Tilings and Patterns: Homework 3 (29.9.2025)

1. Determine the symmetry groups of the following seven strip patterns. (The patterns continue to infinity at both ends):

   pbpbpbpbpbpbpbpbpbpbpbpbpbpbpbpbpbpbpbpbpbpbpbpb

   HHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHH

   FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF

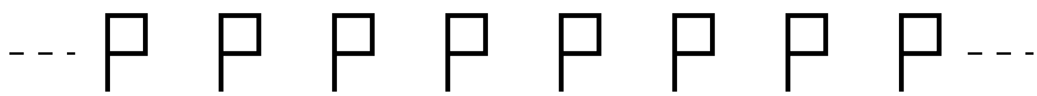
   SSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS

   MWMWMWMWMWMWMWMWMWMWMWMWMWMWMWMWMWMWMWMWM

   DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD

   AAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAA

2. The following pattern has the symmetry group F_{0000} (using the notation from the lectures). For every frieze group F different from F_{0000}, add as little as you can to make it into a pattern whose symmetry group is F.



3. Let ρ be a rotation and P a point. Prove that there is a line ℓ such that for all rotations ρ' with center P, if $\rho'\rho$ is a rotation then its center of rotation is on line ℓ.

4. A subgroup N of group G is called normal if $gng^{-1} \in N$ for every $g \in G$ and $n \in N$. Determine which of the following subgroups of $\mathcal{I}$, the set of plane isometries, are normal.

   (a) The group of translations,

   (b) the group of even isometries,

   (c) the group of rotations around a fixed point P.

5. Construct all those subgroups G of $\mathcal{I}$ that contain all horizontal translations and no other translations. In other words, $\tau_A \in G$ if and only if $A = (r, 0)$ for some $r \in \mathbb{R}$. (Hint: Lemma 2.26 from the notes applies to G.)

6. Given acute triangle $\triangle ABC$, find points P, Q and R on sides AB, AC and BC, respectively, such that $\triangle PQR$ has minimal perimeter. (Recall: acute triangle is a triangle all of whose angles are less than $90°$.) Hint: we solved the problem for a fixed P in assignment 2, problem 6. All you have to do is to determine the optimal point P...

7. (a) Let $A, B, C, D \in \mathbb{R}^2$ be such that $d(A, B) = d(C, D)$. Prove that there exists an isometry α such that $\alpha(A) = C$ and $\alpha(B) = D$.

   (b) Determine the conjugacy classes of the group $\mathcal{I}$ of plane isometries, that is, find the equivalence classes of mutually conjugate isometries.