Effects of a Global Rule on Interacting Cellular Automata

Alyssa Adams\textsuperscript{1,4}, Hector Zenil\textsuperscript{2,4}, Eduardo Hermo Reyes\textsuperscript{3}, Joost Joosten\textsuperscript{3,4}

\textsuperscript{1}Beyond Center, Arizona State University, Tempe, AZ, USA
\textsuperscript{2}Department of Computer Science, University of Oxford, UK
\textsuperscript{3}Department of Logic, History and Philosophy of Science, University of Barcelona, Spain
\textsuperscript{4}Algorithmic Nature Group, LABoRES, Paris, France
Outline

• Motivation
• Cellular automata with a global rule
• Characterizing Complexity
• Results & Discussion
• Future explorations
Motivation

Same initial conditions…

Same two rules…

But different global rule!
Cellular Automata with a Global Rule

With some initial condition…

ECA 1 has:
• Alphabet \{0, 1\}
• Rule \(r_1\) with rule table \(R_1\)

ECA 2 has:
• Alphabet \{0, 2\}
• Rule \(r_2\) with rule table \(R_2\)

Interacting CA has:
• Mixed neighborhood with alphabet \{0, 1, 2\}
• Global rule \(GR\) with rule table \(\{R_1 \cup R_2 \cup MRT\}\)
• \(MRT :=\) mixed rule table
Cellular Automata with a Global Rule

• Mixed Rule Table
  – Outcomes for all possible mixed neighborhoods
  – Neighborhoods not in $R_1$ nor $R_2$

• 12 possible mixed neighborhoods
• $3^{12} \approx 5 \times 10^5$ possible outcome combinations,
  $5 \times 10^5$ possible global rules (GRs)
Methods

• Only 25% of GRs were studied
• 5 initial conditions
  – Initial conditions that already include mixed neighborhoods
  – De Bruijn sequences of order $n = 3$ (for mixed alphabet size $\{0, 1, 2\}$) and of size $k = 5$
  – 242 bits long (periodic boundaries)
• 4 different interacting ECA rules
  – ECA rules 32, 108, 30, 54
  – 10 possible interactions, avoiding redundancy
• 8.8 million cases total, 100 time steps
Characterize Results

• Capture resulting CA output in terms of Wolfram classes with...

• Kolmogorov-Chaitin complexity of a dynamic system\(^1, 2\)

\[
K(r) = \lim_{t \to \infty} \max K(s(i, t))
\]

\[
K(s(i, t)) := \min \text{length}(p) \mid U_t(p(i)) = s
\]

• \(K(s)\) is approachable from below

\(^1\) Zenil 2010, \(^2\) Zenil & Villarreal-Zapata, 2013
Characterize Results

Class 1: Evolution has asymptotic compressibility ratio $= 0$

Class 2: Evolution has compressibility ratio $\leq \frac{1}{2}$

Class 3: Evolution has compressibility ratio $= 1$

Class 4: Evolution has asymptotic compressibility ratio $= 1$

Lempel-Ziv-Welch compression algorithm
Characterize Results

11 segments, each 22 bits long
- Roughly capture regions of different complexity values (low resolution)

Measure asymptotic compression values and compression values at multiple time steps
Interactions of Interest
Future Directions

• Explore the effects of all possible global rules
  – All rule interactions (88)
  – 100 initial conditions
  – 400 billion outputs

• Are there set of GRs that *always* increase/descrease complexity?

• Identify complexity drivers

• Sensitivity to initial conditions