(What follows is the English translation of the subsections 2-A&B of Chapter XXII from my three-volume, 950-page book *The diversity and the unity of the world's languages* [in Finnish] that was published in 2008–2010)

2) SIMPLICITY vs. COMPLEXITY

A) SOME HISTORICAL BACKGROUND

In Pānini's tradition it was already taken for granted that **simplicity** — measured by the **brevity** of grammatical rules — is the overriding goal of the grammar: "Grammarians rejoice over the saving of even the length of half a short vowel as much as over the birth of a son" (cf. Itkonen 1991a: 21).

The same view was represented in American structural linguistics by Harris (1957 [1946]), even if he replaced the term 'simple' by such terms as 'useful' and 'convenient': "The criterion which decides for *-ing*, and against *un-*, as the relevant environment for substitution classes [for verbs] is therefore a criterion of **usefulness throughout the grammar**, a configurational consideration" (p. 143, note 6; emphasis added). "We merely select positions [= substitution frames] in which many morphemes occur, and in terms of which we get the **most convenient total description**" (p. 150; emphasis added). Clearly, the Harris-type simplicity is a property of the entire grammar, not of particular rules (cf. Itkonen 1978: 71– 75).

Chomsky (1957) faithfully repeats the same view: "Notice that simplicity is a **systematic** measure; the only ultimate criterion in evaluation is the simplicity of the whole system" (pp. 55–56). "[W]e may define the phonemes and morphemes of a language as the tentative phonemes and morphemes which ... jointly lead to the simplest grammar" (p. 57).

Chomsky (1957: Chap. 6) envisages three levels of decreasing stringency for linguistic theory. On each application, a 'discovery procedure' would take a corpus as its input and produce the 'correct' grammar as the output. A 'decision procedure' would take as its input a pair constituted by a corpus and a grammar and would decide, 'yes' or 'no', whether this is the correct grammar for this corpus. An 'evaluation procedure' would take as its input a triple constituted by a corpus and two grammars and would, based on a simplicity measure, decide which one is the better grammar. According to Chomsky, the two first procedures as unrealistic while the third consstitiutes an essential ingredient of genuinely scientific linguistics: "We **must** analyze and define the notion of simplicity that we intend to use in choosing among grammars. ... [This theory] **must** tell us how to evaluate such a grammar: it **must** thus enable us to choose between two proposed grammars" (p. 54; emphasis added).

Chomsky (1965) completes the **psychological** turn of generativism. The evaluation procedure is now assumed to be part of the 'language acquisition device: "The device would then select one of these potential grammars by the evaluation measure [=

simplicity measure]" (p. 32). It is important to realize that simplicity is a **relative** notion (and not an absolute one): "[It is a misconception to assume] that 'simplicity' is a general notion somehow understood in advance outside of linguistic theory" (s. 37). This sounds plausible.

For some three decades, 'simplicity measure' was sought after like the Holy Grail. Chomsky (1965: 42) characterizes it in such general terms that it quite expectedly coincides with the Pāninian interpretation: "The obvious numerical measure to be applied to [the evaluation of] a grammar is **length**, in terms of number of symbols" (emphasis added). It felt natural to start by applying this idea to phonology. Chomsky & Halle's efforts culminate in *The sound pattern of English* (1968), with little success.

No simplicity-based evaluation measures were offered in morphosyntax, and the whole idea has been quietly abansoned. It is hard to disagree with Itkonen (1978: 75): "It may be true that it is impossible to devise exact and rigorous discovery procedures for grammars; but no exact and rigorous evaluation procedures for grammars have been devised either. Nor should this be surprising, considering that procedures for purely formally determining the superiority of one theory over another have so far not been devised in any science, empirical or not."

Putnam (1981) reaches the same conclusion: "Let us call a theory which obeys Ockham's razor ... **functionally simple**. ... Ockham's razor seems difficult or impossible to formalize as an algorithm, ..." (p. 133).

B) AN ANALOGY: SIMPLICITY IN LOGIC

Up to now I have been pessimistic or at least sceptical about the possibility of giving an exact definition of simplicity. Therefore it is time to discuss a case where such a definition **can** be given. It could indeed be characterized as the **prototype** of (non-trivial) simplicity. Significantly, it is limited to a narrowly restricted area from which it can**not** be extended (at least in any obvious way) to the to adjacent areas, let alone to linguistics. Nevertheless, it is very important as a **point of comparison**: it shows us quite precisely what it is that we lack. At the same time it makes it easier to grasp the the relation between structural simplicity and numerical simplicity.

What is at issue is the development of the **formalization of propositional logic**. But first we need some background information. What follows is based on Itkonen (2003a: Chap. VI). As far as the present book is concerned, propositional logic will also turn out be useful in the subsequent chapters on complex sentences.

The classical sentential or propositional logic is two-valued. An atomary formula (or sentence-variable) p must be either true (= T) or false (= F). Therefore the negation $\sim p$ is T when p is F, while $\sim p$ is F when p is T. Furthermore, a formula that contains two sentence-variables p and q can have only four distinct truth-value combinations. Their traditional order of presentation is TT, TF, FT, FF. Propositional logic is based on the insight that non-atomary formulae constructed by means of distinct **connectives** are defined depending on which truth-value combinations make them either true or false.

The four standard connectives will now be defined: conjunction $p \,\&\, q$ ('p and q'), disjunction $p \vee q$ ('p or q'), implication $p \rightarrow q$ ('if p then q'), equivalence $p \equiv q$ ('if, and only if, p then q'). In addition, a less usual connective $p| q$, called "Sheffer's stroke", will also be defined:

p	q	p & q	p ∨ q	p → q	p ≡ q	p | q
T	T	T	T	T	T	E
T	E	E	T	E	E	T
E	T	E	T	T	E	T
E	E	E	E	T	T	T

The conjunction $p \,\&\, q$ is true when both p and q are true, and false otherwise. The disjunction $p \vee q$ is false when both p and q are false, and true otherwise. The implication $p \rightarrow q$ is false when the antecedent p is true and the consequent q is false, and true otherwise. The equivalence $p \equiv q$ is true when either both p and q are true or when both p and q are false, and false otherwise. The formula $p \mid q$ is false when both p and q are true, and false otherwise. We see that $p \mid q$ is identical with the negation of the conjunction $\sim(p \,\&\, q)$. This is why the connective | is designated as 'alternative denial' or, more informally, 'not both'. The column under each connective is called its truth-value constellation.

There is no need to accept all these connectives as primitive. Rather, some can be defined by means of others. These are the most common definitions:

a) $p \equiv q \;=\; (p \rightarrow q) \,\&\, (q \rightarrow p)$
b) $p \rightarrow q \;=\; \sim p \vee q$ c) $p \rightarrow q \;=\; \sim(p \,\&\, \sim q)$
d) $p \vee q \;=\; \sim(\sim p \,\&\, \sim q)$
e) $p \,\&\, q \;=\; \sim(\sim p \vee \sim q)$

Equivalence is defined by means of implication and conjunction (more precisely, the former twice at level 1 and the latter once at level 2). Implication is defined in two different ways, namely either by means of negation and disjunction or by means of negation and conjunction. Disjunction is defined by means of negation and conjunction, and — inversely — conjunction is defined by means of negation and disjunction. The so-called truth-table method (or any of its later refinements) establish the definitions a)–e) as 'tautologies', i.e. **logical** truths (more precisely, logical equivalences). The definitions d)–e) are known as "De Morgan's laws".

From the purely **formal** point of view, one is free to choose any pair of connectives as the primitive ones and to use them as the basis for defining the others. From the semantic or, rather, **ontological** point of view, the situation is somewhat different. It is easy to undertand what $p \,\&\, q$ refers to: it is the 'side-by-side' existence of two states of affairs. By contrast, it is more much difficult, or even impossible, to envisage what could be the ontological counterpart of $p \vee q$ (cf. Russell 1967 [1940]: 79). And as for defining the ontological counterpart of $p \rightarrow q$, it has proved to be a hopeless task. Therefore I agree with Hilbert & Ackerman (1928: 8): "Die Darstellung mit → und ~ hat Frege, die mit ∨ und ~ Russell zugrunde gelegt. ... Am **natürlichsten** ist es wohl, von der Darstellung durch & und ~ auszugehen, wie es in Brentanos Urteilslehre geschieht" (emphasis added).

Up to now, definitions need at least **two** primitive connectives. Interestingly, it is possible to define the other connectives by means of **one** single connective. There are two options, either alternative denial $p \mid q$ or 'joint denial' (= 'neither p nor q', with the truth-value constellation EEET), designated by $p \downarrow q$. The former option is chosen in what follows. We get the following definitions:

~p	=	p | p
p & q	=	(p | q) | (p | q)
p ∨ q	=	(p | p) | (q | q)
p → q	=	p | (q | q)

For instance, the identity of the formulae *p & q* and *(p | q) | (p | q)* can be established in the following way; the numbers indicate the progression from the parts to the whole:

p	&	q	(p	|	q)	|	(p	|	q)
1	2	1	1	2	1	3	1	2	1
T	**T**	T	T	E	T	**T**	T	E	T
T	**E**	E	T	T	E	**E**	T	T	E
E	**E**	T	E	T	T	**E**	E	T	T
E	**E**	E	E	T	E	**E**	E	T	E

Both formulae get the same truth-value constellation TEEE, which means that they are just two different ways of saying the same thing.

Now we are in a position to examine, in outline, how the formalization of propositional logic has developed. Russell & Whitehead (1910) presented a system (= R&W system) with **five** axioms (= A) and **two** rules of inference (= R):

A1) (p ∨ p) → p
A2) p → (q ∨ p)
A3) (p ∨ q) →(q ∨ p)
A4) [p ∨ (q ∨ r)] → [q ∨ (p ∨ r)]
A5) (p → q) → [(r ∨ p) → (r ∨ q)]

R1 = Modus Ponens: If A → B is a tautology and A is a tautology, then B is a tautology.
R2 = Rule of substitution: In a tautology a sentence-variable can be uniformly replaced by any formula.[1]

The axioms are proved to be tautologies by means of the truth-table method, and the rules of inference can be proved to be such that if their starting point is a tautology, their end point is a tautology. All and only tautologies (= valid formulae) of propositional logic can be proved (or derived) by means of these axioms and these rules of inference.

Paul Bernays showed in 1926 that the axiom A4) of the R&W system can be derived from the other axioms, which means that it is **redundant**. Accordingly, Hilbert & Ackermann (1928: 22) presented the following **simpler** system (= H&A system) with **four** axioms (and the same two rules of inference as in the R&W system) :

A1′) $(p \vee p) \rightarrow p$
A2′) $p \rightarrow (q \vee p)$
A3′) $(p \vee q) \rightarrow (q \vee p)$
A4′) $(p \rightarrow q) \rightarrow [(r \vee p) \rightarrow (r \vee q)]$

The transition from the R&W system to the H&A system signifies **progress** in the following sense: "a contraction in the axiom set is a gain in **simplicity**" (Sober 1975: 80; emphasis added).

Jan Łukasiewics showed in 1929 that the H&A system with four axioms can be replaced by the following system (= Ł system) with **three** axioms:[2]

A1′′) $(p \rightarrow q) \rightarrow [(q \rightarrow r) \rightarrow (p \rightarrow r)]$
A2′′) $(\sim p \rightarrow p) \rightarrow p$
A3′′) $p \rightarrow (\sim p \rightarrow q)$

The comparison between the R&W system, the H&A system, and the Ł system offers a **prototypical** example of the use of a **simplicity measure**. All three system exemplify the same level of **structural** simplicity/complexity. It is in this — and **only** in this — situation that mere **numerical** simplicity can be used as the criterion for placing descriptions in an order of superiority. Here 'X < Y' means 'X is simpler, and therefore better, than Y'.

Ł system < H&A system < R&W system

As shown by the Chomsky quotation given above, the ideal which one strives after is **numerical** simplicity, i.e. the **number** of descriptive units (e.g. sentence-variables, connectives, and formulae) as well as the **length** of the total description. We have seen that Pāṇini's tradition strives after the same ideal. But this is a realistic ideal only in a context where the descriptions that are being compared with one another are **structurally homogeneous**. If this condition is not fulfilled, there will be problems.

During decades Łukasiewics and his research group attempted to further reduce the number of the axioms, first in Warsaw and then in Dublin. C.A. Meredith showed in 1953 that all and only tautologies of propositional logic can be derived, by means of the familiar rules of inference, from **one** single axiom (cf. Franzke & Rautenberg 1972: 51–52; Haack 1978: 21). It will be designated by 'A-M' (= 'Meredith axiom'), and the corresponding system will be called 'M system':

A-M) $[\![\{[(p \rightarrow q) \rightarrow (\sim r \rightarrow \sim s)] \rightarrow r\} \rightarrow t]\!] \rightarrow [(t \rightarrow p) \rightarrow (s \rightarrow p)]$

But is the M system **simpler** than the preceding axiomatizations? The only meaningful answer to this question is 'yes and no'. The structural homogeneity of the three preceding systems garanteed that it was feasible to operate with one notion of simplicity. But now simplicity has to be divided into two subtypes, numerical and structural. What this means, exactly, will become clearer with the aid of the following example.

Up to now, all axiomatizations have made use of **two** connectives, either disjunction & implication (= R&W and H&A) or negation & implication (= Ł and M).[3] But J.G.P. Nicod showed already in 1917 that propositional logic can be axiomatized by means of one single axiom which makes use of one single connective, namely **alternative denial**. It will be designated by 'A-N' (= 'Nicod axiom'), and the corresponding system will be called 'N system':

A-N) $[p \mid (q \mid r)] \mid [\![[t \mid (t \mid t)] \mid \{(s \mid q) [(p \mid s) \mid (p \mid s)]\}]\!]$

The counterpart of R1 is in the N system as follows:

R1-N) If A | (B | C) is a tautology and A is a tautology, then C i a tautology..

A | (B | C) means 'not both A and B | C', while *B | C* means 'not both B and C'. Therefore it is permissible to make the following transformations of which the latter is based on the second definition, or definition c), of implication (cf. above):

$$A \mid (B \mid C) \quad \Rightarrow \quad {\sim}[A \ \& \ {\sim}(B \ \& \ C)] \quad \Rightarrow \quad A \rightarrow (B \ \& \ C)$$

R1-N, transformed into a formula, yields *{[A → (B & C)] & A} → C*. We see that Modus Ponens (= R1), i.e. *[(A → B) & A] → B*, is entailed by R1-N.

In choosing the primitive connective(s), one has to counter-balance **two** distinct points of view. Or, as Quine (1962 [1940]: 47) puts it:[4]

"In **theoretical** developments ... it is convenient to be able to treat '↓' or 'neither-nor' thus as the sole truth-functional connective. But in **applications** it is convenient to have the signs '~', ['&',] '∨', ['→',] and '≡' as well, for the brevity and clarity which they afford. Adoption of '↓' as the sole connective demands continual use of compounds so cumbersome, indeed, that conventions of shorthand reducing them to manageable length would become a **practical** necessity.

Between the **theoretical** advantage of a single connective and the **practical** advantage of multiplicity, this idea of shorthand effects a complete [!] reconciliation" (emphasis added).

It is generally thought that formal logic, if anything, is of theoretical nature. But, as shown by the Quine quotation, the practical point of view has to be applied in formal logic too, with a corresponding gain in the "brevity and clarity" of the total description.

We reach the same conclusion when the R&W, H&A, and Ł systems are compared to the N system: "Although more **economical** in the respects indicated, the Nicod System can scarcely be said to be **simpler** than such systems as ...H.A. [= Hilbert-

Ackerman System] ... and L.S [= Łukasievics System]. There is only one axiom for N [System], but it is more complicated than any axiom or postulate of any of the other systems. Not only is it longer, but it involves five distinct propositional symbols, *p*, *q*, *r*, *s*, and *t*, whereas the entire set of axioms for any of the other systems can be stated in terms of only three distinct propositional symbols. ... Nicod's Rule [= R1-N] as well as his axiom is **more complicated** than those required in **less economical** systems of logic" (Copi 1967: 269; emphasis added).

If we accept 'simplicity vs. complexity' as the superordinate notion, and if we let the H&A system represent the three first systems, then we see that Copi (1967) assumes **two opposite dimensions** within it:

more economical	<	less economical =	N system	<	H&A system
less complicated	<	more complicated =	H&A system	<	N system

The N system is 'more economical' than the H&A system because it needs only one axiom. At the same time, it is 'more complicated' than the H&A-system because its single axiom is longer than any of the four axioms of the H&A system and because it needs five sentence-variables where the H&A system needs only three. But there is an additional — and very important — criterion which renders an axiom 'complicated', namely **structural** complexity, and more precisely the degree of **hierarchy**. It is revealed by the **number** of the **different types of brackets**. In the H&A system the 'depth' of hierarchy is 3 (starting from the atomary sentence-variables): $p \Rightarrow (\ldots) \Rightarrow [\ldots]$. By contrast, the depth of hierarchy is 5 both in the M system and in the N system: $p \Rightarrow (\ldots) \Rightarrow [\ldots] \Rightarrow \{ \ldots \} \Rightarrow [\![\ldots]\!]$. Moreover, we see that the use of numerical simplicity yields contradictory results. The 'complicated' N system is 'simple' with respect to the number of axioms but 'complex' with respect to the number of sentence-variables.

As can be expected, the 'complicated' nature of the N system is also evident in how theorems are proved. Copi (1967: 270–276) proves altogether 17 N-theorems, including the four axioms of the H&A system, and offers the following general assessment: "it is very difficult indeed" (p. 269) and "it is tedious business" (p. 276). Likewise, it is not easy to prove theorems in the M system.

The notion of simplicity is intertwined with that of **generalization.** Chomsky (1965: 42) notes that "we have a generalization when a set of rules about distinct items can be replaced by a single rule about the whole set". But we have just seen that in reality the situation is more problematical. It is far from clear that simplicity increases if three simple axioms (or rules of inference) are replaced by one complex axiom (or rule of inference). Of course, simplicity may plausibly be claimed to increase if this new axiom is just **a little** more complicated than any of the old axioms (or rules of inference). But what does 'a little' or 'much' mean in his context? This is precisely what nobody knows.

There is one and the same simplicity measure that applies to the R&W, H&A, and Ł systems. This is the ideal situation. But there is no longer any such measure that these three systems would share with the M system, although they all have the same rules of inference and two connectives. And it becomes even more difficult to think of a simplicity measure that could be common to these four systems and to the N system, because the last-mentioned has a different rule of inference and only one connective. Nevertheless, these five systems still share the notion of axiomatization.

The situation becomes even more difficult when such formalizations of propositional logic are taken into consideration which, while covering the same area as the five systems discussed above, do so **without** the notion of axiomatization. First, one has to mention the 'method of natural deduction', which was developed by Gerhard Gentzen since 1934 (cf. Itkonen 2003a: 69–71). It will be called the 'G system'. Its basic idea is, roughly, that if we want to prove the (schematic) formula $A \rightarrow B$ as a tautology, we first assume A as a premise and then derive B by means of rules of inference, after which we 'cancel' the status of A as a premise and transform the derivation into the (tautological) implication $A \rightarrow B$.

Dummett (1981: 432–434) ranks natural deduction higher than any axiomatizations, because (he thinks) it captures the basic insight that what logic is — or should be — investigating is not logical **truth** but logical **consequence**. Stegmüller & von Kibéd (1984: 98–99) offer a similar assessment. It seems natural to assume that, as these scholars see it, axiomatizations of propositional logic are **less simple** than the formalization in accordance with natural deduction.

Second, one has to mention the 'dialogical (or game-theoretic) logic', which has been developed by Paul Lorenzen and Kuno Lorenzin since the late 50's. It will be called the 'L&L system'. It defines the connectives of classical propositional logic in terms of attacks and defences by two discussants (or 'players') and is able to prove the same theorems as the preceding systems. Importantly, however, this is a method, "die ohne den traditionellen Rückzug auf die axiomatische Methode auskommt" (Lorenzen & Lorenz 1978: 19).

From the philosophical point of view, dialogical logic is vastly superior to all other formalizations (cf. Itkonen 1978: 48–49, 2003a: Chap. IV). Because it starts from the linguistic interaction between the speaker and the hearer, it is right to say that "die Dialogregeln rekonstruieren umgangssprachliches Verhalten" (Kamlah & Lorenzen 1967: 161). This is a perfect way to capture the **social** and **normative** nature of both language and logic. It goes without saying that dialogical logic is **more natural** than the other formalizations; and I take this to mean that it is also **simpler**. But of course, this type of simplicity cannot literally be **measured**. An external observer might also say that my judgment contains a subjective element. But the same element is, in my opinion, contained in Dummett's (1981) criticism of axiomatic logic as well.

More precisely, Dummett expresses the (reasonable) view that, instead of asserting that something is true, logic asserts that something is true **if** ... But then, by the same token, he has to ignore the **generalization** that is achieved when (axiomatic) logic is compared to axiomatizations of e.g. geometry, linguistics, and physics (cf. Itkonen 2003a: 70–71). This entails that one cannot arrive at an overview of the entire system, or taxonomy, of the scientific disciplines. Here we have an interesting (metascientific) analogy to the view, mentioned in Section A), that what is decisive is the overall simplicity of the **total** grammar. In any case, it is difficult not to accept here the existence of at least **two** distinct points of view, namely Dummett's and mine. (Notice that my point of view happens to coincide with Frege's, Russell's, Hilbert's, etc.)

No neat picture emerges from what precedes. In any case, some of the results may be summarized with the aid of two distinct scales of decreasing naturalness and/or simplicity, where a straight line | separating two systems indicates some (undefined) degree of incommensurability:

Three different conceptions of formal logic:	L&L <| G <| H&A
Axiomatizations:	Ł < H&A < R&W| M || N

Incommensurability between systems of **formal logic** is tantamount to the absence of a common simplicity measure. Now, in addition, there is a huge increase in incommensurability when we move from formal logic to **psychology of logic** (cf. Itkonen 2003a: Chap. XV).

Formal logic (like mathematics in general) is charactrized by a **prescriptive** research interest: it is the purpose of logicians to enable people to make better inferences than they did before. By contrast, psychology of logic is characterized by a **descriptive** research interst: it is the purpose of psychologists to show, by means of the experimental method, how people infer in fact. Because people differ from one another in this respect, psychology of logic is based on **statistical variation**, which — as we have seen — is totally absent from formal logic.

One of the principal problems in psychology of logic is how test persons perform the four following types of inferences:

Modus Ponens (MP)	Denying the Antecedent (DA)	Affirming the Consequent (AC)	Modus Tollens (MT)
$p \rightarrow q$	$p \rightarrow q$	$p \rightarrow q$	$p \rightarrow q$
p	$\sim p$	q	$\sim q$
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q	$\sim q$	p	$\sim p$

MP and MT are valid inferences: if the premises are true, the conclusion is true. AC is not valid, but it has a meaningful use in testing the hypothesis *p*. DA has no meaningful use.

At first, it may seem easy to measure the mutual simplicity vs. complexity of MP, DA, AC, and MT: one only needs to find out to what extent they are either understood or misunderstood by the test persons. But the situation is — once again — more problematical.

Experimental research shows that MP is generally understood to be valid. It is very surprising, by contrast, that in different types of experiments roughly an equal amount of test persons, i.e. 50 ~ 80%, think the other three inferences are **also** valid, making no systematic distinction between (valid) MT, (meaningful) AC, and (meaningless) DA (cf. Evans 1982: 128–135). Moreover, the situation changes if in the first premise, i.e. $p \rightarrow$

q, either p or q or both are negated. While MP continues to be understood, the correct understanding of MT sinks below 40% (and in one particular test as low as 12%); and, on the other hand, if p is negated, then — curiously enough — test persons exhibit an increasing tendency to misunderstand AC as valid.

As expected, correct understanding of inferences increases if letters or numbers are replaced by meaningful sentences. But then the question arises whether or to what extent the tests measure logical ability (as they should) and not just encyclopedic knowledge.

Changing the first premise from $p \rightarrow q$ into $p \equiv q$ would render both DA and AC valid. Clearly, test persons tend to confuse equivalence and implication. Their performance improves if this confusion can somehow be forestalled. But then the question arises again whether the tests really measure logical ability and not something else: it is part and pacel of logical ability **not** to confuse implication and equivalence.

All in all, the situation turns out to be rather similar in psychology of logic and in formal logic. There is **no absolute** simplicity measure in either domain. In each and every case the interpretation of simplicity dependes on the context and/or the point of view. And it is, of course, quite impossible that there could be a simplicity measure common both to formallogic and to psychology of logic.

But how is the notion of simplicity/complexity handled in the 21st-century linguistics? And how should it be handled? In particular, is there a meaningful distinction to be made between the simplicity of a language and the simplicity of a grammar? And if so, what is it *au juste*? These burning questions will be answered in the next section.

1. It is interesting to note that the programming language PROLOG, which is based on the idea of proving theorems, is ultimately so simple as to make use only of Modus Ponens (= R1) and of a predicate-logic variant of R2 (cf. Itkonen 2003a: Chap. XIII).