3-colouring α the vertices of A_1 obtain the same colour, say $\alpha(v) = 1$ for all $v \in A_1$; and in each of the added triangles the other two vertices obtain different colours, 2 and 3, since they are adjacent to each other and to a vertex of A_1 in G_9^σ, see Figure 6(b).

Each $v \in V$ with $\sigma(v) = 1$ is connected to all $u \in A - A_1$; consequently, $\alpha(v) = 1$ for these vertices. Therefore the set $B_1 = \sigma^{-1}(1) \cap V$ is an independent subset of G_9^σ. Since σ is constant on B_1, B_1 is also independent in G. The vertices in $V - B_1$ (for which $\sigma(v) = 0$) are all adjacent to the vertices in A_1 in G_9^σ, and therefore these vertices are coloured by 2 or 3. The subsets $B_2 = \alpha^{-1}(2) \cap V$ and $B_3 = \alpha^{-1}(3) \cap V$ are independent in G_9^σ. Again, since σ is constant on both B_2 and B_3, these are independent subsets of G. This shows that $\chi(G) \leq 3$. $\square$

6 Open problems

An interesting open problem is to get more insight into those problems which are polynomial for graphs, but become NP-complete for switching classes. As an example, in [7] it is stated that the problem whether a switching class contains a k-regular graph for some k is NP-complete. Another open problem is the bipartiteness problem. It remains to be determined whether bipartiteness of switching classes is NP-complete. The current idea is that it is polynomial.

From the results in this paper one might conclude that "acyclic" problems (like hamiltonian path) become easy for switching classes, but it would also be interesting to find some more general statement. For instance hamiltonian circuit is not "acyclic", but it turned out be easy for switching classes.

Acknowledgements

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