

The **basin of attraction** of an attractor A is the set

$$\mathcal{B}_A = \{c \in S^{\mathbb{Z}^d} \mid \lim_{n \rightarrow \infty} d(G^n(c), A) = 0\}$$

where

$$d(x, A) = \min \{d(x, a) \mid a \in A\}.$$

Proposition. Let A be an attractor determined by inward clopen U , and let \mathcal{B} be its the basin of attraction. Then

(a) $G(\mathcal{B}) \subseteq \mathcal{B}$ and $G(S^{\mathbb{Z}^d} \setminus \mathcal{B}) \subseteq S^{\mathbb{Z}^d} \setminus \mathcal{B}$,

(b) $U \subseteq \mathcal{B}$. In particular, for all $c \in S^{\mathbb{Z}^d}$,

$$\lim_{n \rightarrow \infty} d(G^n(c), \Omega_G) = 0,$$

(c) $\mathcal{B} = \bigcup_{n=0}^{\infty} G^{-n}(U)$,

(d) \mathcal{B} is open,

(e) \mathcal{B} identifies A uniquely: for attractors A_1, A_2 holds

$$A_1 \neq A_2 \implies \mathcal{B}_{A_1} \neq \mathcal{B}_{A_2}.$$