Tilings and Patterns: Homework 6 (Thursday 16.10.2025)

1. Let us define edge-transitivity as follows: an edge-to-edge tiling by polygons is edge-transitive iff for every pair of edges there is a symmetry of the tiling that takes the first edge onto the second edge. We know that the archimedean tilings are vertex-transitive. Determine which archimedean tilings are edge-transitive.

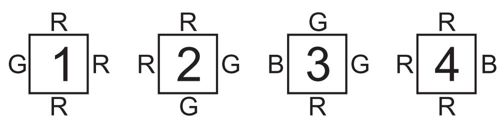
2. A dual of an archimedean tiling is obtained by joining the centers of adjacent tiles. Clearly the dual of every archimedean tiling is monohedral. Draw the duals of the archimedean tilings of types $3{\cdot}3{\cdot}4{\cdot}3{\cdot}4$ and $3{\cdot}6{\cdot}3{\cdot}6$. Are these duals isohedral ? Does there exist an archimedean tiling whose dual is not isohedral ?

3. Prove that there does not exist an edge-to-edge tiling by regular polygons that contains an n-gon with $n > 12$.

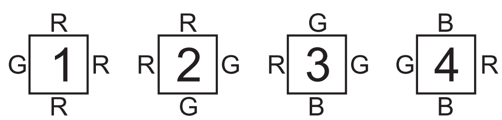
4. Prove that every pentagon with two parallel sides is a prototile of a monohedral tiling.

5. Determine if the given Wang prototile sets (4 tiles, 3 colors) admit valid tilings of $\mathbb{Z}^2$:

   (a)



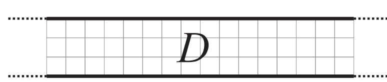
   (b)



6. Let A be a finite set of Wang prototiles, and let $D \subseteq \mathbb{Z}^2$ (possibly infinite). Let $p : D \longrightarrow A$ be a valid tiling of the region D, meaning that any two neighboring tiles in D match in color. For all n, denote $D_n = \{-n, \ldots, n\} \times \{-n, \ldots, n\}$ for the size $(2n+1) \times (2n+1)$ square centered at the origin $(0,0)$. Suppose that for all n there is a valid tiling $f_n : D_n \longrightarrow A$ of D_n such that $f_n(x,y) = p(x,y)$ for all $(x,y) \in D \cap D_n$.

   Prove that p can be extended into a valid tiling of the whole grid $\mathbb{Z}^2$, *i.e.*, that there exists a valid tiling $t : \mathbb{Z}^2 \longrightarrow A$ such that $t|_D = p$.

7. Suppose that a finite protoset A of Wang tiles can tile an infinite horizontal strip $D \subseteq \mathbb{Z}^2$ such that the neighboring tiles match in color inside the strip, and exactly the same finite segments of colors appear on the top and on the bottom boundaries of the strip.



   (In other words: every finite segment of colors on the upper border of the strip appears also somewhere on the lower border of the strip, and vice versa.)

   Prove that this tiling of D can be extended into a valid tiling of the whole grid $\mathbb{Z}^2$.