Recognizing L-convex polyominoes in real time

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I. Introduction
Cellular automata

- 2 dimensions
- Von Neumann Neighborhood
Real Time recognition

- Input is a rectangle of any proportion
- Output on the southwest cell
- Real time is $RT(n, m) = n + m$
Real Time recognition

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- Output on the southwest cell
- Real time is $RT(n, m) = n + m$

**Prop.**

Any language recognizable in real time plus a constant can be recognized in real time.
Polyominoes are finite 4-connected subsets of $\mathbb{Z}^2$.

- L-convex polyominoes
- HV-convex if lines and columns are connected
- L-convex if any two cells can be connected by a path changing direction at most once
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**Theorem**
The language of all L-convex polyominoes can be recognized in real time by a cellular automaton with von Neumann neighborhood.
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**Theorem**

The language of all L-convex polyominoes can be recognized in real time by a cellular automaton with von Neumann neighborhood.
II. Construction
Send signals from each corner.

When signals reach the opposite border, they check the corresponding area and send a secondary signal.

When two secondary signals collide, they check the corresponding area.
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Characterization

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**Prop.**
An HV-convex polyomino is L-convex iff none of the checked areas contain polyomino cells
Characterization

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Prop. An HV-convex polyomino is L-convex iff none of the checked areas contain polyomino cells

Prop. It is sufficient to check from the NE and NW corners
To implement the characterization in real time with CA signals we must be able to:

- decide quickly if there is a polyomino cell in a checked area
- handle collisions between signals from different corners
Horizontal compression
Vertical Compression
Information is sent through the configuration during the compressions so that each cell knows where it is relatively to the polyomino

Compressions take $\frac{n+m}{2}$ steps, but the computation can then work twice as fast.
Preprocessing

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Signal Collisions (1)

Problem: secondary signals from different corners collide

Solution: horizontal primary signals carry a counter of primary signals crossed

The slowdown induced by the counter cannot cause the signal travel time to exceed real time.
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The slowdown induced by the counter cannot cause the signal travel time to exceed real time.
Problem: two secondary signals merge

Solution: ensure that only the left-most corner sends a horizontal primary signal (can be done with information gathered during the preprocessing phase)
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Solution: ensure that only the left-most corner sends a horizontal primary signal (can be done with information gathered during the preprocessing phase)
- Same construction as before
- Need to perform the vertical compression first
- More information needs to be sent during the horizontal compression
III. Open problems
**Question**

Is there a language that can be recognized with a tiling but not with a real time CA with von Neumann neighborhood?