X-ray binaries

Accretion onto a compact object

- Principal mechanism for producing high-energy radiation
- Most efficient method for energy production known in the Universe.

$$\boxed{E_{acc} = G\frac{Mm}{R}}$$

Gravitational potential energy released for body of mass *M* and radius *R* when mass *m* is accreted

$\eta = E_{acc}/mc^2 = GM/Rc^2 = R_s/2R$ - accretion efficiency

Origin of accreted matter

- Given *M/R*, luminosity produced depends on accretion rate $\dot{M}$

$$L_{acc} = \frac{dE_{acc}}{dt} = \frac{GM}{R}\frac{dM}{dt} = \frac{GM\dot{M}}{R}$$

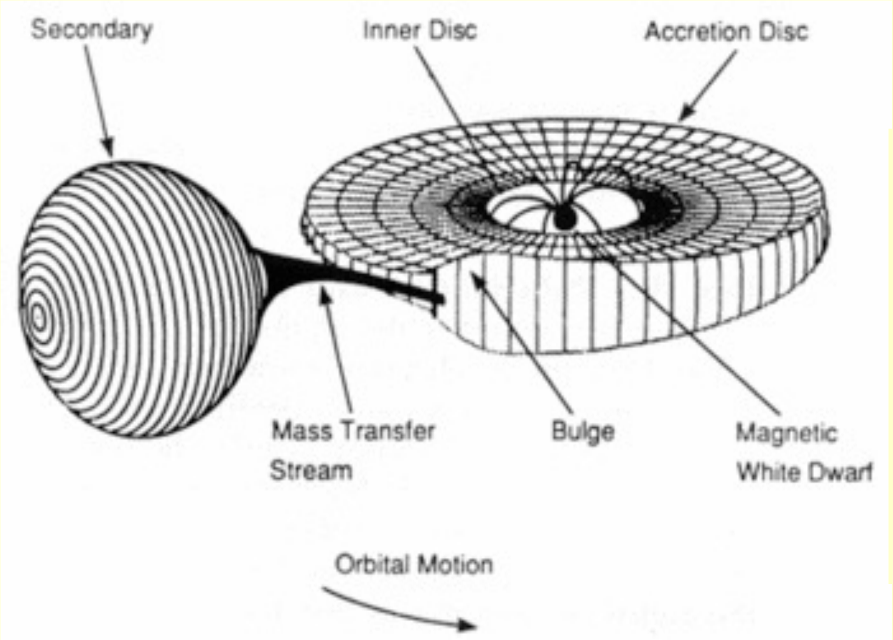
- Where does accreted matter come from? ISM?
  No – captured mass is too small.
  Companion? Yes.

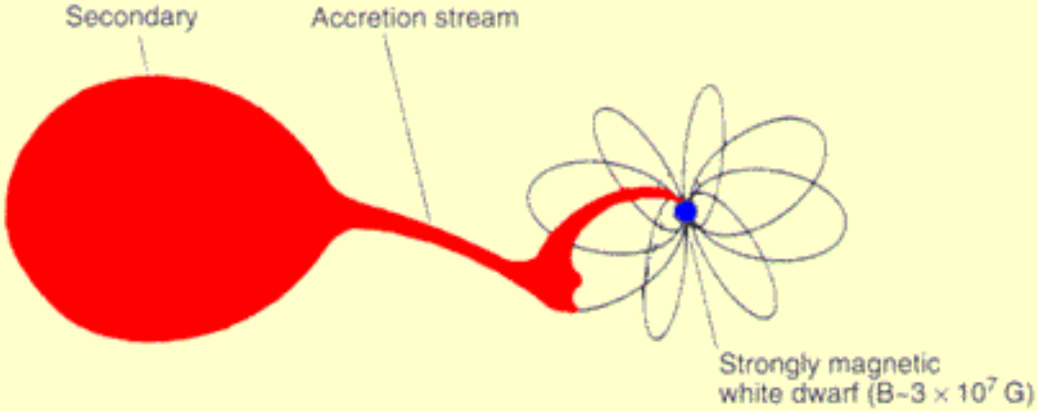
Classification of X-ray binaries

- Based on the nature of the compact object: white dwarf, neutron star, black hole.
- Based on the nature of the companion star: low-mass X-ray binaries (LMXRB) with M_n<1$M_\odot$ and high-mass X-ray binaries (HMXRB) with M_n> a few $M_\odot$.

White dwarfs

	Dwarf novae, cataclysmic variables (about 2000 known)	Intermediate polars (about 100 known; see https://asd.gsfc.nasa.gov/Koji.Mukai/iphome/catalog/alpha.html)	AM Her systems= polars
Magnetic field	weak	intermediate	strong
Accretion disk	yes	partial	no





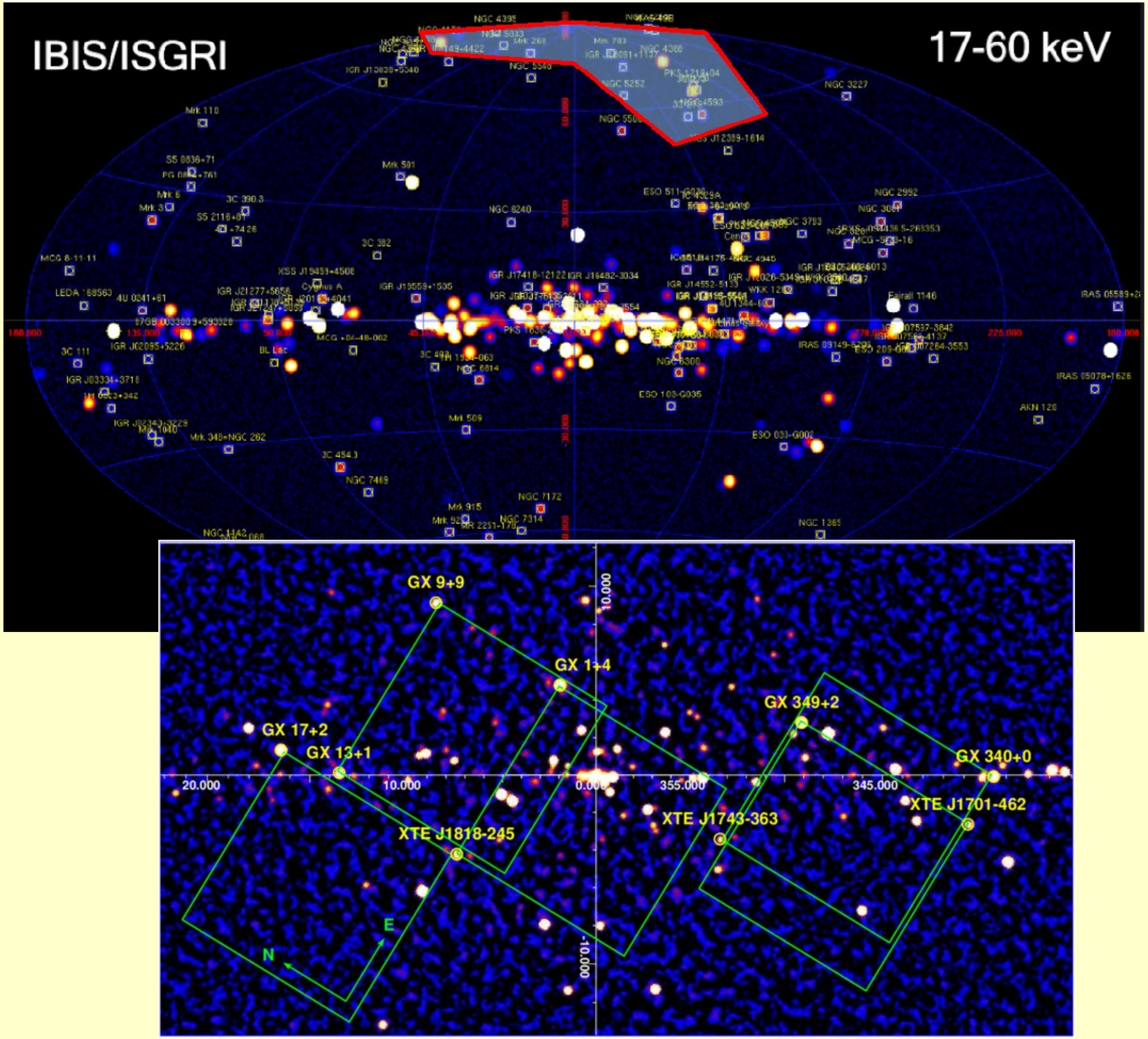
Intermediate polar: Matter flows from the companion star into an accretion disk around the white dwarf, but is disrupted by the white dwarf's magnetic field.

Polar: Matter flows from the companion star and is captured by the strong magnetic field of the white dwarf.
No accretion disk is formed.

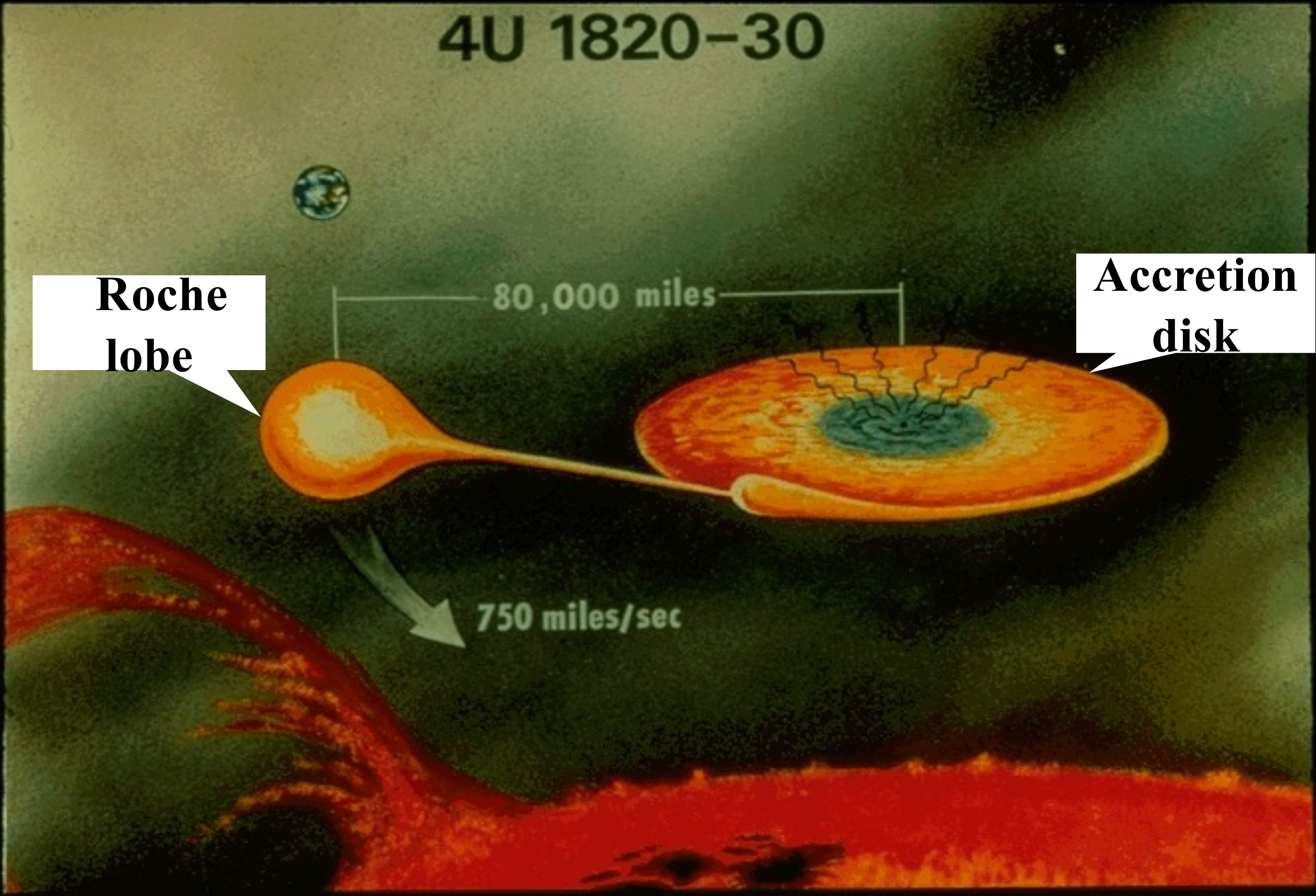
Neutron stars

	LMXRB (low-mass X-ray binaries)	HMXRB (high-mass X-ray binaries)
Magnetic field	Typically weak	Typically strong
Accretion disk	yes. Roche lobe overflow	no. Wind accretion or passing through Be decression disk.
Galactic population	Bulge. Old late-type companions	Disk, galactic arms. Young companions.
Features	quasi-periodic oscillations and ms pulsations	X-ray pulsars
Examples	Sco X-1 SAX J1808.4-3658 (2.5 ms)	Vela X-1 (283 sec) Her X-1 (1.2 sec) Cen X-3 (4.8 sec)

INTEGRAL all-sky survey



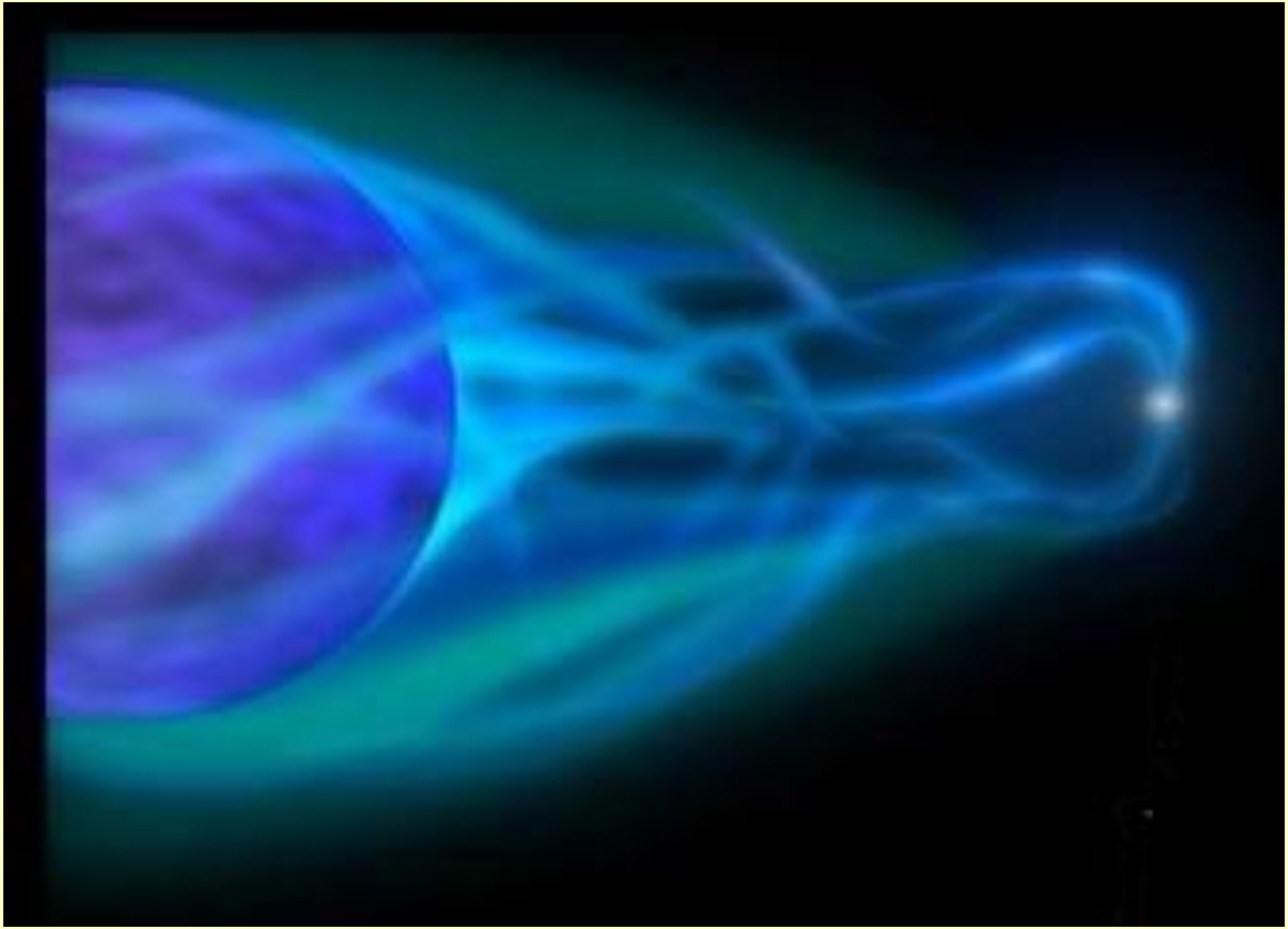
Low-mass X-ray binaries



Roche lobe

Accretion disk

High-mass X-ray binaries



Black holes

	LMXRB	HMXRB
Observational appearance	transients (X-ray novae)	persistent sources
Accretion disk	yes	small?
Accretion type	Roche lobe overflow	Wind (and Roche lobe overflow in some systems)
Examples	GRO J1655−40 GRS 1915+105 V404 Cyg MAXI J1820+070	Cyg X-1 Cyg X-3 ? SS 433 ?

Potential in a rotating frame

Equation of motion in a rotating frame:

$$\frac{d\vec{V}}{dt} = \frac{\partial \vec{V}}{\partial t} + \vec{V} \cdot \nabla \vec{V} = -\nabla \Phi - 2\vec{\Omega} \times \vec{V} - \vec{\Omega} \times (\vec{\Omega} \times \vec{r}) \qquad \left(-\frac{\nabla P}{\rho}\right)$$

Coriolis force

Centrifugal force

For two point masses: $\Phi = -\frac{GM_1}{|\vec{r}-\vec{r}_1|} - \frac{GM_2}{|\vec{r}-\vec{r}_2|}$

$$\vec{\Omega} \times (\vec{\Omega} \times \vec{r}) = \vec{\Omega}(\vec{\Omega} \cdot \vec{r}) - \vec{r}\Omega^2 = -\Omega^2 \vec{\varpi} = -\nabla\left(\frac{1}{2}\Omega^2 \varpi^2\right) = -\nabla\left(\frac{1}{2}(\vec{\Omega} \times \vec{r})^2\right)$$

cylindrical radius

Equation of motion: $\frac{d\vec{V}}{dt} = -\nabla \Phi_R - 2\vec{\Omega} \times \vec{V}$

where Roche potential:

$$\Phi_R = -\frac{GM_1}{|\vec{r}-\vec{r}_1|} - \frac{GM_2}{|\vec{r}-\vec{r}_2|} - \tfrac{1}{2}(\vec{\Omega} \times \vec{r})^2$$

Roche lobe

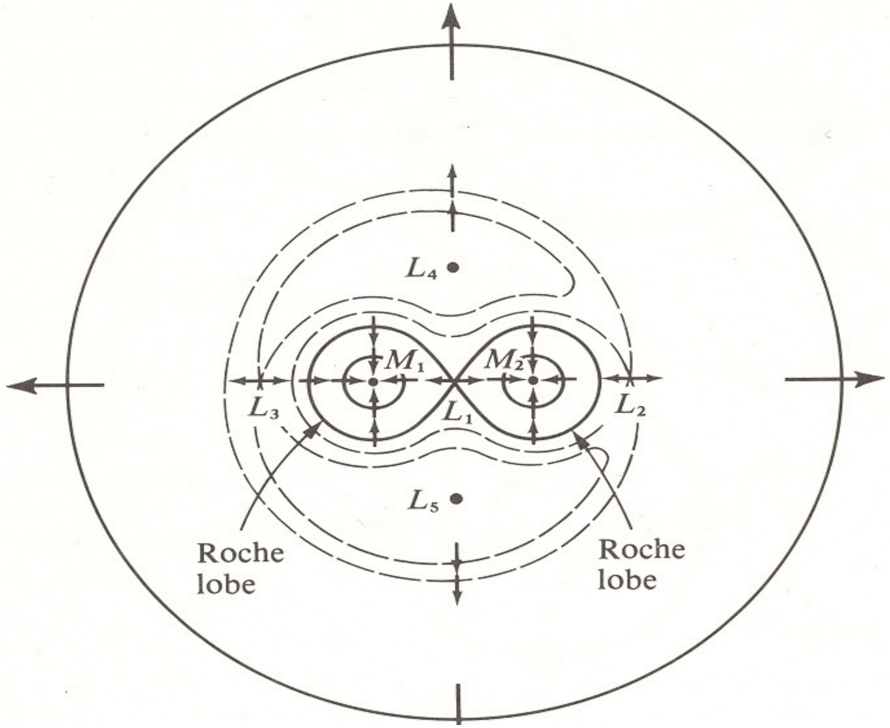
Multiply equation of motion with $\vec{V}\cdot$ $\Rightarrow \frac{dV^2/2}{dt} = -\vec{V}\cdot\nabla\Phi_R = -\frac{d\Phi_R}{dt} \Rightarrow$

$$\Rightarrow \frac{V^2}{2} + \Phi - \frac{\Omega^2\varpi^2}{2} = E_J = \text{const (Jacobi's integral)}$$

L_1, L_2, and L_3 are saddle points and are unstable

L_4 and L_5 are maxima but stable due to Coriolis force

Roche Lobe = equipotential surface (figure 8)



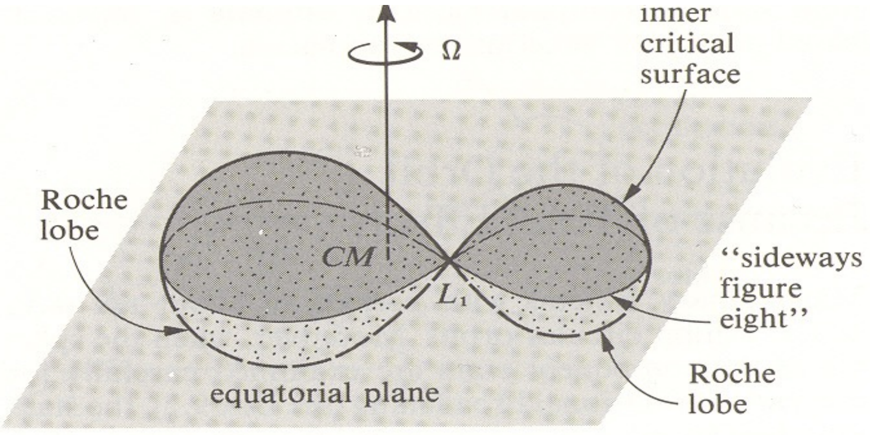


Figure 10.8. A schematic depiction of the three-dimensional structure of the Roche lobes of a binary system with a circular orbit. The rotation of the entire system takes place with angular speed Ω with respect to an axis which passes through the center of mass CM. This axis is perpendicular to the equatorial plane of the system.

Orbit and the Roche lobe size

Size of the orbit. From Kepler's 3rd law

$$4\pi^2 a^3 = G(m_1 + m_2)M_\odot P^2$$

$$a = 1.5\times10^{13} m_1^{1/3}\,(1+q)^{1/3} P_{yr}^{2/3}\,\mathrm{cm}$$

$$a = 2.9\times10^{11} m_1^{1/3}\,(1+q)^{1/3} P_{day}^{2/3}\,\mathrm{cm}$$

$$a = 3.5\times10^{10} m_1^{1/3}\,(1+q)^{1/3} P_{hr}^{2/3}\,\mathrm{cm}$$

$q = M_2/\,M_1$ - mass ratio

Size of the Roche lobe:

$$\frac{R_2}{a} = \frac{0.49q^{2/3}}{0.6q^{2/3} + \ln(1+q^{1/3})}$$

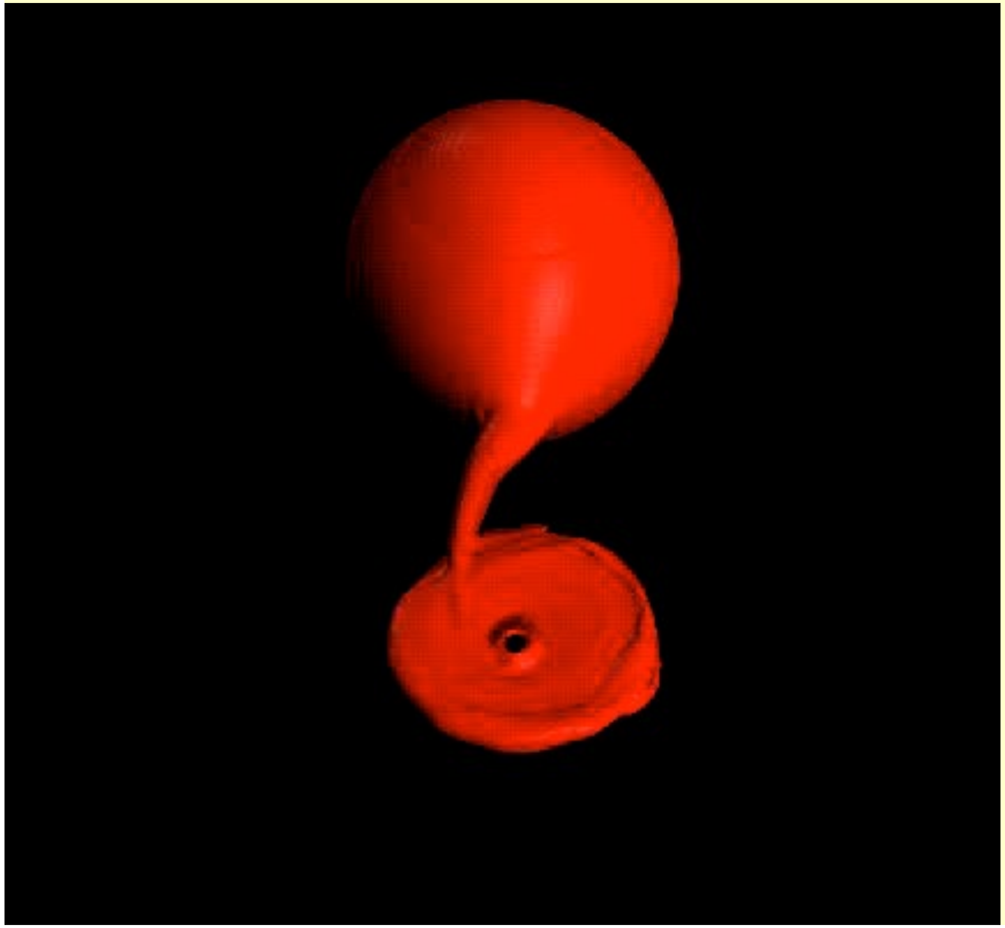
(Eggleton 1983);
for the primary replace q by $1/q$

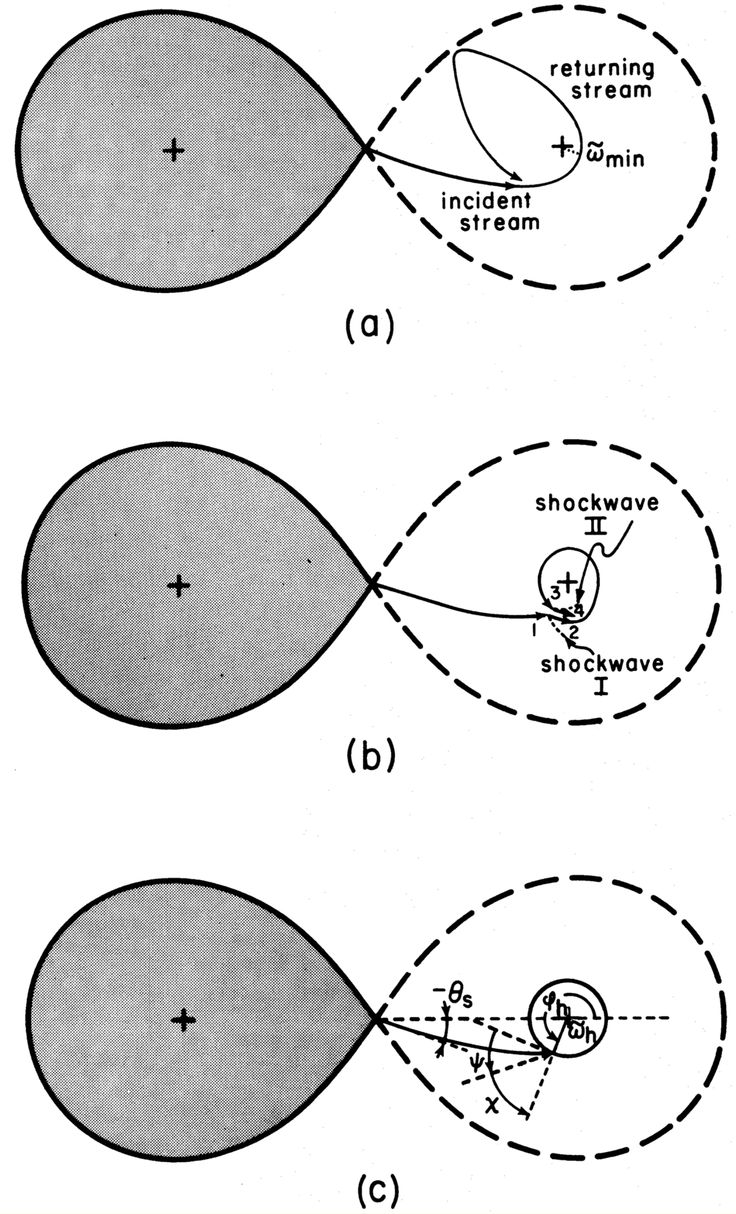
$$\frac{R_2}{a} = \frac{2}{3^{4/3}}\left(\frac{M_2}{M_1+M_2}\right)^{1/3} = 0.462\left(\frac{q}{1+q}\right)^{1/3}$$

(Paczynski 1971)

Accretion via Roche lobe

- Stream trajectory and disk formation for the case of equal masses.
- Coriolis force shifts the stream from the line connecting the stars.





Accretion in Be transients

- A neutron star in a binary system containing a Be star undergoes periodic accretion episodes when it crosses the decretion disk around a Be star.

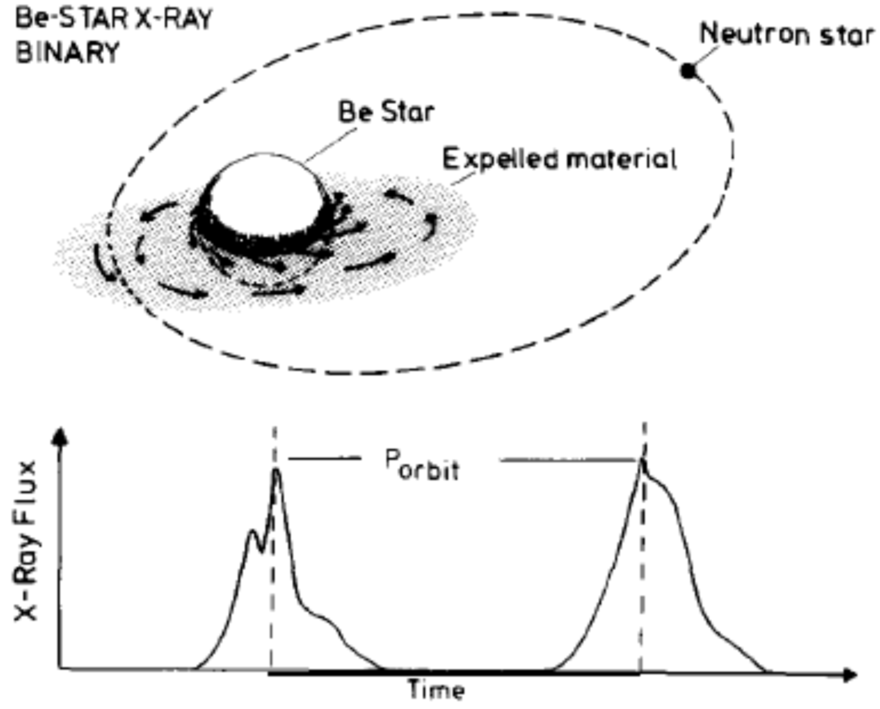


Fig. 17. Schematic model of a Be star X-ray binary system such as X0535+26 and X0332+53. The neutron star moves in a moderately eccentric orbit around the Be star, which is much smaller than its own critical equipotential lobe. The rapidly rotating Be star is temporarily surrounded by matter expelled in its equatorial plane. Near its periastron passage the neutron star enters this circumstellar matter and the resultant accretion produces an X-ray outburst lasting several days to weeks.

Wind accretion

- Stellar wind from OB stars. Mass loss rate is

$\dot{M}_w = 10^{-8}$- $10^{-5} M_\odot$/year

(cf. $\dot{M}_\odot = 3\times10^{-14}\ M_\odot$/year

$$\frac{mV_{rel}^2}{2} = \frac{GmM_x}{R_{\rm acc}}$$

V_{rel} -relative velocity of a compact object and a stellar wind

$$V_{rel}^2 = V^2 + V_w^2$$

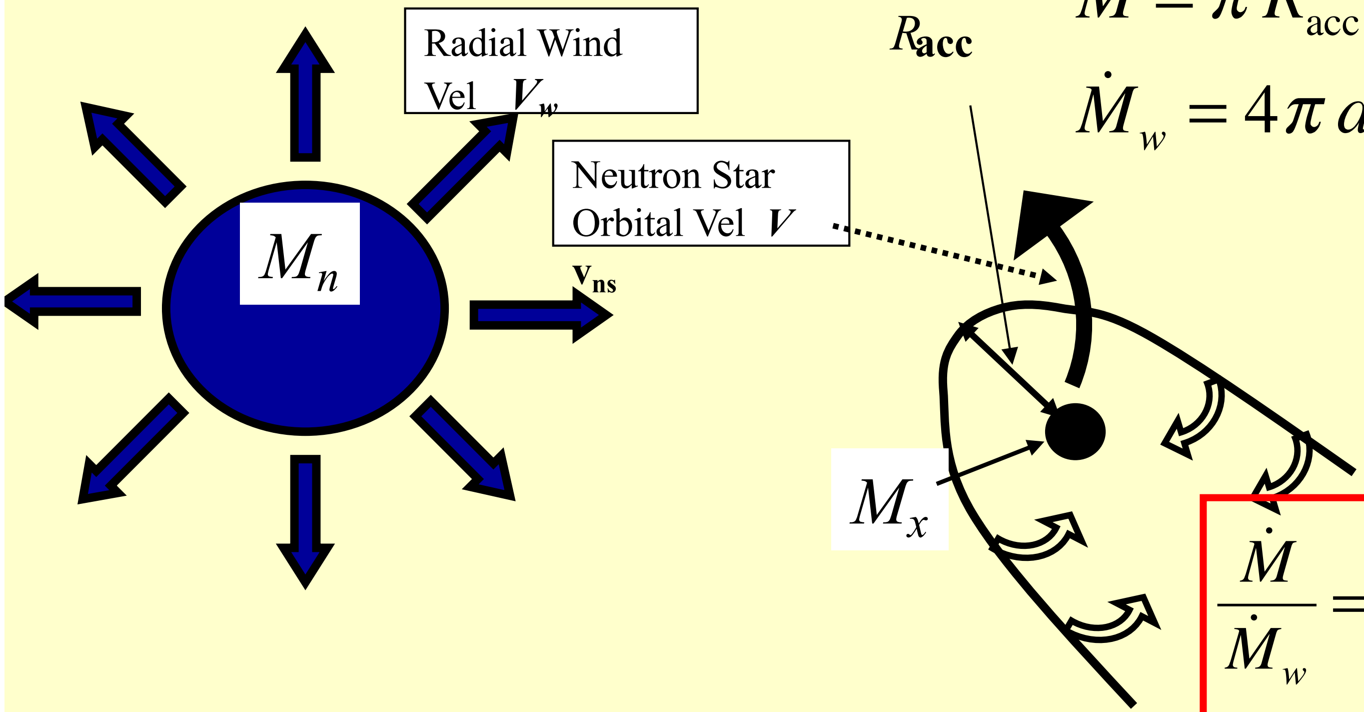
V- orbital velocity of a compact object around a companion

$$V^2 = \frac{GM_n}{a}$$

a – binary separation

$\dot{M} = \pi R_{\rm acc}^2 V_{rel}\,\rho_w$- accretion rate

$\dot{M}_w = 4\pi a^2 V_w \rho_w$- wind loss rate



Derive at home:

$$\boxed{\frac{\dot{M}}{\dot{M}_w} = \left(\frac{M_x}{M_n}\right)^2 \frac{(V/V_w)^4}{\left[1+(V/V_w)^2\right]^{3/2}}}$$

Mass determination in binary systems

$$\frac{a^3}{P^2} = \frac{G(M_x + M_n)}{4\pi^2}, \quad \frac{(a/\mathrm{AU})^3}{(P/\mathrm{yr})^2 (M/M_o)} = 1$$

3rd Kepler's law

$a = a_x + a_n$ – binary separation; P- orbital period

$M_x a_x = M_n a_n$

i - orbital inclination angle

$$K_x = \frac{2\pi a_x \sin i}{P}$$

$$K_n = \frac{2\pi a_n \sin i}{P}$$

projected velocities (amplitudes)

Velocities can be measured from Doppler shifts of optical spectral lines of a companion, or shift of a pulsars frequency

$$M_x > f_n = \frac{M_x^3 \sin^3 i}{(M_x + M_n)^2} = \frac{4\pi^2 a_n^3 \sin^3 i}{GP^2} = \frac{PK_n^3}{2\pi G}$$

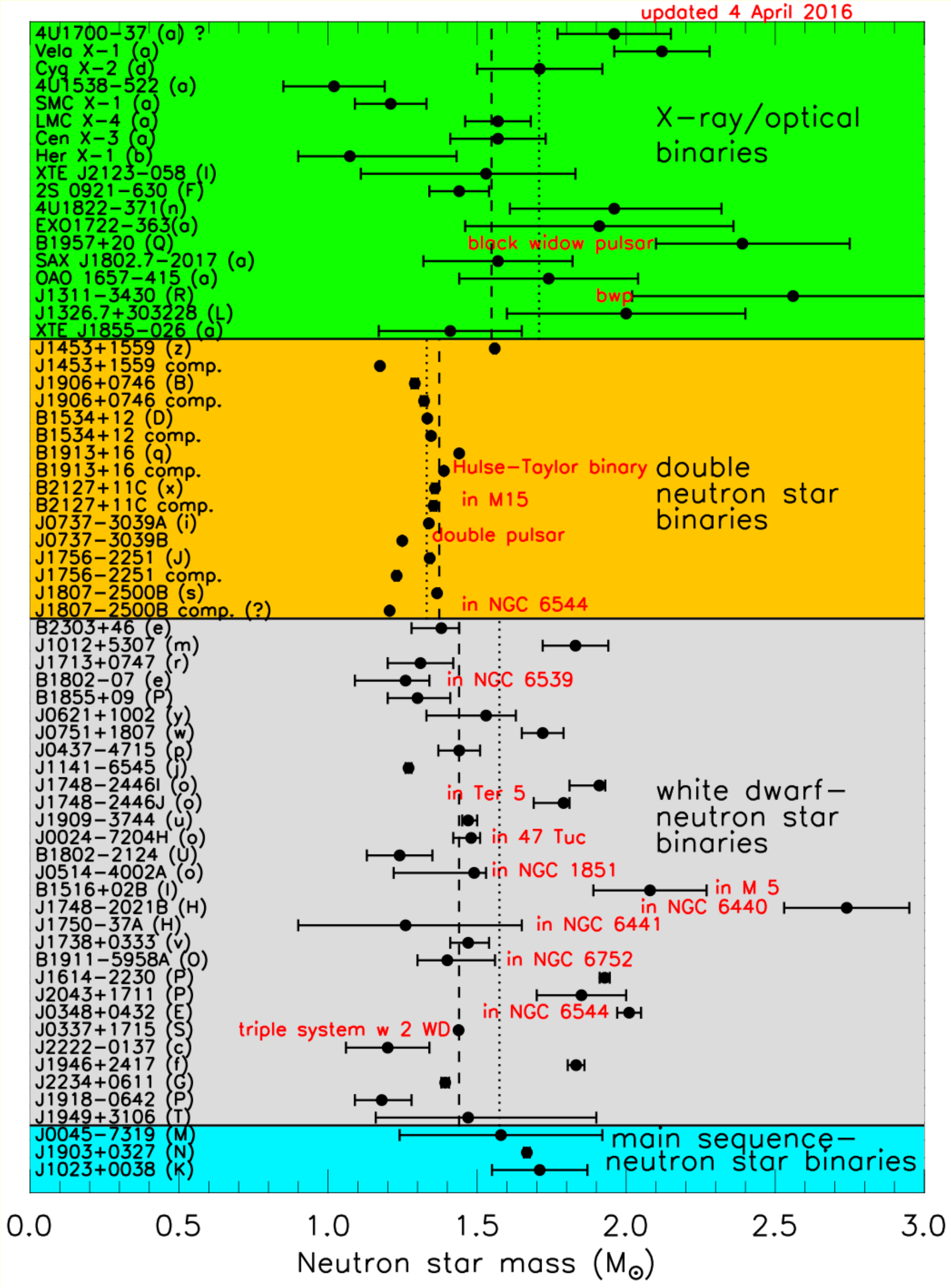
If we know the velocity of the 2nd star, we know a 2nd mass function, similar to the one above, and we can determine the mass ratio (M_x/M_n); we still need the orbital inclination in order to determine the masses individually.

Neutron star masses

Where neutron star masses can be measured, the mass is found to be very close to 1.4 M⊙.

Maximum measured masses are 1.93±0.02 M⊙ in J1614-2230 (Demorest et al. 2010; Fronseca et al. 2016) and 2.01±0.04M⊙ in J0348+0432 (Antoniadis et al. 2013)

Maximum possible mass for a NS is around 2.5 M⊙.



Black hole masses

Name	M_x	M_c	P_{orb} day
A0620-00	9-13	2.6-2.8	0.33
GRO J1655-40	6-6.5	2.6-2.8	2.8
XTE J1118+480	6.4-7.2	6-6.5	0.17
Cyg X-1	14-16 (21?)	20 (40?)	5.6
GRO J0422+32	3-5	1.1	0.21
GS 2000+25	7-8	4.9-5.1	0.35
V404 Cyg	10-14	6.0	6.5
GX 339-4	3-5	5-6	1.75
GRS 1124-683	6.5–8.2		0.43
XTE J1550-564	10-11	6-7.5	1.5
XTE J1819-254	10-18	3	2.8
4U 1543-475	8-10	0.25	1.1
GRS 1915+105	10-18	1.0-1.4	33.5

The Eddington Luminosity

- There is a (not strict) limit to the luminosity that can be produced by a steadily accreting object, known as the Eddington luminosity.

- Effectively it is reached when the inward gravitational force on matter is balanced by the outward transfer of momentum by radiation.

Radiation pressure force

- Radiation may also have an effect on the pressure. Radiation is an inefficient carrier of momentum (velocities have the highest possible value), but when a photon is absorbed or scattered by matter, it imparts not only its energy to that matter, but also its momentum $h\nu/c$.
- The component of photon momentum imparted to an idealized wall is

$$\frac{h\nu}{c}\cos\theta$$

  where θ is the angle the photon trajectory forms with the normal to the wall.

- The number of photons passing through area dA in time dt in interval $d\nu$ and solid angle $d^2\omega$ is

$$\frac{I_\nu}{h\nu}\cos\theta d\nu dA d^2\omega dt$$

Radiation pressure force

- Thus the pressure (momentum per unit area per unit time per unit frequency) may be expressed in terms of the specific intensity integrated over solid angles

$$dP = P_\nu d\nu = d\nu \frac{1}{c} \oint I_\nu \cos^2\theta \; d^2\omega$$

- The total radiation pressure force is then

$$\boxed{P = \frac{1}{c} \int d\nu \; \oint I_\nu \cos^2\theta \; d^2\omega}$$

- For a point source of radiation (i.e. at large distances) $\cos\theta = 1$, and the radiation pressure force reduces to

$$\boxed{\left.\begin{array}{l} P = \frac{1}{c} \oint I \cos^2\theta \; d^2\omega = \frac{1}{c} \oint I \; d^2\omega \\ F = \oint I \cos\theta \; d^2\omega = \oint I \; d^2\omega \end{array}\right\} \Rightarrow P = \frac{1}{c} F = \frac{1}{c} \frac{L}{4\pi r^2}}$$

where r is the distance to the source, F is the radiation flux and L is the source luminosity.

Radiation pressure force

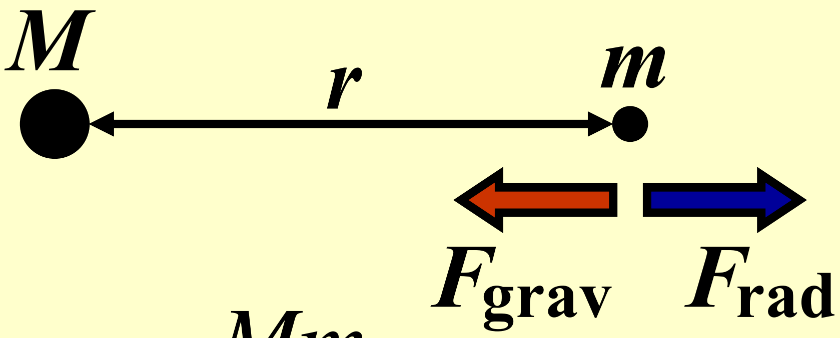
- If, however, we are interested in the momentum transfer rate to the unit length of the atmosphere, then we need to account for the fact that along a ray inclined at angle θ to the normal the path is longer and probability for the photon to be scattered (or absorbed) is also larger. The probability for scattering on radial distance ds is then (σ is the cross-section; n - concentration)

$$\boxed{dp = \sigma n \; ds / \cos\theta}$$

- Thus the momentum transfer rate (i.e. force f) per unit length is

$$\boxed{\frac{df}{ds} = \frac{\sigma n}{c} F}$$

Eddington Luminosity



Accretion rate controlled by momentum transferred from radiation to mass

$$F_{\text{grav}} = G\frac{Mm}{r^2}$$

Outgoing photons from M scatter off accreting material (electrons and protons).

Radiation force:

$$F_{\text{rad}} = P\sigma_T = \frac{\sigma_T}{c}\frac{L}{4\pi r^2}$$

$\sigma_T = \frac{8\pi}{3} r_e^2 \approx 0.665\times10^{-24}\text{cm}^2$ - Thomson cross-section

Eddington Limit

radiation pressure = gravitational pull

At this point accretion stops, effectively imposing a ‘limit’ on the luminosity of a given body.

$$\frac{L\sigma_T}{4\pi r^2 c} = G\frac{Mm}{r^2}$$

Electrostatic forces between e$^-$ and p bind them, so they act as a pair, so $m \approx m_p$

The Eddington luminosity (for pure H plasma) is:

$$L_{Edd} = \frac{4\pi cGMm}{\sigma_T} = 1.3\times10^{38}\,\frac{M}{M_\odot}\text{ erg s}^{-1}$$