Modelling the pulse profiles of accreting millisecond pulsars and X-ray bursters

Juri Poutanena∗ and Marek Gierliński\textsuperscript{b}

\textsuperscript{a}Astronomy Division, P.O. Box 3000, FIN-90014 University of Oulu, Finland
\textsuperscript{b}Department of Physics, University of Durham, South Road, Durham DH1 3LE, UK

The observed variability from the first discovered accreting millisecond pulsar SAX J1808.4−3658 can be explained if the X-ray emission is produced by Comptonization in a hot slab of Thomson optical depth $\tau_{es} \sim 1$ at the neutron star surface. We construct a detailed model of the X-ray production accounting for the Doppler boosting, relativistic aberration and gravitational light bending in the Schwarzschild metric. We show that the black body radiation is strongly beamed along the normal to the slab (a “pencil”–like emission pattern), while the Comptonized luminosity has a broader angular distribution peaking at about 60° from the slab normal (a “fan”–like pattern). Our model reproduces well the pulse profiles at different energies simultaneously, corresponding phase lags, as well as the time-averaged spectrum. The observed soft phase lags result from different radiation patterns of the two main emission components. We determine the radius of the compact object to be $R \sim 11$ km if its mass $M = 1.6 \, M_{\odot}$, while for $M = 1.4 \, M_{\odot}$ the best-fitting radius is $\sim 8.5$ km. The lower limit on the inclination of the system is 65°. We present also simple analytical formulae for computing the light curves and oscillation amplitudes expected from hot spots in X-ray bursters and millisecond pulsars.

1. INTRODUCTION

Coherent oscillations in the frequency range of 200–600 Hz observed during X-ray bursts from a number of low-mass X-ray binaries [11] as well as in the persistent emission from five accreting neutron stars [15] have triggered the efforts to use the information on the amplitude of variability and the folded pulse shape to put constraints on the compactness of the neutron star and thus its equation of state as well as the emission pattern from the neutron star surface. A rather limited photon statistics from X-ray bursters does not allow to reach a high accuracy [6,7,12]. By folding the pulse profile of accreting ms pulsars over a longer observational period (days rather than seconds in the X-ray burst oscillations), one can try to get better constraints on the physical parameters [3,9].

In this paper we describe the basic ingredients of the physical modelling of the X-ray emission from the surface of a rapidly rotating neutron star. We construct a detailed model accounting for relativistic effects. We compare the model with the data on the first discovered accreting ms pulsar SAX J1808.4−3658 and put constraints on the inclination of the system, the position of the emitting region relative to the rotational pole and its emission pattern as well as the stellar radius.

2. RADIATION PATTERN AND THE SPECTRUM

The accreting material following the magnetic field lines close to the neutron star is stopped in the very vicinity of the surface producing a radiation dominated shock [1,4]. We approximate the shock geometry as a plane-parallel slab. Assuming the settling bulk velocity smaller than the thermal electron velocity, we can easily compute the angular distribution of the escaping radiation. The radiation pattern is very different for photons of different scattered orders (see Fig. 1). One sees that the black body (marked with 0) is strongly peaked along the normal to the slab, while photons scattered many times are beamed in the di-
Figure 1. Polar diagram of the normalized radiation flux $\mu I(\mu)$ escaping from an electron scattering slab of Thomson optical depth $\tau_{\text{es}} = 0.7$. Here $\arccos \mu$ is the angle between the normal and the photon direction. Different scattering orders are shown and marked by numbers.

Figure 2. Time-averaged spectrum of SAX J1808.4−3658 as observed by RXTE and the model spectrum.

3. MODEL AND RESULTS

The light curves from a circular spot are computed accounting for the special relativistic effects (Doppler boosting, relativistic aberration and the gravitational light bending in Schwarzschild metric). The model is described in details in [9]. The spot spectrum is assumed to consist of two components: a black body and a Comptonization tail with different angular distribution. Their spectral shapes are taken as in the observed time-averaged spectrum (see Fig. 2). The model parameters are: the pulsar frequency (fixed at $\nu = 401$ Hz for SAX J1808.4−3658), the neutron star mass $M$, stellar radius $R$, inclination $i$, colatitude of the spot centre $\theta$, “optical depth” $\tau$, that describes the angular dependence of the black body intensity $I_{\text{bb}}(\mu) \propto \exp(-\tau/\mu)$, and a parameter $a$ determining the angular distribution of the Comptonized radiation $I_{\text{sc}}(\mu) \propto 1 + a\mu$. This linear dependence of the intensity mimics the angular distribution from the slab geometry (Fig. 1).

A rapid rotation of the star causes significant changes in the pulse profile. We illustrate this by showing in Fig. 3 the light curves of a slowly rotating star (dashed curves) and those modified by the Doppler boosting and aberration for a neutron star of rotational frequency 401 Hz (solid curves). We consider both the black body and the Comptonized emission. The pulse profiles strongly depend on the assumed emission pattern. Light bending reduces the variability amplitude with respect to a less compact star. The Doppler effect can be accounted for analytically by multiplying the curves for a slowly spinning star by $\delta^5$ and $\delta^4$ (where $\delta$ is the Doppler factor) for the black body and the Comptonized emission, respectively (shown by dots, see details in [9]).

The model parameters can be constrained by fitting the observed light curves from SAX
Figure 3. Light curves expected from a slowly rotating star (dashed curves) and that rotating at 401 Hz (solid curves). Parameters: \( M = 1.4 M_\odot \), \( R = 2r_g = 8.4 \text{ km} \), \( i = 80^\circ \), \( \theta = 11^\circ \), \( \tau = 0.16 \) and \( a = -0.78 \) (this parameter corresponds to the scattering optical depth \( \tau_{\text{sc}} \sim 0.7 \)).

J1808.4−3658 (see Fig. 4). The fitted emission pattern of Comptonized and black body radiation confirms our expectations: the black body is beamed along the surface normal while the hard radiation flux (proportional to the intensity times the cosine of the projected area) is more isotropic (Fig. 4b). One of the main result of this study is determination of the radius of the compact star. Depending on the assumed mass the constraints are shown by circles with double error bars (for 90 and 99 per cent confidence intervals) in Fig. 5. For comparison we show also the relations between the compact star mass and its radius for several equations of state of neutron and strange stars. Details are given in [9]. All realistic equations of state lie to the right of the causality line. The line \( R = 2r_g \) (where \( r_g = 2GM/c^2 \) is the Schwarzschild radius) is shown to guide the eye. We constrain the inclination of the system \( i > 65^\circ \). Our model is able to fit the time-averaged spectra, the energy dependent pulse profiles and the observed phase lags, simultaneously. The energy dependence of the pulse profiles arises because the radiation pattern of the two main spectral components is different.

Very similar pulse profiles (but with higher oscillation amplitude) were also observed from the recently discovered fifth ms pulsar XTE J1814-338 [13]. In the framework of the present model, the data can be explained for example, by increasing the colatitude of the spot centre \( \theta \) from 11 to \( \sim 17^\circ \). The presence of the harmonic in the burst oscillations [13] could result from the larger contribution of the Comptonized radiation (with a “fan”–like emission pattern) to the observed flux in this source comparing to other X-ray bursters. This interpretation is supported by the observed hard spectrum [5].

Figure 4. (a) Normalized pulse profiles of SAX J1808.4−3658 in the 3–4 keV (circles) and 12–18 keV (squares) energy band and the best-fitting model light curves (solid and dashed curves). Same parameters as in Fig. 3. (b) The angular distribution of the intrinsic black body (solid curve) and Comptonized (dashed curve) fluxes \( \mu I(\mu) \) in the spot co-rotating frame normalized as \( \int \mu I(\mu) d\mu = 1 \). Only the range of angles between the dotted lines is actually observed. (c) The observed (crosses) and the model (solid curve) phase lags at the pulsar frequency relative to the 3–4 keV band.
Figure 5. Constraints on the radius of the compact star from the light curve of SAX J1808.4−3658.

4. ANALYTICAL LIGHT CURVES AND OSCILLATION AMPLITUDES

Using formalism described in [2] (see [9] for details) one can obtain simple expressions for the light curve expected from a small spot at a rapidly rotating neutron star accounting for Doppler boosting as well as gravitational bending. For example, the bolometric flux from a black body spot is

$$dF = \left(1 - \frac{r_g}{R}\right)^2 \delta^5 \left[\frac{r_g}{R} + \left(1 - \frac{r_g}{R}\right) \cos \psi\right] I_0 \frac{dS}{D^2},$$

(1)

where $I_0$ is the black body intensity (in the spot comoving frame), $dS$ the spot area, $D$ is the distance to the source, and $\cos \psi = \cos i \cos \theta + \sin i \sin \theta \cos \phi$, $\phi$ is the pulse phase. We can also get simple (but very accurate) expression for the oscillation amplitude (here from one spot and neglecting Doppler boosting which does not affect it much):

$$A \equiv (F_{\text{max}} - F_{\text{min}})/(F_{\text{max}} + F_{\text{min}}) = U/Q,$$

(2)

where we defined

$$U = (1 - r_g/R) \sin i \sin \theta,$$

$$Q = r_g/R + (1 - r_g/R) \cos i \cos \theta.$$  

(3)

A spot of angular radius $\rho$ produces amplitude

$$A_\rho = U/\left[Q + \frac{r_g}{R} \tan^2 \frac{\rho}{2}\right].$$

(4)

These results can be generalized to account for Doppler boosting, different emission pattern, and for computations of the amplitudes of other harmonics [8]. The resulting expressions are simple and can be used for analysis of the data on coherent oscillations from ms pulsars and X-ray bursters. Long computations are not needed for this purpose.

REFERENCES

15. Wijnands R., 2003, these proceedings