**Tilings and Patterns: Homework 12 (27.11.2023)**

1. Let $\Sigma$ be a subshift, and let $A = \{c \in \Sigma \mid c \text{ is isolated in } \Sigma\}$. Prove that the *derived set* $\Sigma' = \Sigma \setminus A$ is a subshift.

2. Let $\Sigma = \overline{\mathcal{O}(c)}$ where $c$ is the infinite cross from Example 21 (on page 94) of the notes.

   (a) Determine the isolated points of $\Sigma$.

   (b) Is the derived set $\Sigma' = \Sigma \setminus A$ transitive, where $A$ is the set of isolated points from (a) ?

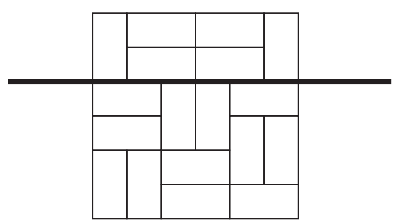
   (c) Determine the set $A'$ of the isolated points of the derived set $\Sigma'$, and form the second derivative $\Sigma'' = \Sigma' \setminus A'$.

3. Prove that $\mathcal{O}(c)$ is a subshift if and only if $c$ is strongly (=two-way) periodic.

4. Prove that the following problem is semi-decidable: "Are all configurations of a given SFT strongly (two-way) periodic ?" (This problem was shown to be undecidable in Problem 5, homework set #9. Note also Theorem 6.29 from the lecture notes.)

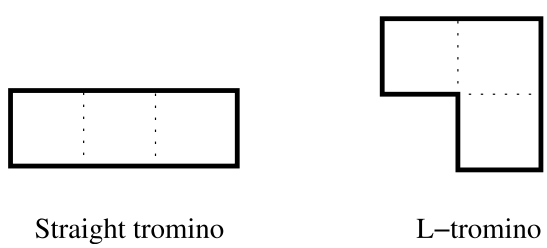
The last three problems are "recreational" problems about tilings with finitely many tiles:

5. Consider finite tilings of a $6 \times 6$ board using 18 dominoes. Prove that every such covering necessarily contains a *fault line*, that is, a straight line that does not cut through any domino. (Hint: How many dominoes are needed to block each line, and how many lines can one domino block?)



An example of a fault line

6. A tromino consists of three unit squares glued together. There are two variants: the straight tromino and the L-tromino:




   (a) Place 21 straight trominoes on a standard $8 \times 8$ checkerboard so that they cover all but one square of the board. (The pieces may be placed in horizontal and vertical orientations.)

   (b) Do the same with 21 L-trominoes, and prove that in this case the uncovered square can be chosen to be any square of the board.

7. Prove that it is not possible to tile a $10 \times 14$ rectangle using 35 copies of the $1 \times 4$ rectangle (the *straight tetromino*).