

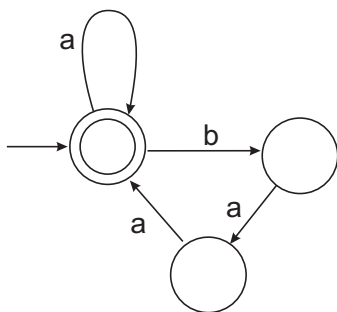
Automata and Formal Languages. Homework 2 (21.9.2026)

1. Construct an NFA (with as few states as you can) that recognizes the following languages:

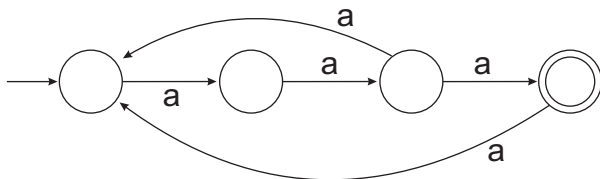
- (a) $L = \{w \in \{a, b\}^* \mid w \text{ contains at most two } a\text{'s}\}$
- (b) $L = \{w \in \{a, b\}^* \mid w \text{ contains an even number of occurrences of } ab \text{ as a subword}\}$
- (c) $L = \{w \in \{a, b\}^* \mid \text{the first and the last letter of } w \text{ are identical}\}$

2. Describe the words accepted by the following two NFA:

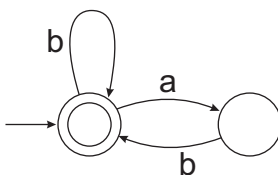
(a)



(b)

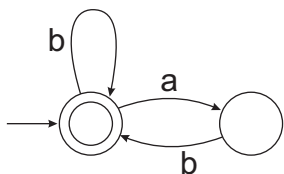


3. Determine how many different words of length n does the following NFA accept:

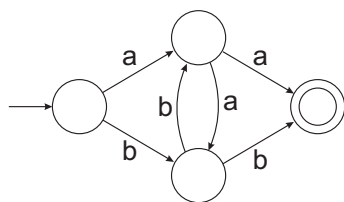


4. An NFA for language L is called *unambiguous* if every $w \in L$ has exactly one accepting computation path. Determine which of the following three NFA are unambiguous:

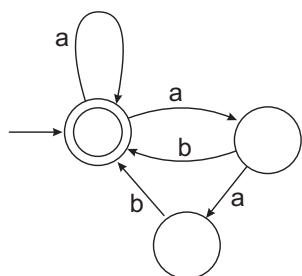
(a)



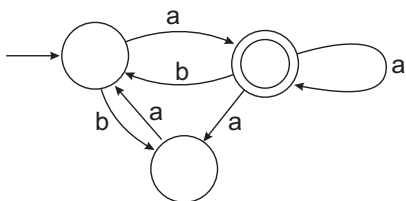
(b)



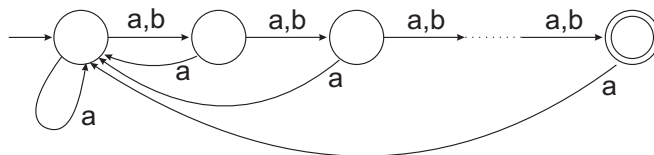
(c)



5. Use the powerset construction to construct a DFA that recognizes the same language as the following NFA:



6. The NFA in problem 5 above has three states, and the DFA obtained after the powerset construction contains $8 = 2^3$ states, all of which are reachable from the initial state. Prove that for every positive integer n there exists an n -state NFA with input alphabet $\{a, b\}$ such that the power set construction yields a 2^n -state DFA, whose states are all reachable from the initial state. (Hint: Consider, for example, the n -state NFA below.)



7. Recall the definition of synchronizing words from the homework of last week. Let $A = (Q, \Sigma, \delta, q_0, F)$ be an n -state DFA that has synchronizing words.

- (a) Prove that for any $q, p \in Q$ there exists a word w such that $|w| \leq \frac{n(n-1)}{2}$ and $\delta(q, w) = \delta(p, w)$.
- (b) Prove that there exists a synchronizing word whose length is at most

$$\frac{n(n-1)(n-2)}{2} + 1.$$