Automata and Formal Languages. Homework 6 (Thursday 17.10.2024)

- **First exam on Tuesday, Oct 22, at 10:15-13:15 in room M2. Material: lecture notes up to page 76 (beginning of Section 3.5).**
- **No lectures or demos on the week of the exam.**

1. Construct context-free grammars that generate the following languages:
   (a) $(ab + ba)^*$,
   (b) $\{(ab)^n a^n \mid n \geq 1\}$,
   (c) $\{w \in \{a, b\}^* \mid\ w \text{ is a palindrome }\}$,
   (d) $\{w \in \{a, b\}^* \mid\ w \text{ contains exactly two } b\text{'s and any number of } a\text{'s }\}$.

2. Construct context-free grammars that generate the following languages:
   (a) $\{a^n b^m \mid n \leq m \leq 2n\}$,
   (b) $\{a^n b^m \mid m \neq n \text{ and } m \neq 2n\}$.

3. It is easy to see (and will be proved in the class) that the family of context-free languages is closed under union, concatenation and the Kleene closure. Use this fact to give another proof for the fact that every regular language is context-free.

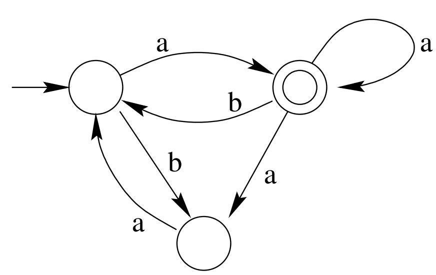
4. Consider the grammar $G = (\{S, A, B\}, \{a, b\}, P, S)$ with productions

$$\begin{array}{lcl} S & \longrightarrow & SAB \mid \varepsilon \\ A & \longrightarrow & aA \mid \varepsilon \\ B & \longrightarrow & Bb \mid \varepsilon \end{array}$$

   (a) Give a leftmost derivation for $aaba$.
   (b) Draw the derivation tree corresponding to the derivation in (a).

5. Are the following grammars ambiguous or unambiguous ? Prove.
   (a) $G = (\{S\}, \{a, b\}, P, S)$ with productions $S \longrightarrow aSa \mid aSbSa \mid \varepsilon$.
   (b) $G = (\{S\}, \{a, b\}, P, S)$ with productions $S \longrightarrow aaSb \mid abSbS \mid \varepsilon$.

6. Let L be the language recognized by the following NFA:



   (a) Construct a context-free grammar G that generates language L.

(b) Is your grammar G ambiguous or unambiguous ? Prove.

(c) Prove that no regular language is inherently ambiguous.

(d) Construct an unambiguous context-free grammar for L.

7. Let $G = (\{S, A, B\}, \{a, b\}, P, S)$ be the context-free grammar with productions

$$\begin{array}{lcl} S & \longrightarrow & aAbS \mid bBaS \mid \varepsilon, \\ A & \longrightarrow & aAbA \mid \varepsilon, \\ B & \longrightarrow & bBaB \mid \varepsilon, \end{array}$$

in P.

(a) Prove that G is unambiguous.

(b) Prove that $L(G) = \{w \in \{a, b\}^* \mid \; w \text{ contains equally many } a\text{'s and } b\text{'s} \}$.