Automata and Formal Languages. Homework 5 (14.10.2024)

1. Prove that taking the quotient of a regular language with a regular language is an effective operation. In other words, show that there is an algorithm to construct the regular language L/R when given as inputs regular languages L and R.

2. Show that there are algorithms to determine

   (a) if a given regular language L is prefix-closed, i.e., $uv \in L \Longrightarrow u \in L$,

   (b) if a given regular language L is prefix-free, i.e., $uv \in L, v \neq \varepsilon \Longrightarrow u \notin L$,

   (c) if all words of a given regular language L are non-squares, i.e, $ww \notin L$ for all $w \in \Sigma^*$.

   (Hint: Can you relate properties like $L/\Sigma^* \subseteq L$ or $L/\Sigma^+ \cap L = \emptyset$ or $\mathrm{Half}(L) = \emptyset$ to these questions ?)

3. Show that there are algorithms to determine

   (a) if a given regular language L is a code (see Problem 4 of the homework set 1 for the definition of codes).

   Hint: Use effective closure properties. Can you relate the language $L^*((L/L) \cap \Sigma^+) \cap L^*$ to this question ?

   (b) if a given NFA is unambiguous (see Problem 4 of the homework set 2 for the definition of unambiguity).

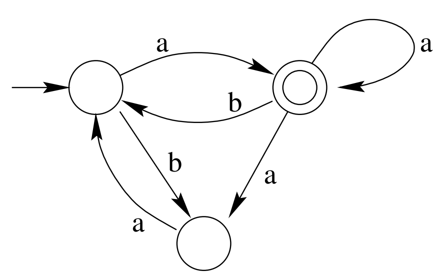
4. Let L be the language represented by the regular expression $a^*b + ba$.

   (a) Find regular expressions for the languages $\mathrm{Ext}(\varepsilon, L)$, $\mathrm{Ext}(a, L)$, $\mathrm{Ext}(ab, L)$, $\mathrm{Ext}(b, L)$ and $\mathrm{Ext}(bb, L)$.

   (b) Determine the equivalence classes of the relation R_L.

   (c) Based on (b), construct the minimum state DFA for the language L.

5. Find the minimum state DFA for the language recognized by the following NFA:



6. Let $A = (Q, \Sigma, \delta, q_0, F)$ be a DFA with $n = |Q|$ states. Recall that states $p, q \in Q$ are *distinguishable* if there is a word $w \in \Sigma^*$ such that $\delta(p, w) \in F$ and $\delta(q, w) \notin F$, or vice versa, and we say that word w distinguishes p and q. Prove that any distinguishable p and q are distinguished by a word whose length is at most $n - 2$.

7. Prove that for every $n \geq 1$ there exists an NFA with n states such that the equivalent minimum state DFA has 2^n states. (Hint: recall Problem 6 in the second homework set.)