1. Use PIE to count the number of integer solutions of \( x_1 + x_2 + x_3 + x_4 = 14 \), where \( 1 \leq x_i \leq 8 \) for all \( i \). (Present the solution in details.)

2. Count the number of permutations \( \alpha \in S_8 \) with \( \alpha(2i) \neq 2i \) for all \( 1 \leq i \leq 4 \).

3. Use PIE to prove that

\[
\sum_{k=1}^{m} (-1)^{k-1} \binom{n}{k} \binom{n-k}{m-k} = \binom{n}{m}.
\]

4. Determine the ménage number \( M_n \) as the number of ways of seating \( n \) couples at a circular table such that men and women alternate, and no one sits next to his or her spouse.

5. Two shuffled decks of \( n \) cards \([1, n]\) are dealt one card at a time from each deck. What is the probability that the same card is never dealt at the same time from both of the decks?

6. Let \( P \) be a finite poset.

   (a) Let \( \alpha = \zeta - \delta \). Show that \( \alpha^k(x, y) = \alpha \ast \cdots \ast \alpha(x, y) \) (where \( \alpha \) is \( k \) times) is the number of chains of length \( k \) in \( P \) from \( x \) to \( y \).

   (b) Let \( P \) be a finite poset. Denote \( \zeta^2 = \zeta \ast \zeta \). Show that \( \zeta^2(x, y) \) is the number of elements in the interval \([x, y]\).

**Additional theorem.** (P. Hall) Let \( P \) be a finite poset with a zero \( 0 \) and a maximum element \( 1 \). Then

\[
\mu(0, 1) = \sum_{i=1}^{n} (-1)^i c_i,
\]

where \( c_i \) is the number of chains, \( 0 < x_1 < \cdots < x_{i-1} < 1 \) and \( n \) is the length of the longest chain (without repetitions).

7. Find the Möbius function of the finite poset of Example 6.1.

8. Let \( A \subseteq [1, n] \) be a \( t \)-set. Show that there are

\[
f(A) = \sum_{k=t}^{n} \binom{n-t}{k-t} (n-k)!( -1)^{k-t}
\]

permutations \( \alpha \in S_n \) for which \( A \) is the set of fixed points. Conclude that the number of permutations on \([1, n]\) that have exactly \( t \) fixed points is

\[
\sum_{\substack{A \subseteq [1, n] \\ |A|=t}} f(A) = \binom{n}{t} \sum_{k=t}^{n} \binom{n-t}{k-t} (n-k)!( -1)^{k-t}.
\]