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THE USE AND MISUSE OF THE PRINCIPLE OF AXIOMATICS IN LINGUISTICS

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Certain fundamental questions concerning the methodology of grammatical descriptions are discussed in this paper. Linguistics and logic are shown to have important characteristics in common. It is also shown, on the other hand, that the principle of recursivity, borrowed from logic, has been applied rather blindly to linguistics.

1. Introductory remarks

Today we are witnessing an increasing convergence, both in aims and in methods, between linguistics and logic. This state of affairs inevitably raises the following question: If linguistics is an empirical science and logic is a formal science, as is commonly assumed, how, then, is the above-mentioned convergence or even overlapping to be understood? I have provided a general answer to this question in Itkonen 1974, where I argue, among other things, that synchronic linguistics is a case of conceptual analysis, or 'explication' in a technical sense, and therefore must be kept strictly apart from empirical sciences, if by 'empirical science' we mean a science genuinely in accord with the methodology of natural science. [1] In this paper I shall show that, in accordance with my general position there is an important methodological level where linguistics (in the sense of synchronic-grammatical description) and logic are similar to each other and different from all natural or quasi-natural sciences. Secondly I shall point out certain respects in which linguistics and logic differ below this level of abstractness. Some of these differences will be seen to undermine the general applicability of logical methods to the description

[1] For other statements to the same effect see Itkonen 1970, 1972, and 1975.

of natural languages. In particular, it will turn out that the role of recursivity in natural language grammars has been generally misunderstood.

Unlike descriptions of logical semantics, linguistic-semantic descriptions of the customary type do not make use of interpretation functions relating linguistic expressions to (sets of) things which exist in one or more 'possible worlds'. In addition, it is trivially true of syntactic descriptions, both in logic and in linguistics, that they do not make use of interpretation functions. Consequently, within linguistics, both syntactic and semantic descriptions are *sentences* (i.e. sentence-forms) of a given theoretical-descriptive language, that is, 'sentences' which, displaying their respective internal structures, appear at different stages of derivations created in accordance with the rules of generative grammars. It is a well-known fact that generative grammars are definable as axiomatic systems (cf. sect. 3). Therefore, when comparing linguistics to logic, I shall disregard the method of logical semantics and restrict my attention to the axiomatic method, as it is applied either within linguistics or within logic.

2. General characterization of the subject matter of grammatical descriptions

An empirical science investigates events and states in space and time. If linguistics is to be an empirical science, it must be possible to identify those spatio-temporal phenomena that are supposedly investigated by a natural language grammar. At first glance, it may seem natural to say that actual utterances occurring in space and time constitute the data of linguistics. However it is clear that instead of studying everything that ever gets uttered, linguists concentrate upon *correct* utterances or, rather, correct *sentences*. So the question is now, whether the concept of correct sentence can be defined in terms of space and time. No genuine answer is given e.g. by the formulation of 'instance functions' that are decreed to match utterances with sentences (Kasher 1972). Any concept whatever may be connected, in a similarly abstract manner, with some spatio-temporal phenomena. Therefore, the formulation of instance functions tells us nothing about the nature of linguistic concepts.

It should also be clear that 'correct sentence' (or, for that matter, 'correct speech act') is a *normative* concept, not a theoretical one. It can be given theoretical definitions (which contain theoretical concepts like 'deep structure'), but this does not mean that it has ceased to be normative or has been driven out of existence. Because the concept of correctness is correlative with that of rule, and a rule determines what one ought to do, the question of defining correctness in terms of space and time, reduces to the question whether it is possible to define what one ought to do in terms of (factual reactions to) what one does as a matter of fact. It is a general philosophical truth that this cannot be done, the reason being, roughly, that in contradistinction to inanimate things, man can do anything whatever, either by mistake or by decision; for instance, he can say anything, whether correct or not, and react in whatever way, whether normally or abnormally, to what is being said. Therefore, instead of investigating utterances and reactions to them, a grammar investigates the normative linguistic knowledge on the basis of which utterances and reactions are recognized as correct or incorrect, or as normal or abnormal.

Knowing the correctness of a form is tantamount to knowing the rule which determines its correctness. As an example, let us consider some of the rules which make 'The girl smokes' into a correct sentence: (a) The definite article precedes the noun (= **girl the smokes*). (b) The ending of the 3rd person singular is *-s* (= **the girl smokel*). (c) The noun precedes the verb (= **smokes the girl*). (d) The subject of the verb must be spatio-temporal (= **the number smokes*). Rules of this type are known with absolute certainty, which means that sentences referring to them are infalsifiable, or necessarily true.

To qualify as empirical, a sentence or a theory must meet the following general condition: "...*criteria of refutation* have to be laid down beforehand: it must be agreed which observable situations, if actually observed, mean that the theory is refuted" (Popper 1963: 38). Now, let us consider the sentence 'In English the definite article precedes the noun'. Which observable situations, if actually observed, would mean that this sentence is refuted? We realize immediately that there can be no such situations. Of course, we can imagine and even produce forms apparently contradicting the rule, e.g. 'man the', but they cannot refute the corresponding sentence, because they are (known to be) *incorrect*, whereas the sentence is about *correct* forms only. Consequently, we cannot even imagine a situation which could possibly mean that our sentence is refuted and therefore it must be unempirical or necessarily true. (Of course, it is not necessarily true in the same way as, e.g., 'All bachelors are unmarried'.) At most, we can imagine that English contains a rule according to which forms like 'man the' are correct: in this imaginary English our sentence would apparently be false. But clearly it is not an *observable situation*, i.e. a situation definable in terms of space-time coordinates, that English contains this or that rule. Moreover, should English contain the above-mentioned rule, we would then know it with the same certainty as we know the actual rule today. And, in any case, assuming the existence of such a rule would mean *changing* English from what it is now, a maneuver clearly in contrast with the assumptions of our *synchronic*-grammatical description of English.

We have to do here with intuitive, atheoretical knowledge of language Because of its necessary character, it is a case of *conceptual* knowledge. When rules (of language), which determine correct sentences as conceptual possibilities, are definitely known, it might be said that the social control of knowledge is absolute. On the other hand, there are areas of atheoretical knowledge of language where the social control is less than absolute and where, consequently, one cannot know with any great amount of certainty whether a given form is correct or incorrect. Every change of language is preceded by such a state of uncertainty. In such cases, observation of actual occurrences and, hence, statistical description are in order.

It is generally assumed that rules of language can be known only in a more or less unreliable way. This is indeed a necessary consequence of the view that linguistics is an empirical science. However, it is easy to see that we have here a confusion between (atheoretical) rules of language and (theoretical) rules of grammar. As can be seen from the examples given above, there are rules of language which are known without any possibility of doubt. In this respect, such rules are similar to rules of logic and mathematics, and true sentences referring to any of these types of rules (e.g. 'In English the definite article precedes the noun', '$\sim$(p & $\sim$p)', '3 × 6 = 18') are equally necessarily true. It is a common characteristic of all such and similar sentences that we exempt them from doubt and, instead, use them as a *criterion* to find out whether or not a given person understands English, arithmetic, or logic. It is perfectly possible empirically that someone would deny the truth of any such sentence. Significantly, however, this would not tell us anything about these sentences; it would only tell us something about the one who denies their (necessary) truth. On the other hand, it should be clear that it is not only possible but even necessary to doubt the truth of the rules of the grammar which a linguist proposes as part of his theoretical description of a given natural language.

3. Grammars and systems of logic vs. theories of empirical science

An axiomatic system consists of a set of axioms and of a set of rules of inference by means of which theorems are derived from axioms and/or from theorems previously derived. Generative grammars, from type 0 to type 3, can be subsumed under the general notion of axiomatic system (see e.g. Wall 1972: 197–212). This remains true even if Chomsky-type phrase structure rules and transformations are replaced, respectively, by McCawley's "node admissibility conditions" and Lakoff's "local and global derivational constraints". The basic similarity between axiomatic systems can be brought out by comparing two systems, one in the rewriting notation and the other in the language of predicate calculus, both of which generate the same set of theorems, viz. all and only sentences of the type $a^n b^n$ (see fig. 1). Both derivations say the same thing: *aabb* 'is' an *S* or belongs to the class *S*.

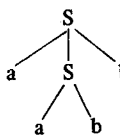
Axiom: S (A) Rules of inference: S → ab (R1) S → aSb (R2)	Axioms: ab ∈ S (A1) (x) (x ∈ S ⊃ axb ∈ S) (A2) Rules of inference: Modus Ponens (MP) Universal Instantiation (UI)
Derivation of *aabb*:  	Derivation of *aabb*: 1. (x) (x ∈ S ⊃ axb ∈ S) (A2) 2. ab ∈ S ⊃ aabb ∈ S (1, UI) 3. ab ∈ S (A1) 4. aabb ∈ S (2, 3, MP)

Fig. 1. A comparison between rewriting notation and predicate calculus.

'Axiomatic system' is a concept which contains 'axiomatic theory' as a special case. The former is merely a procedure for mechanically passing from strings to other strings, whereas the strings characteristic of the latter must admit of truth or falsity.

The fact that generative grammars are a subclass of axiomatic systems does not, as yet, prove anything about the possible similarity between linguistics and logic, because the best-developed natural sciences have also been axiomatized, and it might be claimed, and is in fact claimed by transformationalists, that the generative grammar of a given language is the axiomatic theory of this language, analogous in every relevant respect to the theory of physics, for instance. Therefore I must show that generative grammars of natural languages are fundamentally different from axiomatizations of natural sciences and fundamentally similar to axiomatic systems of logic.

The axiomatization of a natural science generates a set of universal and/or statistical hypotheses as its theorems. These sentences purport to refer to something in the external world, namely higher-level or lower-level regularities obtaining in that particular domain of reality which is investigated by the science to be axiomatized. If such regularities exist, the theorems, or hypotheses, are true; otherwise they are false. If they are false, then one or more of the axioms from which they were derived are – by Modus Tollens – also false, and must be modified accordingly. In other words, it is the purpose of the axioms and of the rules of inference to generate *true* sentences as theorems, and the criterion for the truth of a sentence lies *outside* it, i.e. in the external world.

Now, compare this situation to that prevailing in linguistics. The sentences generated by the grammar of a natural language do not, of course, refer to anything in the external world which would be the subject matter of linguistics in the same way as, for example, regularities of such and such a type are the subject matter of physics. Rather, these sentences, with their characteristic meanings and presuppositions, are *themselves* the subject matter of linguistics. That is to say, it is the purpose of a natural language grammar to generate, not *true* sentences, but (syntactically and semantically) *correct* sentences, as its theorems, and the criterion for the correctness of a sentence lies *in* the sentence itself, not outside it.

In the last-mentioned respect, logic seems at first glance to occupy an intermediate position between linguistics and natural science. Just like the axiomatization of a natural science, an axiomatic system of logic is not interested in generating just correct sentences. Rather, it is in both cases a *precondition* for the sentences to be generated as theorems that they be (formally) correct or 'well-formed'. On the other hand, a system of logic is not interested in generating sentences which are simply true. Rather, it purports to generate sentences which are *valid*, or *logically* true. Validity is truth in 'all possible worlds', which means that reference to the external world has no bearing on the validity of sentences. Consequently, and most importantly, the criterion for the validity of a given sentence, just like that for the correctness, lies *in* the sentence itself. (Moreover, since validity is given a formal definition, logic may concentrate directly upon sentence-formulae, instead of dealing with sentences exemplifying formulae.) In this decisive respect, logic and linguistics are similar to each other and different from natural science.

From the above, it follows that if the grammar of a language is considered as an axiomatization of L, it is a self-referential axiomatization because its theorems neither refer to nor are pictures of anything, but are *themselves* the object of axiomatization. In other words, the grammar does not *speak* about but *shows* its subject matter, i.e. the structures and interrelationships of the sentences of L. The axiomatization of a logical language is self-referential in the same sense. Of course, a generative, self-referential grammar requires its own referential metagrammar which, using ordinary language, gives an interpretation of the generative apparatus.

Even if a grammar of L may be taken as its axiomatization, it cannot be taken as an axiomatic *theory*, for the simple reason that, unlike the sentences constituting a theory, the rewriting rules constituting a (generative) grammar are neither true nor false. (This rather trivial truth is emphasized in Lieb 1974.) Moreover, considered in their function of rules of inference, the rewriting rules relate strings of grammatical symbols, whereas the rules of inference of axiomatic theories relate (hopefully) true sentences. Of course, what transformationalists have meant – although they have not been able to express it clearly – is that the *metagrammar* is a theory which claims, either truly or falsely, that such and such rewriting rules express generalizations about L and generate all and only correct sentences of L or, more realistically, generate relatively many correct and relatively few incorrect sentences of L. But then it must be admitted that, as theories, the existing metagrammars are far from axiomatic.

Now, because metagrammars deal with truth or falsity, and not just with correctness or incorrectness, it may seem that we have been forced to admit that, after all, linguistics is just one empirical science among others. However, this is not so; or more precisely, it is so only on the assumption that the preceding argument has turned logic too into an empirical science. As we saw above, the metagrammar claims, either truly or falsely, that the grammar generates all and only correct sentences. But in precisely the same way the metalogic, or the interpretation of a system of logic, claims, either truly or falsely, that the system generates all and only valid formulae (cf. sect. 5).

Observations analogous to those just made in connection with the axiomatization of a science can be made in connection with the explanation of an event. According to the Hempel/Oppenheim model, an explanation typical of natural science is reconstructed as an inference whose premises refer to general regularities and particular events (=antecedent conditions), and whose conclusion refers to that event in the external world which is to be explained: if the inference is valid and the premises are true, the event is considered as having been explained (Hempel 1965: 335–338). The event to be explained and the conclusion of the purported explanation are of course *two separate* entities. On the other hand, insofar as it is assumed that a grammar 'explains' the sentences that it generates, the situation is entirely different because what is to be explained, i.e. sentences of a natural language, and the conclusions of purported explanations, i.e. end strings of grammatical derivations, are *identical*. Consequently, a grammar can be said to 'explain' the sentences only in the sense in which a system of logic can be said to 'explain' its theorems. The 'explanatory' character of both types of systems consists in the fact that they show how, starting from one or more axioms, one gradually arrives, by means of antecedently given rules of inference, at a given sentence or a formula.

On the other hand, if linguistic explanations are construed at the level, not of grammar, but of metagrammar, rewriting rules (e.g. 'S → NP+VP') and lexical rules (e.g. '*the girl* = NP', '*smokes* = VP') might be interpreted as premises of an empirical explanation, i.e. as universal hypotheses and statements of antecedent conditions, respectively. Together, they would 'explain' the fact that 'The girl smokes' is a correct sentence (Wang 1972). Here too, however, the analogy to empirical explanations is defective. Empirical explanations are about events in space and time, but the fact that 'The girl smokes' is a correct sentence, is not in space and time; nor can it be defined in terms of spatio-temporal phenomena (cf. sect. 2). Just as obviously the fact that *the girl* is a noun phrase is not in space and time and therefore cannot be taken as an antecedent condition. More generally, what we have here is not explanation in Hempel/Oppenheim's sense but rather classificatory systematization of concepts, i.e. presenting the properties a correct sentence must have in order to be what it is. This is not how natural science operates. Rather, one is reminded of how the properties of different (correct) figures are systematically presented in geometrical analysis.

In short, grammatical explanations do not fit the Hempel/Oppenheim model (Itkonen 1972, 1974: 213–222). The same point is also made in Andresen 1974. Furthermore, since explanation is generally considered as structurally identical with (scientific) prediction and, on the other hand, prediction constitutes the basis for confirmation or disconfirmation, i.e. testing, it follows that the notions of prediction and testing, too, are different in linguistics and natural science.

4. On the similarity between linguistics and logic

Logic has developed out of the need to formalize the knowledge about rules for correct inferring and thinking, rules which, taken together, determine the concept 'valid formula'. In other words, logic is necessarily based on logical intuition,[2] but it is precisely the purpose of axiomatization to transcend the inevitable limits of intuition: axiomatization offers a way to extend the intuitively known rules of thinking to cases where intuition as such is powerless, or to extend logical knowledge beyond mere logical intuition. The specific character of the axiomatization of logic resides in the fact that the object and the method of axiomatization are practically identical: rules of thinking are axiomatized by using those very same rules.[3] It is a well-known truth, thanks to Gödel, that truths about a system cannot generally be proved within the system itself, or that truth is a much wider concept than provability. This means, among other things, that the validity of the rules of thinking used in constructing an axiomatization of logic cannot itself be exhaustively proved within this axiomatization. Of course, it can be proved within metalogic, but this procedure only moves the problem onto a higher level; that is, we have here the familiar idea of a potentially infinite hierarchy of metalanguages. The only way to guarantee the acceptability of any single axiomatization is to construct it by using rules of thinking which are rules in the strongest possible sense, i.e. which are so self-evident that no rational person calls them into question. The reference to rationality shows that, within logic, both the object and the method of axiomatization are thoroughly *normative* matters.

Up to a certain point, the situation is similar within linguistics. The grammar of a natural language is an axiomatization, or systematization,

[2] Cf. Blanché 1962: 63: "Every formal axiomatic system is in effect bounded on all sides by the domain of intuition".

[3] I disregard here throughout such nonstandard logics as many-valued logic which have no clear intuitive interpretation and thus cannot in any clear sense be called axiomatizations of intuitive logical knowledge. However, even many-valued logic necessarily makes use of some well-known logical principles and, in any case, the ordinary-language *interpretations* of many-valued logic rely, again, on all the principles of standard logic.

of *correct* sentences of this language. Sentences, constructions, and forms are correct because they conform to *rules* (of speaking) which make them correct. Rules are known (or knowable), which means that, in contradistinction to regularities in nature, rules coincide with the knowledge pertaining to them. The rules in question determine the concept 'correct sentence in L'. Consequently, a natural language grammar is an analysis of this concept. Notice that, in an analogous way, every conceptual analysis is an analysis of *normative* knowledge because every concept may be misused or misunderstood, i.e. used or understood against the *norm* which determines – and is, or could be, known to determine – the correct way of using or understanding it. There is a further similarity between logical and linguistic axiomatizations: since all formal systems, including generative grammars, must be interpreted ultimately in terms of ordinary language and since, on the other hand, ordinary language is the subject matter of linguistics, it follows that (correct sentences conforming to) rules of language both constitute the object of linguistic axiomatization and figure in its interpretation, i.e. in the metagrammar.

By now, the reason why linguistic and logical axiomatizations and explanations are different from axiomatizations and explanations typical of natural science, has become abundantly clear: Linguistics and logic are concerned with the description of (normative) *knowledge*, whereas the natural sciences are concerned with the description of physical *events*, and if anything at all is clear, then it is the fact that knowledge and event exist in thoroughly different ways; and as thoroughly different entities, knowledge and event necessitate characteristically different methods of description. It is true that, in actual practice, the natural scientist is more concerned with establishing deductive relationships between higher-level and lower-level regularities than with investigating particular events; and it might be claimed that regularities are similar to rules, and different from events, in that they do not simply exist in space and time. However, the natural sciences *ultimately* deal with particular events (Hempel 1965: 423), because these are primary with respect to regularities insofar as they determine which general hypotheses about regularities are (assumed to be) true. By contrast, in addition to the fact that rules, unlike regularities, are known, they are also primary with respect to actions insofar as they determine which actions are (known to be) correct.

Since the knowledge analyzed, in their own ways, by linguistics and logic is about rules, it is knowledge about what (correct) actions *can* be done, and not about what actions, whether correct or incorrect, *are* done as a matter of fact. Factual actions are located in space and time, as are factual events investigated by natural science, and therefore they share with the latter the properties stemming from man's epistemological limitations vis-à-vis space and time. That is, although knowledge of factual actions and knowledge of factual events are different in kind, due to the conceptual difference between action and event, they are nevertheless equally hypothetical.

5. Testability of grammars and of systems of logic

The claims about the 'empirical' character of grammatical descriptions are based, above all, on the alleged fact of their 'empirical testability'. A grammar is 'tested' by finding out whether it generates *all* and *only* correct sentences of L (with their 'correct' structural descriptions). Now, the same requirements are imposed upon axiomatic systems of logic, except that the crucial concept is not 'correctness' but 'validity'. Once a formal definition of validity has been given, it is required that the system in question be both 'sound' and 'complete'. It is sound if it generates *only* valid formulae, and it is complete if it generates all valid formulae. The system is then 'tested' by finding out whether it generates all and only valid formulae. Moreover, even if the system is *provably* sound and complete, it may still be tested by finding out whether validity as defined within the system corresponds in each case to the *intuitive* notion of validity, that is, whether the theorems of the system (which have been proved as formally valid) are intuitively valid, and whether all intuitively valid truths formalizable in the language in question are theorems of the system. No general proofs concerning these last-mentioned properties of axiomatic systems can be given. Such a proof woulf require a *formal definition of intuition*, or of intuitive validity, which either is a contradiction in terms or leads directly into infinite regress.

One can get a good picture of the testability in logic by following, for instance, the development of von Wright's system of deontic logic (von Wright 1971). Although the different versions of this system have all been provably sound and complete, they have been disconfirmed – with the severity of disconfirmations decreasing with the development of the system – by showing that they either generate intuitively invalid formulae or fail to generate intuitively valid formula (for details, see Itkonen, forthcoming). At a sufficiently high level of abstraction, the analogy to the testing of generative grammars should be obvious.

6. The differing objectives of linguistics and of logic

I have outlined above the basis for what I think is an undeniable similarity between linguistics and logic. From now on, I shall turn to discussing their differences.

If formal logic had *no* connection with intuitive, atheoretical notions of inference and necessary truth, as they are expressed or expressible in natural language, then it would simply have no point. Logical intuition (which is a type of social and socially controlled knowledge) constitutes, then, the ultimate, irreducible basis for the whole of logic.

However, while linguistics investigates the actual rules of language, logic could not possibly remain content with investigating the rules people actually use to make inferences or, more generally, to form necessarily true sentences. Rather, starting from certain selective aspects of actual inferring (which is just one very narrow aspect of actual speaking), formal logic develops new, more controllable ways of inferring and, to this end, it has to invent new kinds of languages.

The precise relation, within logic, between innovation and adherence to ordinary discourse has been formulated by Nagel as follows:

> "The explicit formulation of canons of inference serves to clarify vague intent; ... The adoption of a system such as is found in Whitehead and Russell's *Principia Mathematica* is in effect the adoption of a set of regulative principles for developing more inclusive and determinate habits for using language than are illustrated in everyday discourse. No known recent system of formal logic is or can be just a faithful transcription of those inferential canons which are embodied in common discourse ..." (Nagel 1949: 205).

It is also noteworthy that Nagel repeatedly stresses the importance of those 'norms', 'ideals', or 'objectives' which guide and justify the construction of formal systems, in particular of their rules of inference. Formal systems are constructed for a *definite purpose*. To put it roughly, it is their purpose to enable people to make inferences which, because of their length and/or complexity, could not otherwise be made or, more generally, to state the validity of formulae whose validity could not otherwise be stated because of their length and/or complexity. It is precisely because logic does not attempt to describe people's actual logical knowledge in the same way as linguistics describes their actual linguistic knowledge, that (experimental) psychology of logic is totally irrelevant to the system-construction in logic, whereas psychology of language has, or could have, a direct bearing on linguistic descriptions.

It could be said that logical languages come into being together with their 'grammars', i.e. axiomatizations, because they are constructed so as to be axiomatizable with as few axioms and rules of inference as possible. By contrast, natural languages had of course existed for a very long time, before any attempt was made to provide grammars for them. In addition, in modern generative or axiomatizing grammars no rules of inference have such a priviledged status as Modus Ponens has in logic. In fact, the precise nature of grammatical rules is still an open question. Nor is it known, even approximately, how many rules would be needed to generate the sentences of a given natural language.

7. Comparing correctness and validity

A linguistic grammar investigates *correct* sentences whereas logic investigates *valid* formulae which, just as well as invalid ones, must be *well-formed* (cf. sect. 3). Taken in itself, correctness corresponds to well-formedness, but as the special concern of linguistics, it corresponds to validity, which is the special concern of logic. Rules of formation generate all and only well-formed formulae whereas rules of transformation (or inference), together with the axioms, generate all and only valid formulae. Because of the artificial character of logical languages, their rules of formation can be stated quite trivially, whereas within linguistics the analogous task is exceedingly difficult and has, with respect to any one language, been accomplished so far only to a quite restricted extent.

It is understandable that linguists have not been much interested in axiomatic theories, because these are concerned with transferring truth from axioms to theorems by means of appropriate rules of transformation, whereas linguistics is concerned with correctness, not with truth, of sentences, that is, with rules of formation rather than rules of transformation. The relevance of axiomatics to linguistics became apparent only after a new and more comprehensive notion of the rule of inference was supplied under which both rules of formation and rules of transformation could be subsumed. In the most general terms, a rule of inference is an n-place relation on strings of a given alphabet, and the n-th coordinate of a given ordered n-tuple is said to follow from the preceding ones. For instance, Modus Ponens is a three-place relation consisting of ordered triples of the type $(A \supset B, A, B)$, for instance the triple of the strings 2, 3, and 4 in the right-hand derivation in fig. 1. Similarly, a rule like

'S → aSb' is a two-place relation consisting of ordered pairs of the type (xSy, xaSby), where x and y are any strings of a's or b's; specifying the axiom as 'S' weeds out all unacceptable strings not complying with the structure $a^n Sb^n$, and the other rule 'S → ab' stops the recursion.

The most obvious difference between systems of logic and generative natural language grammars is that the latter are so-called *extended* axiomatic systems, i.e. they contain non-terminal or auxiliary symbols, and only the last strings of derivations consist of terminal symbols in a proper order and are thus sentences of the language to be axiomatized (Wall 1972: 199–204). By contrast, in the customary type of axiomatic system both the axioms and the theorems are not only well-formed but also true. In axiomatization of empirical subject matter it is empirical truth which is, hopefully, transferred from axioms to theorems, whereas in axiomatization of logic the transferable property is logical truth, or validity. This latter fact is stated in an axiom of modal logic 'N(p ⊃ q) ⊃ (Np ⊃ Nq)', which says that if 'p ⊃ q' is a tautology, then if 'p' is a tautology, 'q' is a tautology too. Generative grammars contain no transferable property comparable to empirical truth or validity. They concentrate upon correctness, but neither the starting point of derivation, or 'S', nor indeed any of the steps preceding the terminal string is an actual correct sentence. And whether the terminal string is in fact a correct sentence, depends on the linguist's skill. The fact that generative grammars are extended axiomatic systems also entails that, contrary to what is the case in logic, theorems already proved cannot be inserted into derivations to facilitate the proof of new theorems; rather, the derivation of each particular sentence extends all the way back to the axiom 'S'. However, there is a general heuristic principle which has much the same effect as allowing theorems already proved to appear in derivations: Rules which have been corroborated by the fact that they generate *only* correct sentences of a given type are allowed to be iterated indefinitely in order to generate *all* (longer) correct sentences of the same type. This is in fact the familiar 'clear case principle', or the principle of letting the grammar decide the correctness.

It has sometimes been said that phrase structure rules generating deep structures (of whatever type) are comparable to formation rules whereas grammatical transformations which ultimately lead to surface structures are comparable to rules of transformation (or inference). We have already seen why this analogy is defective. Formation rules generate well-formed formulae, but deep structures generated by phrase structure rules cannot in any clear sense be called 'correct'; at least they are not correct in the same sense as surface structures. Moreover, in logic each application of a rule of inference yields a new (valid) theorem, whereas only those grammatical transformations which may figure as the last step of a derivation yield correct sentences.

If we nevertheless accept the above-mentioned defective analogy, then it follows that deep structures acquire the status of axioms. If, moreover, we accept the principle that transformations must be meaning-preserving, then there is in generative grammars too a transferable property, namely meaning. However, not only does this analogy between truth and meaning, or validity and meaning, rest on a defective analogy, but it is defective in itself. The meaning identity of deep and surface structures is not a verifiable fact – as is the validity of axioms on the one hand and of theorems on the other – but a *stipulation*. Deep structures are not sentences, but abstract entities needed to put in evidence similarities and differences in form and meaning between sentences. It is not at all clear in which sense deep structures themselves could be said to have a determinate meaning, witness Chomsky's and McCawley's disagreement as to the 'correct meaning' of a deep structure like 'John felt [John be sad]' (McCawley 1973: 5–6). But if the meaning is not determinate, then it does not make sense to ask whether or not transformations preserve it. The stipulation in question is a descriptive device, and it remains an open question whether it is desirable to make use of such a device. This is due to the fact that while validity is one and the same in axioms and theorems, the number of different meanings is enormous, and one cannot foresee how they should be best described. Consequently, it is not very revealing to compare phrase structure rules and grammatical transformations to formation rules and rules of inference in logic.

8. The fallacy of total formalization: a critique of Bar-Hillel's 'categorial grammar'

Validity must be ultimately based on logical intuition, i.e. intuitive (and atheoretical) notion of necessity. However, in every system of logic validity is given a formal definition, and special methods are developed to 'test' whether or not a given formula is (formally) valid. If the method is purely mechanical and gives a yes-or-no answer after a finite number of steps, it is called a decision precedure. For formulae of propositional logic and of its modal or deontic extensions, the method of truth-tables or some analogous method provides a decision procedure. On the other hand, there is no decision procedure for formulae of predicate logic and of its extensions. However, it is known which type of quantified relational formulae transcend the capacity of any decision procedure, and in practice it can always be 'established' (even if not 'decided'), for instance by means of the so-called tree method (Jeffrey 1967), whether or not a given formula is valid. It is also possible to establish the validity or invalidity of a formula by finding out whether or not it is a theorem of the system in question. This presupposes, however, that the validity of the axioms and the validity-preserving nature of the rules of inference have been established first. Hence the notions of validity and theoremhood remain, in principle, separate notions, as can also be seen from the fact that the questions of completeness and soundness are separate questions: it is two different things to ask whether a formula is a theorem, if it is valid, and to ask whether it is valid, if it is a theorem.

It is natural to ask to what extent the above concepts and distinctions are applicable to natural language grammars. It is when we try to answer this question that, I think, certain rather important differences will emerge.

The question of theoremhood has as its counterpart the question whether or not a given grammar generates a given sentence. In whatever way this is found out, it is an entirely straightforward matter. Notice, however, that this point of view implies nothing as to the *correctness* of the sentence which the grammar can be shown to generate. Similarly, the question about the correctness of a given sentence seems straightforward enough. That is, it does not seem to be possible to give any general and formal definitions of correctness and, correspondingly, there does not seem to exist any formal way of 'deciding' or 'establishing' the correctness of a given sentence. Rather, the only way to do this seems to be simply by means of linguistic intuition.

Although this conclusion may seem rather obvious, it runs counter to some basic tenets of transformational theory and must be defended against possible criticism. But first it is instructive to examine as extreme a proposal as possible to the effect that it is possible, after all, to give a purely formal definition of correctness, along with a procedure for deciding the correctness of sentences. The so-called categorial grammar as presented in Bar-Hillel 1962 contains such a proposal. It is the aim of

a categorial grammar to identify the (correct) sentences of a language by determining their syntactic structures. The fundamental grammatical categories are sentence and noun, and derived categories are set up for lexical units other than nouns according to their syntactic functions, i.e. according to their ability to combine with other lexical units belonging to such and such categories. A derived category expresses both the category with which it combines and the category which results from this combination. For instance the category symbol $\frac{S}{\overleftarrow{n}}$ for an intransitive verb says that it combines with a noun to its left to form a sentence. By using 'rules of cancellation', it is possible to eliminate two identical contiguous categories such that they meet the arrow condition and one is self-dependent while the other is the denominator of a categorial 'fraction'. '(Correct) sentence' is defined as a string of words which, once put into the categorial notation by the 'assignment function', cancels to the *S*-symbol by repeated application of cancellation rules (Bar-Hillel 1962: 554). For instance, the sentence 'Poor John ran away' may be represented categorially as follows (Lyons 1968: 227–231):

$$\frac{n}{\overrightarrow{n}} \cdot n \cdot \frac{S}{\overleftarrow{n}} \cdot \frac{\left(\frac{S}{\overleftarrow{n}}\right)}{\overleftarrow{\left(\frac{S}{\overleftarrow{n}}\right)}}$$

Rules of cancellation may be applied only to such contiguous categories which are, within a constituent structure, immediately dominated by a common category. Thus, in our example, 'John' and 'ran' cannot be the first input to a rule of cancellation. Two cancellations give as the result:

$$\frac{n}{\overrightarrow{\cancel{n}}} \cdot \cancel{n} \cdot \cancel{\frac{S}{\overleftarrow{n}}} \cdot \frac{\left(\frac{S}{\overleftarrow{n}}\right)}{\overleftarrow{\cancel{\left(\frac{S}{\overleftarrow{n}}\right)}}}$$

The elimination of the two remaining *n*'s cancels the sentence to the *S*-symbol, which means that the sentence is correct. Correctness is (formally) defined in terms of the cancellation rules which constitute the decision procedure for correctness.

It is easy to see that the adequacy of this kind of formalization is wholly dependent on how well we have carried out the categorial classification of the words in the natural language under description. And, once again, *this* question cannot be answered by formal methods but only by reference to our everyday linguistic intuition. For example, if someone should categorize 'to hit' as an intransitive verb, i.e. as $\frac{S}{n}$, then his grammar would decide that 'John hit' is correct and 'John hit the ball' is incorrect. We have here a clear conflict between formal and intuitive variants of a certain concept (here 'correctness') and, as always in such cases, it is the intuitive variant that wins. A formalization may be perfect on its own conditions, but precisely those conditions can always be critically evaluated in light of the (intuitive) knowledge which is to be formalized.

What was just said holds of course for all types of formalization, but there is in addition a significant difference between linguistics and logic as to the number of those formalizing steps which a full description contains, and hence, as to the number of intuitive judgements corresponding to each step. A decision procedure presupposes a formalization of the entities to which it is to be applied. Now, the important point is that in logic the number of such decisions can be kept, intuitively speaking, quite low. The reason for this lies in the *selective* nature of logic, that is, in the fact that logic, by definition, shapes its subject matter in such a way that it becomes accessible to the functioning of *recursivity*: the whole of the intended subject matter is thought of as formalizable – and in successful cases it is formalized in fact – by repeated application of quite a few formal principles which have been proposed, evaluated, and then decided on in light of logical intuition.

On the other hand, linguistics has no similar *liberty of shaping its own subject matter*, given that it is undeniably the task of linguistics to describe natural language as it actually is, and not to make it, for whatever purpose, into something which it is not. If we consider the case of categorial grammar, we notice first of all that every lexical unit of the language under description must be assigned to one category or another; and because every natural language contains at least one thousand words, most of which are polysemous, the categorization necessitates several thousands of separate decisions as to adequate formalization. Secondly, the categorization is based on the types of correct combinations which a lexical unit may form with other similar units; and since one and the same unit (even with one and the same meaning) may obviously occur in several types of environment, it follows that the number of requisite

intuitive judgements or decisions as to adequate formalization increases accordingly. Thirdly, it is far from clear that the principles governing the combinatory capacities of lexical units and phrases are the same as those governing the analogous capacities of such sublexical units as phonemes, on the one hand, and of such supralexical units as clauses, on the other. At least, it is not revealing to categorize, for instance, all subordinate clauses as cases of $\frac{S}{S}$ or $\frac{S}{S}$. Therefore, those extremely many categorizations which would have to be formulated within the standard domain of categorial grammar must be supplemented by new types of categorization at other linguistic levels. It should be clear that the total number of decisions contained in the formalization of natural language would be incalculable and, hence, in no way comparable to the quite restricted number of decisions needed, for instance, in the formal construction of predicate logic. Consequently, a decision procedure which would have to rest on an incalculable number of previous decisions would be rather different from decision procedures as used in logic. In fact, categorial grammar provides a 'decision procedure' for (correct) sentences roughly in the same way as any dictionary provides a 'decision procedure' for (correct) words.

Furthermore, irrespective of the complexity of the task of formalizing natural language, there is a definitive objection against defining correctness as in Bar-Hillel's categorial grammar. Together with the general convention regarding the application of cancellation rules, the assignment of lexical units to different categories is tantamount to enumerating the rules of a given categorial grammar. Now, because the correctness of a sentence is defined solely in terms of successful application of cancellation rules to the string of categories representing the sentence, it follows that *there is no grammar-independent way of establishing the correctness of sentences*. It is immediately clear that if this principle is accepted, grammatical description becomes an entirely arbitrary and uninteresting undertaking. It is certainly a basic requirement which every interesting grammar must at least try to meet that it provide a description of all and only correct sentences of the language in question. But if correctness is defined solely in terms of the grammar, then the grammar, i.e. *any grammar whatever*, automatically and necessarily describes all and only 'correct' sentences. And this does not make sense. Consequently we must have, on the one hand, a grammar-independent notion of correctness and, on the other, a grammar which tries to capture this notion.

The same criticism can be made from the standpoint of logic. The questions of soundness and completeness are central (meta-)logical questions. But, of course, it makes sense to ask whether the formulae of a system are its theorems, if they are valid, i.e. whether the system is complete, only if there is an independent way of establishing validity on the one hand and theoremhood on the other. If validity were defined in terms of theoremhood, the questions of soundness and completeness would cease to be meaningful questions. To make this observation applicable to linguistics, one only needs to replace 'valid' and 'theorem of G' by 'correct' and 'capable of being described by G'.

A categorial grammar is a 'recognition grammar': it starts from the lexical level and arrives, via increasingly abstract structures, at the sentence symbol 'S'. It is a well-known fact that recognition grammars are mathematically equivalent to 'production grammars' which proceed in the opposite direction. Generative grammars of the standard type are production grammars, in this sense. Reformulated in the generative terminology, the attempt to define correctness in terms of grammar means that all and only sentences, generable by a given grammar are correct. Curiously enough, Lyons has given a definition of 'grammaticality' which is in agreement with this fallacious notion of 'correctness':

> "...: whether a certain combination of words is or is not grammatical is a question that can only be answered by reference to a particular system of rules which either generates it (and thus defines it to be grammatical) or fails to generate it (and thereby defines it to be ungrammatical)" (Lyons 1968: 153).

This notion of grammaticality is identical with generability. However, Lyons's position is not quite as incomprehensible as it might seem to be at first glance. On the one hand, he operates with the notion of intuitive and, hence, grammar-independent 'acceptability'. On the other hand, he is forced to contradict his own definition and to use the term 'grammatical' in an absolute sense, i.e. in a sense in which it is not relativized to any particular grammar.

Because categorial grammar is based on the assumption that natural languages exhibit a contiguous immediate-constituent structure (Bar-Hillel 1962: 552), it is of course an inadequate model for grammatical description, except perhaps in the base component of the grammar (Lewis 1972). I have discussed categorial grammar so extensively only because it makes the far-reaching methodological claim, in a rather explicit form, that it is possible to give a purely formal definition of correctness in natural language, along with a decision procedure for correctness.

9. Why definitions of correctness and decision procedures for correctness are impossible

In the most general terms, validity is defined as truth in all possible worlds or on all possible interpretations. On the one hand, this definition then can be given a formal expression in the framework of each particular system. On the other hand, it can be used in proving the completeness of a given logical system. For instance, the Henkin-type proof of completeness depends crucially on the fact that 'A is not valid' means 'A has a falsifying model', which is in turn equivalent to '~A has a verifying model'.

When we consider the possibility of a similar general definition of correctness, we must notice first of all that, unlike logical languages, natural languages are continuously subjected to unconscious change: every synchronic state of any natural language is the result of innumerable changes which have been brought about, in an essentially unpredictable fashion, by widely diverging and often conflicting forces. As a result, every natural langauge is full of asymmetries and exceptions which, in the light of diachronic analysis, can be identified as residues of changes reaching back to different stages in the history of the language. At any given moment there are forces at work which aim at greater uniformity in certain parts of the language but, as a result, they bring about greater diversity in some other parts, which again gives rise to new attempts at unification; and so on.

When we consider natural languages in such a *realistic* way, it appears as a practical necessity that there can be no single (formal) property, however complex, which all and only correct sentences of a given synchronic state would share. All that can be said is that that is correct what is (intuitively) known to be correct. Of course, the situation is entirely different in the case of those artificial languages which transformationalists have been so fond of using as 'simplified' or 'idealized' models of natural languages. For instance, nothing is easier than to construct a 'language' whose correct sentences are decreed to be of the type $a^n b^n$. There is a grammar-independent way of deciding whether a given sentence satisfies this formal definition of correctness, namely counting the *a*'s and *b*'s and verifying their mutual positions by simply looking at them. Thus, *aaabbb* is correct whereas *aaabbbb* and *ba* are not. Given this grammar-independent definition of correctness, it is a nontrivial task to construct a grammar which generates all and only sentences satisfying it. Once such a grammar has been constructed, impeccable proofs of soundness and completeness can be given. It is easy to prove that, for instance, the grammar presented in fig. 1 is sound and complete in the required sense. Therefore, from the methodological point of view, this grammar-conception is clearly superior to Bar-Hillel's categorial grammar.

Artificial languages have been expressly constructed in such a way that general definitions of validity or correctness can be given for them. This in turn makes it possible that proofs of soundness and completeness can be given for the grammars of such languages. However, similar definitions and proofs cannot be given in connection with natural languages simply because – as the epithet 'natural' indicates – they have not been constructed with a similar purpose, or with any conscious purpose; and it is just not true that natural languages would have unconsciously developed general and formal notions of correctness analogous to the notions of validity which logicians have been developing consciously.

Although correct sentences may be formally described, correctness itself seems to be a primitive notion which cannot be defined in terms of any other notion. To put it otherwise, the normative force of a rule lies in the rule itself, not in something outside it: If there is disagreement as to the correctness of a given form or action, one has to try to convince the others by appealing to the relevant rules, and not by appealing to socio-economic or psychological or neurophysiological factors which might be said to underlie, in one sense or another, the rules in question (Nagel 1949: 206–207). Doing this would in fact involve an elementary sort of category mistake. It does not seem possible, then, to define correctness in a way which does not make circular use of the very notion of correctness. For instance, correctness cannot be defined simply in terms of general agreement and social control, because correct is precisely what people agree to *be correct*. It is a characteristic feature of paradigmatic or unequivocal correctness that true statements about it are known to be true and, hence, are infalsifiable or 'necessarily true' (cf. sect. 2). I have argued that writing a linguistic grammar consists in the attempt at establishing a systematic correspondence between intuitively necessary rule-sentences, *inter alia* sentences about correctness, and formally necessary sentences about the generative capacity of a grammar, a process structurally similar to 'explication' as defined in Pap 1958 (Itkonen 1974: ch. IX, esp. 271). However, this does not, of course, amount to giving a *definition* of correctness.

The lack of a general and formal definition of correctness entails the

lack of a decision procedure for correctness. Yet Chomsky and Wall, among others, claim that such a decision procedure exists as a matter of fact; they equate correctness with generability, on the assumption that the grammar in question is the 'real' one:

"It is reasonable to suppose, for example, that for any arbitrarily given string of phones, morphemes, words, or whatever, a person who commands a natural language grammar G can determine, if he is given enough time and memory aids, whether or not the string is grammatical. This amounts to saying that the speaker-hearer has available to him a decision procedure for membership in L(G) and, if so, the language is a recursive set (cf. Chomsky, 1965, pp. 31–32 and footnote 18, p. 202)" (Wall 1972: 236).

It seems to me that the term 'decision procedure' is being used here in a misleadingly metaphorical way. At present, we have absolutely no idea about what the 'natural language grammar G' is like which the speaker-hearer commands *as a matter of psychological fact*. Consequently, we have absolutely no idea about what kind of procedure it is that a speaker uses in determining the membership of a given sentence in the set L(G), i.e. the set of the (correct) sentences generable by G and constituting the language L, nor do we indeed have any idea about what kind of set L(G) is. When discussing the artificial language whose (correct) sentences are decreed to be of the type $a^n b^n$, we noticed that the correctness of these sentences can be decided in a grammar-independent way, namely by simply *looking at* them. But for this to be possible, it is presupposed, of course, that we know what we are *looking for*, i.e. that we know the general definition of correctness in this language. However, in the case of natural language this is precisely what we do not know. In short, it is not reasonable to speak either of a grammar-dependent or of a grammar-independent decision procedure for correctness. Notice, finally, that the entire assumption underlying Chomsky's and Wall's positions, namely that the speaker-hearer is always able to determine the correctness or grammaticality of an arbitrarily given string of words, is quite obviously false, as attested by the current controversies about the correctness of sentences that transformationalists have been using as evidence for their theories (cf. Botha 1973: 178–185). In fact, if we leave aside speculations resulting from the (false) analogy of artificial languages, we see at once that there is no reason to suppose that there could be a 'procedure' which exhaustively divides all sentences of a natural language either into correct or into incorrect ones. Correct is what is agreed to be such and controlled accordingly. But since language is a *natural* normative system, its rules, unlike those of chess or of formal logic, are characteristically *open,* with the consequence that, as we move towards more and more complex and unusual cases, there inevitably comes a moment when it is impossible to say whether or not a given form is correct.

10. A critique of the clear case principle: the fallacy of grammatical induction

In the two previous sections I have been defending the grammar-independent, intuitive notion of correctness. As a particular type of social knowledge, linguistic intuition is under social control precisely in the same way as prescientific or atheoretical knowledge of any other well-structured social institution about which preople have acquired intimate knowledge by participating in it. At one extreme, such knowlege is certain; we can label some intuitions as 'uncertain' only because we know what a certain intuition is. Knowledge is uncertain always at the (theoretical) level of description and sometimes at the (atheoretical) level of the object of description. It is always, and only, when knowledge is uncertain that additional criteria for knowledge are needed. In linguistics, it is the task of psycholinguistic or of sociolinguistic research to provide such criteria.

In the 'standard' version of transformational grammar, the grammar-dependent notion of correctness is justified as follows:

"... what is grammatical is whatever has to be hypothesized as such in order to explicate the properties and relations of sentences that are antecedently construed as grammatical" (Katz and Bever 1974: 51).

As a consequence, 'grammaticality' is generally defined in terms of the "formal property of generation in an optimal grammar" (op. cit.: 15–16). The rules of such a grammar admit of no variation or gradation, that is, they are 'absolute formulations':

"Chomsky ... views each and every string of the language as belonging to one or the other of the two categories 'grammatical' or 'ungrammatical'. For him, the middle range of 'undecidable cases' reflects not some inherent gradient in the phenomena which a descriptively adequate rule must represent but simply incomplete knowledge on the part of the linguist" (op. cit.: 11).

The rationale for the idea that grammars can be used to determine the correctness of sentences lies in the assumption that even if a sentence is so complex or otherwise so unclear that our linguistic intuition says nothing about its correctness or incorrectness, it *must* nevertheless be correct, if it can be shown to be generated by repeated application of recursive rules each of which, taken in itself, would contribute to generating a (simple) sentence which according to our intuition is definitely correct. (Thus it is admitted, in any case, that atheoretical, grammar-independent intuition is the ultimate criterion of correctness.) This argument amounts to the claim that there is an analogy between mathematical or logical induction and what might be called 'grammatical induction'. The question about the grammar-dependent notion of correctness, then, reduces to the question whether such an analogy exists as a matter of fact.

To take an example from mathematics, the following formula expresses a mathematical truth because for every integer n, it can be proved that if it holds for n, then it holds for its successor:

$$(n)\left[(0 + 1 + \ldots + n) = \frac{n \cdot (n+1)}{2}\right].$$

Now, the first difference between mathematical induction and grammatical induction consists in the fact that the former concentrates on transferring a clearly defined mathematical property from one integer to its successor whereas, as we have seen in the two previous sections, correctness, which grammatical induction ought to transfer from one sentence to another, is not a clearly defined property. Secondly, the whole concept of mathematical induction would not have come about, in the first place, were it not possible, at any given moment, to stop the induction and to *check*, by a method *independent from* the inductive proof itself, whether or not the proof is in fact correct. That is, we can replace n by any integer and then simply *compute* whether or not the equation is true for this value of n. For instance, if we substitute 3 for n, we get

$$0 + 1 + 2 + 3 = \frac{3 \cdot (3+1)}{2},$$

which is obviously correct. By contrast, *we have no similar method of checking whether an arbitrarily complex sentence generated by recursive* *rules of our grammar is or is not correct*. Ex hypothesi, the sentence is too complex to be judged on the basis of our linguistic intuition. It was precisely for this kind of situation that we invented the grammatical induction, or the convention that the number of the applications of a given rule does not affect the correctness of the outcome. But if we are now asked whether a given extremely complex sentence which has been generated by our grammar is *in fact* correct, then we do not know what to say. Of course, it would be entirely vacuous to point to the fact *that* it has been generated by the grammar, because the question is precisely whether the alleged correlation between generability and correctness holds *as a matter of fact*. What is needed is the possibility of an independant check analogous to that exemplified in our mathematical example. In linguistics, however, this possibility does not exist.

Precisely the same remarks can be made concerning the difference between logical induction and grammatical induction. The soundness of a system of logic is proved in two steps: first it is shown that the axioms are valid; then it is shown that the rules of inference, when applied to valid formulae, produce valid formulae. From this it follows that every theorem of the system is valid, because a theorem is the last line of a sequence of which each line either is an axiom or results from applying a rule of inference to a valid formula. In such a way the validity of any given step in an indefinitely long derivation can be proved inductively. However, there obtains the possibility to check whether the logic we have been using in our proof is in fact correct. That is, at any given moment it is possible to stop the logical induction and to check by an *independent* method, e.g. the tree method, whether or not the theorem in question is in fact valid. We can verify the validity of theorems by using the *same* method as the one by means of which we originally established the validity of the axioms. In linguistics, by contrast, there is no analogous possibility: it is by means of intuition that the correctness of simple sentences and, hence, the suitability of the rules generating them was established, but the complex sentences which are generated by repeated application of these (recursive) rules and whose correctness is at stake here are precisely too complex to be judged on the basis of intuition. Furthermore, while validity is a formal notion, correctness is not. Consequently it is not even clear what it means, precisely, to say that grammatical rules transfer correctness from one (intuitively clear) sentence to another (intuitively unclear) sentence. It may be the case that the meaning of such a claim is not just unclear but even self-contradictory.

Within transformational grammar, no noncircular meaning can be given to the requirement that the grammar generate all and only correct sentences of L, so long as correctness is defined, to an unspecified extent, in terms of generability. This requirement clearly aims at something essential, but it cannot be made sense of until an *independent*, that is, *grammar*-independent criterion of correctness has been made available. Not surprisingly, the only such criterion proves to be *linguistic intuition* (supplemented, in uncertain cases, by psycholinguistic and sociolinguistic evidence). Thus, by taking our clue from the clear distinction between validity and theoremhood, we have come to reject the grammar-dependent notion of correctness in favor of the intuitive notion of correctness.

11. A critique of the use of recursivity and of the related notion of 'grammaticality'

If we accept the conclusion of the last section, we must reject that particular notion of 'grammaticality' which has been one of the cornerstones of transformational grammar from the beginning. At the same time, we must reject as unjustified the extensive use which transformational grammar makes of recursivity.

In transformational grammar, grammaticality is a concept largely independent of actual linguistic intuition. It is a matter of 'competence', or something that the 'ideal speaker' supposedly knows. Intuition is a matter of 'performance', performance being a perversion of competence, resulting from the 'limitations', i.e. the actual psychological constitution, of non-ideal speakers. For instance, the sentence 'The rat the cat the dog chased killed ate the malt' is 'perfectly grammatical' (Chomsky and Miller 1963: 286), and so are all more complex sentences formed by repeated application of the same self-embedding rule of relativization. This is the principle of grammatical induction which we discussed above. To take another example, consider the grammar of conditional sentences:

$$S \rightarrow \text{if } S \text{ then } S$$
$$S \rightarrow \Delta$$

Varying the direction of recursion, we get, at the depth of three applications of the recursive rule, e.g. the following structures:

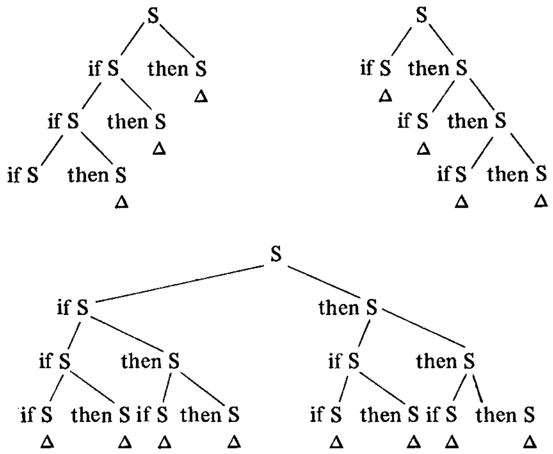


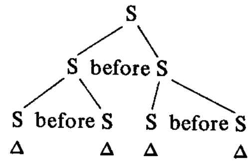
Fig. 2.

That is, we get the following sentence-forms: 'If if if S then S then S then S', 'If S then if S then if S then S', "If if if S then S then if S then S then if if S then S then if S then S'. Sentences exemplifying either one of the first two sentence-forms are beyond the intuitive comprehension of normal speakers, and sentences exemplifying the third sentence- form are beyond the intuitive comprehension of any speaker. We can convince ourselves of the correctness of such sentences (supposing, for the moment, that we can do such a thing) only by verifying that the recursive *if*-rule has been correctly applied. That is, we must have recourse to some formal method of representation, e.g. a tree diagram, already at this quite shallow level of complexity. We must *infer* inductively the (alleged) correctness of such sentences and, moreover, we have no independant way of checking whether the result of our inference is in fact correct (cf. sect. 10). Here, logic with its well-established rules of inference provides the computational ideal. The reference to logic is quite explicit in the passage where Chomsky mentions, as a further argument for the grammaticality of excessively long and/or complex sentences, that "we can even state quite simply the conditions under which they can be true" (Chomsky 1957: 23). This argument, however, assumes the truth of precisely that claim which has to be proved, namely that methods of logic are directly relevant and applicable to the description of natural language.

To give one more example, consider the grammar of *before*-sentences:

$S \rightarrow S$ before S
$S \rightarrow \Delta$

Here, it seems to me, intuitive incomprehension is reached already at the depth of two applications of the recursive rule:



or: "S before S before S before S"

Fig. 3.

Chomsky and Miller equate sentences such as these with sentences 'Birds eat' and 'Black crows are black' because all of these sentences. though 'grammatical', are equally unlikely to be ever uttered (Chomsky and Miller 1963: 287). This is a surprising argument, considering that Chomsky has taken great pains to show that there is no connection between the grammaticality of a sentence and its probability of occurrence (Chomsky 1957: 16). And in any case, Chomsky and Miller overlook the crucial difference that 'Birds eat' and 'Black crows are black' understood immediately whereas the artificially complex sentence-types which we have been discussing here cannot be understood without the help of some kind of computational machinery. Because sentences of the latter type are not understood as such, there can be no obvious *linguistic* reason why they should be taken as grammatical. Rather, the reason must be the unquestioned assumption that the same computational principles which are valid in logic and mathematics are also valid in linguistics, in other words, that there is an analogy between logico-mathematical and grammatical induction. We have seen, however, that there is no such analogy. Why should we, then, accept the general computational nature of natural language grammars? Furthermore, remember that a system of logic is constructed for *a definite purpose*: it is meant to generate among its theorems valid formulae which are so complex that not even the best logician, unaided by formal machinery, could grasp their validity. Now, even if we leave aside the fact that the analogy between logical induction and grammatical induction breaks down because of the difference inherent in the notions of validity and correctness, we may still ask whether the *purpose* of constructing natural language grammars is comparable to the purpose of constructing systems of logic: Is it, or should it be, the purpose of a grammar to generate correct sentences which are so complex that not even the best linguist, unaided by formal machinery, could utter or understand them? In my opinion, the answer is definitely 'No'. And the reason is simply that once we explicitly formulate the purpose of generative grammars in this way, we see that it just does not make sense. By contrast, the analogous purpose constitutes the very essence of logic and mathematics.

Using Habermas's terminology, we might say that the *research interests* of linguistics and logic are quite different here. It is a *mistake* to assume that linguistics must imitate logic and to invent, to this end, a non-psychological 'ideal speaker' who speaks an indefinitely complex language similar to the artificial languages recursively generated by systems of logic. The 'ideal speaker' possesses no properties over and above those belonging to an axiomatic system; in fact, *the two are identical*. Consequently, what we have here is an attempt to introduce, without any justification, the principle of axiomatics into linguistics. More precisely, the sole 'justification' for this maneuver is to rechristen axiomatic systems as 'ideal speakers', thus making them more palatable to ordinary practicing linguists.

I have been arguing here that before we accept the principle of axiomatics as part of the theoretical apparatus of natural language descriptions, we must carefully examine to what extent its acceptance is *genuinely* justified. In particular, this means examining whether there is a sufficient analogy between grammatical and logical induction. We have seen that this analogy is non-existent.

These questions have never been raised within transformational grammar, probably because of Chomsky's early preoccupation with the theory of recursive functions. At the time, it may have seemed a rather obvious step to extend the recursive treatment from artificial languages to natural languages, because the common term 'language' seemed to guarantee the similarity between the two kinds of the research objects. This assumption is wrong, however, because – as we have seen – natural languages and artificial languages are 'languages' in quite different senses. Another reason for emphasizing the need for recursivity must be sought in the fact that it was generally mistaken for creativity. This is curious, because a recursive device, e.g. the grammar of fig. 1, is a purely mechanical one, and therefore the very opposite of what is meant by 'creative'. On the other hand, the extremely large number of actual or (realistically) possible utterances in a language can be accounted for by incorporating a quite restricted amount of recursivity into the grammar (cf. below).

12. Implications for constructing psychologically real grammars

Sentences which exceed a certain limit of length and/or complexity cannot be uttered or understood because of the limitations of the human memory (as well as for other psychological reasons). Now, in transformational grammar it is an article of faith that "such grammatically irrelevant conditions as memory limitations, distractions, shifts of attention and interest" belong to linguistic performance and need not be accounted for in linguistic descriptions (Chomsky 1965: 3). But surely it is a quite elementary mistake to place memory limitations on a par with shifts of attention and interest. The latter phenomena may or may not occur in actual cases of language use, whereas the former predetermine the very nature of language use. Memory limitations are a necessary, inherent characteristic of the human mind, and a psychological theory which fails to account for this fact is about as inadequate as a biological theory of man which fails to account for man's mortality. (In effect, it is misleading to speak of 'memory limitations'; it is more correct to speak simply of memory, that is, factual memory.) Only idiosyncratic memory limitations are comparable to shifts of attention and belong to performance. Furthermore, there is clear evidence contradicting the claim that the hypotheses about memory limitations operative in the use of language could be just borrowed from general psychology. Rather, linguistic memory seems to function according to principles of its own (Whitaker 1974).

It is obvious that transformational grammar, with its unrealistic notion of competence, is not a psychologically real theory: the competence of the 'ideal speaker' is a 'mental grammar', but – as we saw in sect. 11 – this grammar is an imitation of axiomatic systems which have been constructed for the description of *artificial* languages.

It has often been claimed that any limitations on the permissible length of sentences would be arbitrary (e.g. Chomsky 1957: 23). But, of course, there is nothing arbitrary about this. It is, rather, an entirely empirical question how long and/or complex sentences people can understand or recall. Although the 'core' of language is constituted by cases which are, beyond any possibility of doubt, known to be what they are, psycholinguistic and sociolinguistic research is needed to ascertain the facts in those areas where linguistic knowledge is less than absolute. In a sense, then, this kind of research is needed to establish the difference between natural language and non-natural language. Of course, such a difference can only be an approximate one, but it would be absurd to deny its reality for this reason, just as it would be absurd to deny that there is a difference between rich and poor only because it cannot be defined with the accuracy of one penny (Ziff 1974).

From the grammatical point of view, length and complexity of sentences result from the application of recursive rules. Therefore, to the extent that length and complexity are not part of language, recursivity is not part of grammar. Whitaker draws the same conclusion:

> "More specifically, we may incorporate the restrictions imposed by linguistic memory into the early rules of G in order to disallow an indefinite number of recursions on the #S# node, or... to disallow an indefinite number of self-embedded sentences" (Whitaker 1974: 86).

It is a remarkable fact that transformationalists have carefully avoided describing e.g. conditional or temporal sentence-types, in connection with which recursivity rapidly leads to incomprehension, and have, instead, concentrated upon the description of those rather few sentence-types which remain relatively 'transparent' even after several applications of recursive rules. However, in these cases recursivity is not a specifically syntactic phenomenon, because the recursive rules involved, mainly the right-branching rules of *that*-clause formation and relativization, generate, essentially, long *lists* of sentences. Moreover, man's factual memory places limitations even on the application of transparent recursive rules, because from a given point onwards the speaker or the hearer can no longer identify or recall what he has said or heard, unless he starts using such means of verification as counting or writing down (Ziff 1974). But all such and similar means of verification, including the use of auxiliary diagrams in more complex cases (cf. sect. 11), are elements external to spontaneous use of language. That is, these problems arise, not in the context of actual use of language, but only when the attempt is made to apply to linguistics those computational and recursive principles which have been borrowed

from logic or mathematics. With a little exaggeration, it might even be said that logic, in an interesting sense, starts precisely at that level of the depth of recursion where linguistics, in a psychologically real sense, ends. For instance, consider how Schnelle uses the 'possible words' semantics in the description of natural language. He examines the hypothetical situation in which a person who in the real world is a woman wishes to be (in the first dream world) a man who wishes to be (in the second dream world) a woman who ..., etc. On the basis of this alternating series of possible worlds, he notes, it can be inferred, that the possible individual in the 2116th dream world is a woman (who wishes to be a man, etc.) (Schnelle 1973: 227). This analysis would supposedly be the semantic description of the natural language sentence containing 2116 relative clauses and purporting to express what is the case in the first 2116 dream worlds. – All this may be interesting from the logical point of view, but it is certainly not relevant to linguistics.

If the grammar is to generate all and only intuitively correct sentences, plus those sentences which, as a result of psycholinguistic and/or sociolinguistic reserach, can be shown to be relatively or reasonably correct, then its recursive capacity must be strongly limited. In fact, I think it would be possible to replace the mathematical notion of recursivity by rules determining the possible combinations, both in qualitative and in quantitative respects, of the different types of clauses in a given language. In a sense, this would mean a return to the classification of clauses (e.g. temporal, causal, concessive, relative) as in traditional grammar. Together with the combinatory freedom of words in phrases and clauses, all these combinatory possibilities would guarantee that the number of novel sentences of the language is *practically* (but not theoretically) infinite in spite of the fact that the length of a single sentence seldom exceeds, say, one hundred words.

13. A critique of Bever's distinction between grammar and perception

In a series of papers, most recently in Bever 1974, Bever has defended that notion of grammaticality which I have been objecting to here. Referring to the example from Chomsky and Miller 1963 mentioned above, he draws a distinction between grammaticality and acceptability: incomprehensible or unacceptable sentences generable by rules of the grammar are posited as grammatical, their unacceptability being a result of their perceptual complexity. This conception is said to be based on the 'formal naturalness' of grammars: sentences are taken as grammatical or ungrammatical depending of whether their inclusion into or exclusion from language would simplify or complicate the grammar. However, not only are unacceptable (or simply incorrect) sentences taken as grammatical, but also acceptable (or correct) sentences are taken as ungrammatical, provided they cannot be generated without adding new rules to the grammar. In fact, Bever purports to give a precise definition of analogy by defining acceptable but ungrammatical forms as analogical. This conception only makes transformational grammars even more incapable than before to account for creativity and change, given that analogy is a necessary component of both and is now being excluded from the grammar. I consider Bever's concept of analogy as a *reductio ad absurdum* of his position.

Bever also claims that, given the forms which are both acceptable and grammatical, he can *predict*, on perceptual grounds, the unacceptability of more complex, grammatical forms. But it might just as well be said that, given the correct forms, or the rules determining their correctness, we can predict that forms breaking the rules, in one way or another, will be (perceived as) incorrect, complexity being just one type of rule-breaking.

Throughout, Bever justifies his analyses by referring to the resulting simplicity of the grammar. It is questionable, however, whether this kind of justification is itself justifiable. A grammar with unrestricted recursivity is no doubt simpler than one with different kinds of restrictions on recursivity. But this kind of simplicity is the simplicity of axiomatic systems, and the question is precisely, whether such systems offer adequate models for linguistic descriptions. In my opinion, they do not, or do so only to a quite limited extent (cf. sect. 10–12).

14. Concluding remarks

Wittgenstein's proof of the impossibility of private languages shows that language is inseparable from the public *use* of language (cf. Itkonen 1974: ch. II, 1). Use of language consists in *actions* conforming to rules of speaking. *Intentions* are an inseparable part of actions. One intends to do, e.g. to say, only what one *can* do. One can do only what one *understands*, i.e. what one understands oneself as doing. What one can do,

constitutes one's *competence*, in a psychologically real sense of this word. Consequently, sentences too long or too complex to be understood or recalled do not belong to linguistic competence. It is the task of pragmatic grammars to examine this kind of (conscious) competence, which comprehends both the realistically possible speech acts and, at a more restricted level, the form and meaning of sentences contained in such acts, just as it is the task of psycholinguistics to discover the (unconscious) mechanisms underlying this competence. Since speech acts are typically free, intentional actions, it is the 'free will' (whose functioning is explicable with the aid of practical syllogism) which puts the underlying psychological mechanisms into motion.

This notion of linguistic competence (which is characterized at greater length in Itkonen 1974: ch. XI) has obvious similarities both to Habermas's and to Hymes's notions of 'communicative competence'. The so-called concrete or natural phonology, as represented by Anttila and Linell among others, may be mentioned as an example of linguistic analysis operating with the notion of a psychologically real linguistic competence.

Transformationalists are free to continue doing axiomatizing linguistics and to reject the supplementary criteria offered by psycholinguistics and sociolinguistics. What is required of them is that they acknowledge the nature of their commitment to axiomatics and, *eo ipso*, give up the claim to psychological reality. (They have never even raised the claim to sociological reality.) This research strategy is possible and, in a sense, legitimate but it is contrary to the trend towards greater integration of knowledge, which sees language as part of the human mind and of the human society.

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