

Automata and Formal Languages. Homework 6 (Thursday 15.10.2026)

- First exam on Monday, October 19, at 9:30-12:30 in room M3. Material: lecture notes up to page 76 (beginning of Section 3.5).
- No lectures or demos on the week of the exam.

1. Construct context-free grammars that generate the following languages:

- (a) $(ab + ba)^*$,
- (b) $\{(ab)^n a^n \mid n \geq 1\}$,
- (c) $\{w \in \{a, b\}^* \mid w \text{ is a palindrome}\}$,
- (d) $\{w \in \{a, b\}^* \mid w \text{ contains exactly two } b\text{'s and any number of } a\text{'s}\}$.

2. Construct context-free grammars that generate the following languages:

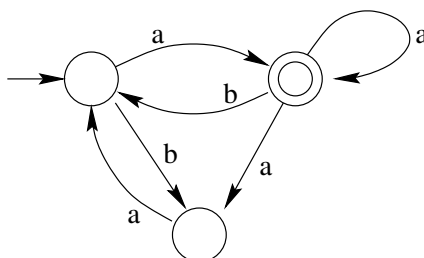
- (a) $\{a^n b^m \mid n \leq m \leq 2n\}$,
- (b) $\{a^n b^m \mid m \neq n \text{ and } m \neq 2n\}$.

3. It is easy to see (and will be proved in the class) that the family of context-free languages is closed under union, concatenation and the Kleene closure. Use this fact to give another proof for the fact that every regular language is context-free.

4. Consider the grammar $G = (\{S, A, B\}, \{a, b\}, P, S)$ with productions

$$\begin{aligned} S &\longrightarrow SAB \mid \varepsilon \\ A &\longrightarrow aA \mid \varepsilon \\ B &\longrightarrow Bb \mid \varepsilon \end{aligned}$$

- (a) Give a leftmost derivation for $aaba$.
 - (b) Draw the derivation tree corresponding to the derivation in (a).
5. Are the following grammars ambiguous or unambiguous? Prove.
- (a) $G = (\{S\}, \{a, b\}, P, S)$ with productions $S \longrightarrow aSa \mid aSbSa \mid \varepsilon$.
 - (b) $G = (\{S\}, \{a, b\}, P, S)$ with productions $S \longrightarrow aaSb \mid abSbS \mid \varepsilon$.
6. Let L be the language recognized by the following NFA:



- (a) Construct a context-free grammar G that generates language L .

- (b) Is your grammar G ambiguous or unambiguous ? Prove.
- (c) Prove that no regular language is inherently ambiguous.
- (d) Construct an unambiguous context-free grammar for L .

7. Let $G = (\{S, A, B\}, \{a, b\}, P, S)$ be the context-free grammar with productions

$$\begin{aligned} S &\longrightarrow aAbS \mid bBaS \mid \varepsilon, \\ A &\longrightarrow aAbA \mid \varepsilon, \\ B &\longrightarrow bBaB \mid \varepsilon, \end{aligned}$$

in P .

- (a) Prove that G is unambiguous.
- (b) Prove that $L(G) = \{w \in \{a, b\}^* \mid w \text{ contains equally many } a\text{'s and } b\text{'s}\}$.