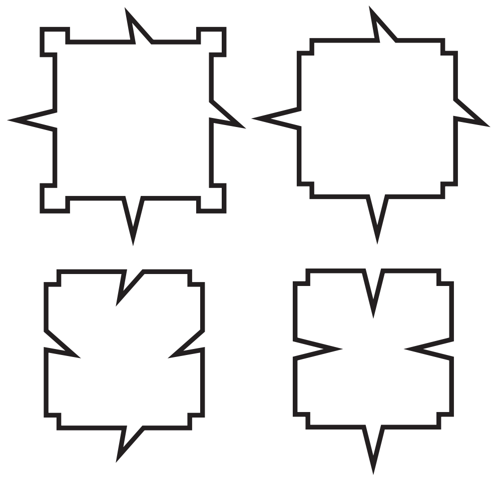
Tilings and Patterns: Homework 7 (9.10.2023)

1. Prove that the following six polygons form an aperiodic set (i.e., admit a tiling, but admit no tiling with a wallpaper symmetry):

   Note that reflected and rotated copies of the tiles may be used. (Hint: There is a connection to the Robinson's aperiodic set of Wang tiles. But observe that the square shaped markings at the corners provide a parity constraint that is different from the parity tiles of the Robinson's tile set in the lectures.)

2. Show that 28 Robinson's tiles without the parity constraint do not form an aperiodic set. (You may try to find a periodic tiling by hand – but not spend too much time on it. I needed a computer search.)

3. Let P be a finite set of Wang prototiles where the colors are given by incoming and outgoing arrows, as in the Robinson's tile set. There can be several arrows in the tiles. Suppose, however, that each tile in P has more outgoing than incoming arrows. Prove that P does not admit a valid tiling.

4. Find the balanced sequences that represent numbers
   (a) $5/2$,
   (b) $3/7$.

5. Prove that the balanced sequence of real number x has the following property: for every positive integer k, the sum of any k consecutive elements of the sequence is either $\lfloor kx \rfloor$ or $\lceil kx \rceil$. (Hence the name balanced.)

6. Prove that for every real number x and positive integer k, the sequence $B(x)$ contains at most $k+1$ different segments of length k.

7. (a) Construct a set of 6 Wang tiles that multiply balanced representations of numbers $x \in \left[\frac{1}{3}, \frac{2}{3}\right]$ by 3. In other words: all tiles of the set multiply by 3, and for each x in the given interval the tiles admit a tiling of a horizontal strip whose bottom edges read $B(x)$ and the top edges read $B(3x)$.
   (b) Construct a set of 6 Wang tiles that multiply balanced representations of numbers $x \in \left[\frac{2}{3}, 2\right]$ by $\frac{1}{2}$.
   (c) Is the union of the sets from (a) and (b) an aperiodic set of prototiles ? Explain.