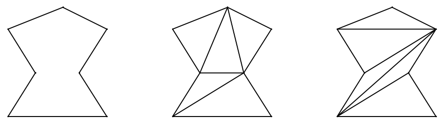
Graph Theory: Problem Set 10

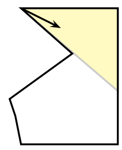
March 29 (2018)

DEFINITION. A **polygonal triangulation** T is a polygon that is divided into triangles by nonintersecting diagonals. The **graph of** T has the vertices of the polygon, and the edges are the edges and the diagonals of the triangulation in a natural way.



1 Let G be a graph of a polygonal triangulation. Show that $\chi(G) \leq 3$.

DEFINITION. An **Art Gallery** consists of a polygonal room with straight walls. We seek for the minimum number of guards needed to watch the paintings on the walls. Each guard is placed in a corner of the gallery, and every point of every wall must be visible to at least one guard.



2 (CHVÁTAL 1975) Consider an art gallery with n corners. Show that we need at most $n/3$ guards.

3 Let G be a maximal planar graph with a plane embedding $P(G)$ and a colouring $\alpha\colon V_G \to \{1,2,3\}$ (that need not be proper). A face with all three colours is called **polychromatic**. Show that the number of polychromatic faces is even.

4 Let G be a planar graph. Show that G has an *induced* bipartite subgraph H such that $\nu_H \geq \nu_G/2$.

5 Show that the polynomial $k^2 - 6k + 8$ does not divide the chromatic polynomial of any planar graph.

DEFINITION. Every graph G has a plane drawing such that no three edges intersect at the same point. The **crossing number** $\times(G)$ is the minimum number of intersections of edges in such a plane drawing of G. Therefore G is planar if and only if $\times(G) = 0$, and, for instance, $\times(K_5) = 1$.

6 Show that $\times(K_6) = 3$.