Tilings and Patterns: Homework 4 (6.10.2025)

1. The reverse side of this paper contains six wallpaper patterns. Determine the symmetry group of each pattern.

2. (a) Let γ be a glide reflection and ρ a rotation by 120°. Show that $\gamma\rho\gamma\rho^{-1}\gamma$ is a reflection.

   (b) Let ρ be the rotation by 60° around point P. Show that $\tau\rho$ is a rotation by 60° around point $\rho\tau(P)$.

3. Show that the wallpaper group W_3^1 is generated by three reflections.

4. The *orbit* of point P under group G is the set $\{\alpha(P) \mid \alpha \in G\}$.

   (a) Determine the orbit of P under the wallpaper group $G = W_3$, where P is a center of a 3-fold rotation. Then determine the symmetry group of the orbit.

   (b) For all rosette groups C_n and D_n, determine the sizes (=number of elements) of all orbits.

5. Determine all possible symmetry groups that a finite set of 7 points can have.

6. Let us define a fundamental domain of a symmetry group G as follows: it is a compact $s \subseteq \mathbb{R}^2$ that has a nonempty intersection with the orbit of every point, and no interior point of s belongs to the same orbit with another elements of s. (The fundamental domain hence contains representatives from all orbits, and only the boundary points of the fundamental domain may belong to the same orbit.)

   Find some fundamental domains for W_6 and W_4.

In this class we are mostly dealing with tilings of the infinite plane. However, many neat "puzzle" -type problems can be posed concerning finite tilings. Several examples will be included in the homework problems in the coming weeks. We start with the following classical example.

7. A domino is the 1×2 rectangular tile obtained by gluing two unit squares into each other. Given an 8×8 checkerboard with a pair of diagonally opposite corner squares removed, prove that it is not possible to tile (i.e., to cover completely without overlaps) this board with dominoes, each of which covers exactly two squares.