Automata and Formal Languages. Homework 7 (28.10.2024)

1. Construct a Chomsky normal form grammar that is equivalent to the grammar $G = (\{S, A, B, C, D, E, F\}, \{a, b\}, P, S)$ where P contains productions

$$\begin{array}{lcl} S & \longrightarrow & DC \mid aCb \\ A & \longrightarrow & CC \mid AB \\ B & \longrightarrow & Saa \mid \varepsilon \\ C & \longrightarrow & \varepsilon \\ D & \longrightarrow & SB \\ E & \longrightarrow & aF \mid EA \\ F & \longrightarrow & bEa \end{array}$$

2. Let G be a grammar, and let $w \in L(G)$ be of length $n \geq 1$. Determine the length of the derivations of w if
   - (a) G is in the Chomsky normal form,
   - (b) G is in the Greibach normal form.

3. Let $M = (\{q, f\}, \{a, b\}, \{Z_0, A\}, \delta, q, Z_0, \emptyset)$ be a PDA where all non-empty values of δ are given below:

$$\begin{array}{lcl} \delta(q, a, Z_0) & = & \{(q, AZ_0), (q, Z_0)\} \\ \delta(q, a, A) & = & \{(q, AA)\} \\ \delta(q, \varepsilon, A) & = & \{(f, A)\} \\ \delta(f, b, A) & = & \{(f, \varepsilon)\} \\ \delta(f, \varepsilon, Z_0) & = & \{(f, \varepsilon)\} \end{array}$$

   - (a) Write down an accepting calculation (the ID's) for input word $aaaabb$, using the empty stack acceptance mode. Is M deterministic ?
   - (b) Which of the following words are in $N(M)$: bab, abb, aab ? Describe in English the language $N(M)$.

4. Construct a PDA that recognizes (in the acceptance mode of your choice) the language

$$L = \{w \in \{a, b\}^* \mid \ w \text{ contains equally many } a\text{'s and } b\text{'s}\}.$$

5. Let $G = (\{S\}, \{a, b\}, P, S)$ be a grammar where P contains productions $S \longrightarrow aSS \mid b \mid \varepsilon$.
   - (a) Use the construction in the proof of Theorem 78 to find a single state PDA M that recognizes the language $L(G)$ with the empty stack acceptance mode.
   - (b) Write an accepting calculation (all ID's) for input $ababb$, using the PDA you constructed in (a).
   - (c) Use the construction in Lemma 82 to find convert the PDA you constructed in (a) back into an equivalent context-free grammar G'.

(d) Write down a leftmost derivation for $ababb$ using the new grammar G'.

6. Consider the Greibach normal form grammar $G = (\{S, A, B\}, \{a, b\}, P, S)$, with productions

$$\begin{array}{rcl} S & \longrightarrow & aABS \mid bS \mid b \\ A & \longrightarrow & aBA \mid bS \\ B & \longrightarrow & bAS \end{array}$$

in P. Construct a PDA A with a single state and without any ε-moves such that $N(A) = L(G)$.

7. Let $G = (V, T, P, S)$ be a grammar in the Greibach normal form. Show how to construct an equivalent grammar whose productions are all of the forms

$$A \longrightarrow a, \qquad A \longrightarrow aB \qquad \text{or} \qquad A \longrightarrow aBC$$

where $A, B, C \in V$ and $a \in T$. (Hint: a good choice for new variables might be $\{[w] \mid w \in V^+ \text{ and } |w| < n\}$ where n is the length of the longest right side of a production in P.)