Automata and Formal Languages. Homework 3 (30.9.2024)

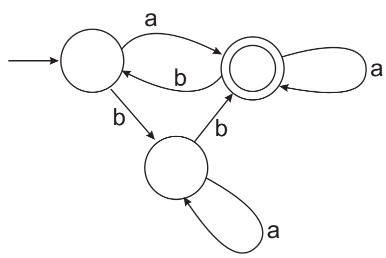
1. Find a regular expression that defines the language

   (a) $L = \{w \in \{a,b\}^* \mid\ w \text{ contains at least one } a \text{ and at least one } b\ \}$

   (b) $L = \{w \in \{a,b\}^* \mid\ w \text{ contains } ab \text{ or } ba \text{ (or both) as a subword}\ \}$

   (c) $L = \{w \in \{a,b\}^* \mid\ w \text{ contains both } ab \text{ and } ba \text{ as a subword}\ \}$

2. Use the language equation method to find a regular expression for the language recognized by the DFA



3. For any language L, let use define its reverse

$$L^R = \{w^R \mid w \in L\}$$

   to be the set of the mirror images of its words. Prove that if L is a regular language then L^R is also a regular language. (Hence the family of regular languages is closed under reversal.)

4. For $w \in \{0,1\}^*$, let us denote by w_2 the natural number whose binary expansion w is, that is,

$$(a_n a_{n-1} \dots a_1 a_0)_2 = a_0 + 2a_1 + 4a_2 + \dots + 2^n a_n.$$

   Construct NFA for the following languages.

   (a) $\{a^k \mid\ k \text{ is a multiple of } 3\ \}$,

   (b) $\{w \in \{0,1\}^* \mid\ w_2 \text{ is a multiple of } 3\ \}$

5. Let us generalize (b) of the previous problem. Show that for any fixed positive integer N the language

$$L_N = \{w \in \{0,1\}^* \mid N \text{ divides } w_2\ \}$$

   is a regular language.

6. Use the pumping lemma to prove that the following languages over the alphabet $\{a,b\}$ are not regular:

   (a) $\{w \in \{a,b\}^* \mid w^R = w\}$,

(b) $\{a^n b^m \mid m \leq n\}$.

7. Show that the following languages do not satisfy the pumping lemma and are hence not regular, (Do not use closure properties to modify the language!)

   (a) $\{a^n b^m \mid m \neq n\}$,

   (b) $\{a^n b^m \mid \ n$ and m are relatively prime, i.e., $gcd(n, m) = 1\ \}$. (Here, *gcd* stands for the greatest common divisor.)