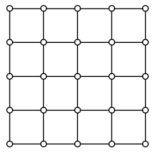
Graph Theory: Problem Set 5

February 15 (2018)

1 (**Gomory's Theorem** (1973)) Consider the 8×8 chessboard, and remove any white square and any black square. Show that one can cover the broken board using 1×2 domino pieces (that can be rotated).

2 Is the $k \times k$-**grid graph** hamiltonian?



3 Let G be a disconnected triangle-free graph (i.e., it has no induced C_3). Show that its complement graph $\overline{G}$ has a Hamilton path.

4 Show that each self-complementary graph G has a Hamilton *path*. (A graph G is **self-complementary** if it is isomorphic with its complement $\overline{G}$.)

Hint. Use Chvátal's theorem for Hamilton *cycles* by introducing a new vertex.

5 Show that a tree T of order $n \geq 2$ can have at most one perfect matching.

6 How many perfect matchings does the complete bipartite graph $K_{n,n}$ have?

7 (SUMNER (1974)) Let G be a connected clawless graph of even order. (A graph is **clawless** if it has no induced $K_{1,3}$.) Show that G has a perfect matching.

Sousselier's riddle (1963): President of the Petersen Memorial Club has a problem while arranging a meeting. Indeed, each member positively detests all other members except for a few friends. Therefore one should seat the members at the round table so that everyone had a friend next to him on both sides. The president consulted a graph theorist: 'Ooy, can't do', said the graph theorist, 'unless one - any of you - fails to show up.'

How many members were there in the club?

Problem of Knight's battle. Consider a knight moving on a chess board of $n \times n$ squares:

$$V_G = \{v_{ij} \mid i, j \in [1, n]\}.$$

This gives a graph on V_G with edges $uv \in E_G$ if the knight can move in one step from the square u to the square v. Then for odd n, G is not hamiltonian.