Ordered Sets (2015)

Problem Set 8 (March 13)

There are no lectures on Tuesday 10th of March.

1 Let L be a lattice. Show that a subset $P \subseteq L$ is a prime ideal if and only if $L \setminus P$ is a prime filter of L.

2 Prove Lemma 2.47: A proper subset I of a lattice L is a prime ideal if I is closed under finite joins and

$$x \wedge y \in I \iff x \in I \text{ or } y \in I\,. \tag{2.5'}$$

3 Prove Lemma 2.39: Let I_i be a set of ideals of a lattice L for all $i \in A$. Then also $\bigcap_{i \in A} I_i$ is an ideal of L if it is nonempty. In particular, every subset $X \subseteq L$ has the smallest ideal containing X:

$$(X] = \{I \mid I \in \mathrm{Id}(L) \text{ and } X \subseteq I\}\,.$$

4 Let L be a lattice that satisfies the ascending chain condition (ACC). Show that for each nonempty subset $A \subseteq L$, there exists a finite subset $F \subseteq A$ such that $\bigvee A = \bigvee F$.

5 Consider Theorem 2.36. Show that the relation Ψ is an equivalence relation.

6 Prove Theorem 2.36.