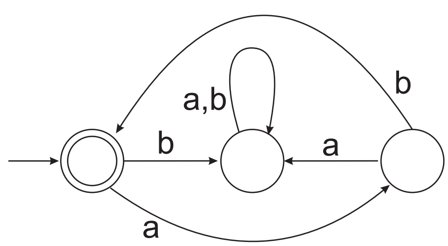
Automata and Formal Languages. Homework 1 (9.9.2024)

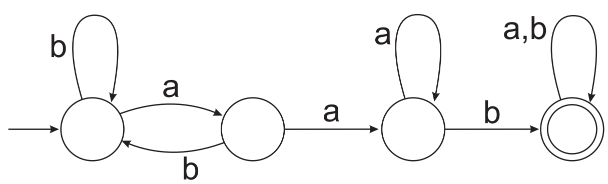
1. Find
   (a) two different words of length five that have exactly same proper subwords,
   (b) a word over alphabet $\{a, b\}$ that is not a concatenation of two palindromes,
   (c) a word of length five that contains as subword all words of length two over the alphabet $\{a, b\}$.

2. Construct a DFA that recognizes the language
   (a) $L = \{w \in \{a, b\}^* \mid \ w \text{ contains at least one } a\ \}$
   (b) $L = \{w \in \{a, b\}^* \mid \ w \text{ starts with } b \text{ or } ab\ \}$
   (c) $L = \{w \in \{a, b\}^* \mid \ w \text{ ends in } aa\ \}$

3. Describe the words accepted by the following two DFA:
   (a)



   (b)



4. A language $L \subseteq \Sigma^+$ is a *code* if

$$u_1 u_2 \ldots u_n = v_1 v_2 \ldots v_m \text{ where all } u_i, v_i \in L$$

implies that $n = m$ and $u_i = v_i$ for all $i = 1, \ldots n$. In other words, it is not possible to express any word in two different ways as a concatenation of elements of the code. Determine which of the following languages are codes.
   (a) $L = \{a, ba, ab\}$,
   (b) $L = \{ab, bab\}$,
   (c) $L = \{ab^n \mid n \geq 0\}$.

5. Let L be a regular language recognized by a DFA with n states.

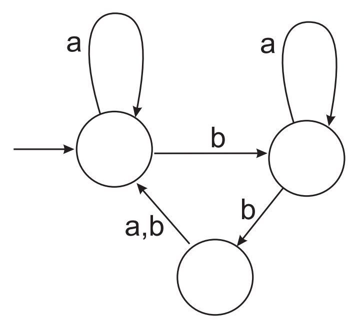
   (a) Prove that $L \neq \emptyset$ if and only if $\exists w \in L$ such that $|w| < n$.

   (b) Prove that L is infinite if and only if $\exists w \in L$ such that $n \leq |w| < 2n$.

6. Prove that if u and v are words such that $uv = vu$ then there exists a word w such that $u = w^n$ and $v = w^m$ for some $n, m \geq 0$. (Hint. Use mathematical induction on the length of the word.)

7. Let $A = (Q, \Sigma, \delta, q_0, F)$ be a DFA. Word $w \in \Sigma^*$ is called *synchronizing* for A if $\delta(q, w) = \delta(p, w)$ for all $q, p \in Q$.

   (a) Construct a DFA that recognizes the synchronizing words of the DFA



   (b) Prove that the synchronizing words of any DFA form a regular language.