ERRATUM:
MINIMIZERS OF THE VARIABLE EXPONENT, NON-UNIFORMLY
CONVEX DIRICHLET ENERGY

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http://mathstat.helsinki.fi/analysis/varsobgroup/

The original paper contains a flaw in Lemma 4.4. This lemma should be written as follows.

\textbf{Lemma 4.4} Let $p$ be a bounded log-Hölder continuous exponent on an open set $\Omega \subset \mathbb{R}^n$. Then

$$|u_\delta(x)|^{p(x)} \leq C \left( \varrho L^p(\Omega)(u) + 1 + |\Omega| \right)^{p'/p} \left( \int_{B(x,\delta)} |u(y)|^{p(y)} \, dy + 1 \right)$$

for all $x \in \Omega$, $\delta \in (0, \infty)$, and $u \in L^{p(x)}(\Omega)$.

The proof of Lemma 4.4 is as in the paper. Lemma 4.4 has been used twice, both times in the proof of Theorem 4.6.

\textbf{Corrections for the proof of Theorem 4.6}

First time Lemma 4.4 has been used in the beginning of page 185. Here the right hand side of Lemma 4.4 is estimated by the maximal operator. Surely this works also for Lemma 4.4 above. Second time Lemma 4.4 is used between inequalities (4.3) and (4.4). Here we have to change our arguments:

Thus we have shown that

$$\limsup_{\delta \to 0} \int_F |\nabla u_\delta|^{p(x)} \, dx \leq \int_{F \setminus Y} |\nabla u|^{p(x)} \, dx + \limsup_{\delta \to 0} \int_Y |\nabla u_\delta|^{p(x)} \, dx.$$  

This inequality holds for every $\delta' > 0$. Now we fix $\varepsilon > 0$ and restrict our attention to so small values of $\delta'$ that $|Y' \setminus Y| < \varepsilon$ and

$$\int_{Y' \setminus Y} |\nabla u|^{p(x)} \, dx < \varepsilon.$$  

We estimate the remaining integral from (4.3) by another split:

$$\int_Y |\nabla u_\delta|^{p(x)} \, dx = \int_{Y' \setminus Y_\delta} |\nabla u_\delta|^{p(x)} \, dx + \int_{Y\setminus Y'} |\nabla u_\delta|^{p(x)} \, dx.$$
For the first integral we use almost the same method as before, except that we do not use the maximal function as an intermediary (because \( p \) is not now bounded away from 1 as \( \delta \to 0 \)). We have

\[
|\nabla u_\delta(x)|^{p(x)} \leq C \int_{B(x,\delta)} |\nabla u(y)|^{p(y)} \, dy + C
\]

for \( x \in Y' \setminus Y_\delta \) by Lemma 4.4. Therefore

\[
\int_{Y' \setminus Y_\delta} |\nabla u_\delta|^p \, dx \leq C \int_{Y' \setminus Y_\delta} \int_{B(x,\delta)} |\nabla u(y)|^{p(y)} \, dy \, dx + C |Y' \setminus Y| \leq C \epsilon.
\]

Thus the right hand side is bounded by \( C \epsilon \).

The rest of the proof of Theorem 4.6 is as in the paper. □

There is a minor mistake in Proposition 6.3 as well. The correct version should be as follows.

**Proposition 6.3** Let \( p \) be a bounded continuous exponent and \((\lambda_j)\) be a sequence decreasing to 1. Let \((u_\lambda_j)\) be a sequence of \( p(\cdot) \)-solutions in \( \Omega \) with boundary values \( f \in W^{1,p(\cdot)}(\Omega) \cap L^{\infty}(\Omega), \delta > 1 \), so that \( u_\lambda_j \to u \) in \( L^{p(\cdot)}(\Omega) \) and \( u_\lambda_j \twoheadrightarrow u \) in \( W^{1,p(\cdot)}_{loc}(\Omega \setminus Y) \). Then

(i) \( u \in BV^{p(\cdot)}(\Omega) \); and

(ii) \( \mathcal{E}_{BV^{p(\cdot)}(\Omega)}(u) \leq \liminf_{j \to \infty} \mathcal{E}_{L^{p(\cdot)}_{loc}(\Omega \setminus Y)}(\nabla u_\lambda_j) \) for every open set \( E \subset \subset \Omega \).

The proof is as in the paper, except we may assume that the open neighborhood \( U \) of \( E \cap Y \) is a subset of \( E \).

Proposition 6.3 is used in the proof of Theorem 7.1 and we have to slightly modify its proof: Proposition 6.3 has to been used in an open set \( U \) not in a compact set \( F \). Theorem 7.1 holds as it is stated in the paper.