Cellular Automata. Homework 12 (20.4.2026)

1. Let G be a surjective CA. Prove that the attractors of G are exactly those non-empty clopen sets U that satisfy $G(U) = U$. Also prove that if U is an attractor then its complement $S^{\mathbb{Z}^d} \setminus U$ is an attractor or the empty set. (Hint: use the balanceness property to prove that in surjective CA all clopen inward sets U satisfy $G^{-1}(U) = U$.)

2. An attractor A that is a subshift (i.e. $\tau(A) = A$ for every translation τ) is called a *subshift attractor*.

   (a) Prove that if a CA has a minimal attractor A then A is a subshift attractor.

   (b) Prove that the basin of attraction for any subshift attractor is dense. (In other words, every cylinder contains a configuration that is in the basin of attraction.)

3. Determine all equicontinuity points of the traffic CA (=elementary CA 226), and determine if the CA is sensitive to initial conditions.

4. Determine all equicontinuity points of the majority CA (=elementary CA 232), and determine if the CA is sensitive to initial conditions.

5. Suppose G is a CA whose neighborhood vector contains $\vec{0}$ and all unit coordinate vectors $(0, \dots 0, \pm 1, 0, \dots 0)$. Suppose also that G has a *spreading state* $q \in S$ with the property that $f(x_1, x_2, \dots, x_m) = q$ if some $x_i = q$.

   (a) Show that G is not sensitive.

   (b) Show that G is equicontinuous if and only if it is nilpotent.

   (c) Show that G has a minimal attractor, and determine that attractor.

6. Let G be the elementary CA number 108.

   (a) Show that 00 is a 2-blocking word and that 01110 is a 3-blocking word.

   (b) Show that words 1111, 0110110, 010110, 011010 and 10101 are 2-blocking.

   (c) Show that all words of length 7 are 2-blocking.

   (d) Determine all equicontinuity points of G. Is G stable ?

7. Let G be the following one-dimensional *Coven's CA*. The state set is $S = \{0, 1\}$, the neighborhood is $N = (0, 1, 2)$, and the local rule f is defined by $f(a, 1, 0) = 1 - a$ for all $a \in S$ and $f(a, b, c) = a$ for $a, b, c \in S$ if $bc \neq 10$. In other words, a cell swaps its bit if and only if it is followed by the pattern 10 in the two next cells on the right.

   Prove that 000 is a 2-blocking word.

   (Hint: Prove that, for every $n \in \mathbb{N}$, the only pre-image pattern of $001^{2n+1}0$ that begins with 00 on the left is $001^{2n+3}0$. Conclude that the two leftmost cells of pattern 000 remain indefinitely in states 0.)