1. How many permutations are there in $S_n$ (on $X = [1, n]$) that have a cycle decomposition $\alpha = \alpha_1\alpha_2$ into one $k$-cycle $\alpha_1$ and one $(n-k)$-cycle $\alpha_2$?

2. Consider two indistinguishable standard dices $D$ and $D'$ (with painted digits $1, 2, \ldots, 6$ on their faces). Count using Cauchy-Frobenius the number of different outcomes when the dices are tossed once. (Without C-F we know that the answer is $21 = (1/2) \sum i$ by considering the pairs $(i, j)$ with $j \leq i$, but pretend you did not understand this.)

3. Consider an equilateral triangle $T$. In how many ways one can colour the edges of $T$ with four colours, when two colourings are regarded the same if there is an isometry (rotation or reflection of $T$) that transforms one to the other.

4. Prove Theorem 8.4: Two permutations $\alpha, \beta \in S_n$ are conjugates if and only if $\text{Type}(\alpha) = \text{Type}(\beta)$.

5. Let $G$ be a finite group, and denote by $C(a) = \{bab^{-1} \mid b \in G\}$ the conjugacy class of the element $a \in G$. Show that $|C(a)|$ divides $|G|$.

6. Consider a cube, where the vertices are labelled by circles and squares as in the figure. Show that the rotational symmetry group $G$ of this cube has 12 elements.

7. Assume $G$ is a transitive permutation group on a finite set $X$ with $|X| \geq 2$, i.e., $G$ has only one orbit. Show that there exists an $\alpha \in G$ that has no fixed points.

8. Let $G$ be a permutation group on $X$. Show that for all $\alpha \in G$ and $x \in X$,

$$G_{\alpha(x)} = \alpha G_x \alpha^{-1}.$$