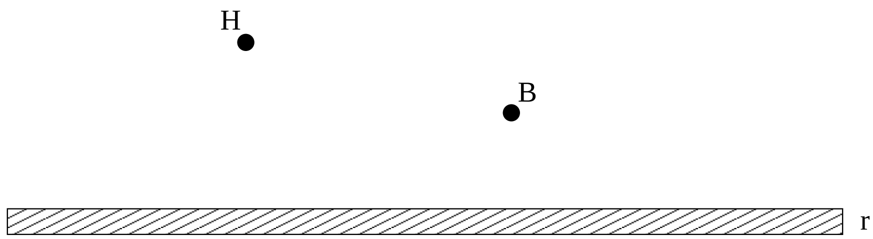
Tilings and Patterns: Homework 1 (4.9.2023)

1. Solve the following problems (use a certain isometry in your solutions):

   (a) Members of a family range in height from 90 cm to 180 cm. What is the smallest possible height of a vertical wall mirror such that each family member can see himself/herself completely in the mirror, assuming that eyes are 10 cm below the top of the head.

   (b) Determine the shortest route from the house H to the barn B that visits river r along the way:



2. Prove the "onto" -part of Theorem 2.1. In other words, show that for every isometry α and point P there must exist a point X such that $\alpha(X) = P$. (Do not use any theorems proved in the class that depend on Theorem 2.1.)

3. (a) Does the set of all translations form a subgroup of $\mathcal{I}$?

   (b) Does the set of all reflections form a subgroup of $\mathcal{I}$?

   (c) Does the set of all rotations form a subgroup of $\mathcal{I}$?

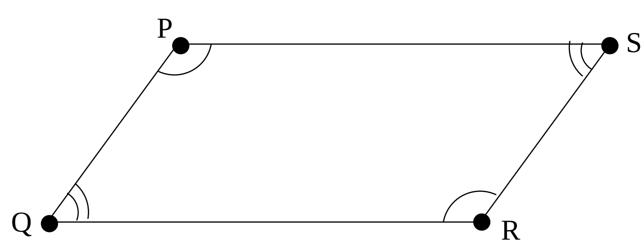
   (d) Does the set of all glide reflections form a subgroup of $\mathcal{I}$?

4. Determine all isometries of finite order, that is, all isometries α such that $\alpha^n = \iota$ for some positive integer n. You may use the fact that translations, reflections, rotations and glide reflections are the only isometries of the plane.

5. Let $\sigma_{(a,b)}$ denote the halfturn about point $P = (a, b)$.

   (a) Give a formula for the coordinates of $\sigma_{(a,b)}(x, y)$.

   (b) Show that the composition $\sigma_R\sigma_Q\sigma_P$ of three halfturns about non-collinear points P, Q and R is the halfturn σ_S about point S, where S is the unique point such that $PQRS$ is a parallelogram:



   (c) Prove that $\sigma_R\sigma_Q\sigma_P$ is a halfturn also when P, Q and R are on the same line.

6. Find all triangles such that given three non-collinear points P, Q and R are the midpoints of the sides of the triangle.

7. Show that the affine function

$$\begin{pmatrix} x \\ y \end{pmatrix} \mapsto \frac{1}{5}\begin{pmatrix} 3 & 4 \\ -4 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

   from $\mathbb{R}^2$ to $\mathbb{R}^2$ is an isometry. Determine the type (translation, reflection, rotation or glide reflection) of the isometry.