Physics of X-ray pulsars

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Neutron stars

- Size: about 10-15 km.
- Mass: 1.25-2.0 solar masses are accurately measured in pulsars in binary systems

$$E_{\rm grav} \sim GM^2/R \sim 5 \times 10^{53}\ {\rm erg} \sim 0.2\, Mc^2,$$

$$g \sim GM/R^2 \sim 2 \times 10^{14}\ {\rm cm\ s^{-2}},$$

Mean mass density:

$$\bar{\rho} \simeq 3M/(4\pi R^3) \simeq 7 \times 10^{14}\ {\rm g\ cm^{-3}} \sim (2-3)\,\rho_0$$

Normal nuclear density:

$$\rho_0 = 2.8 \times 10^{14}\ {\rm g\ cm^{-3}}$$

Neutron stars

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Remnants of massive stars

Ginzburg(1964) and Woltjer(1964) proposed that the magnetic flux ($\sim BR^2$) of a star is conserved.

Magnetic field of NS

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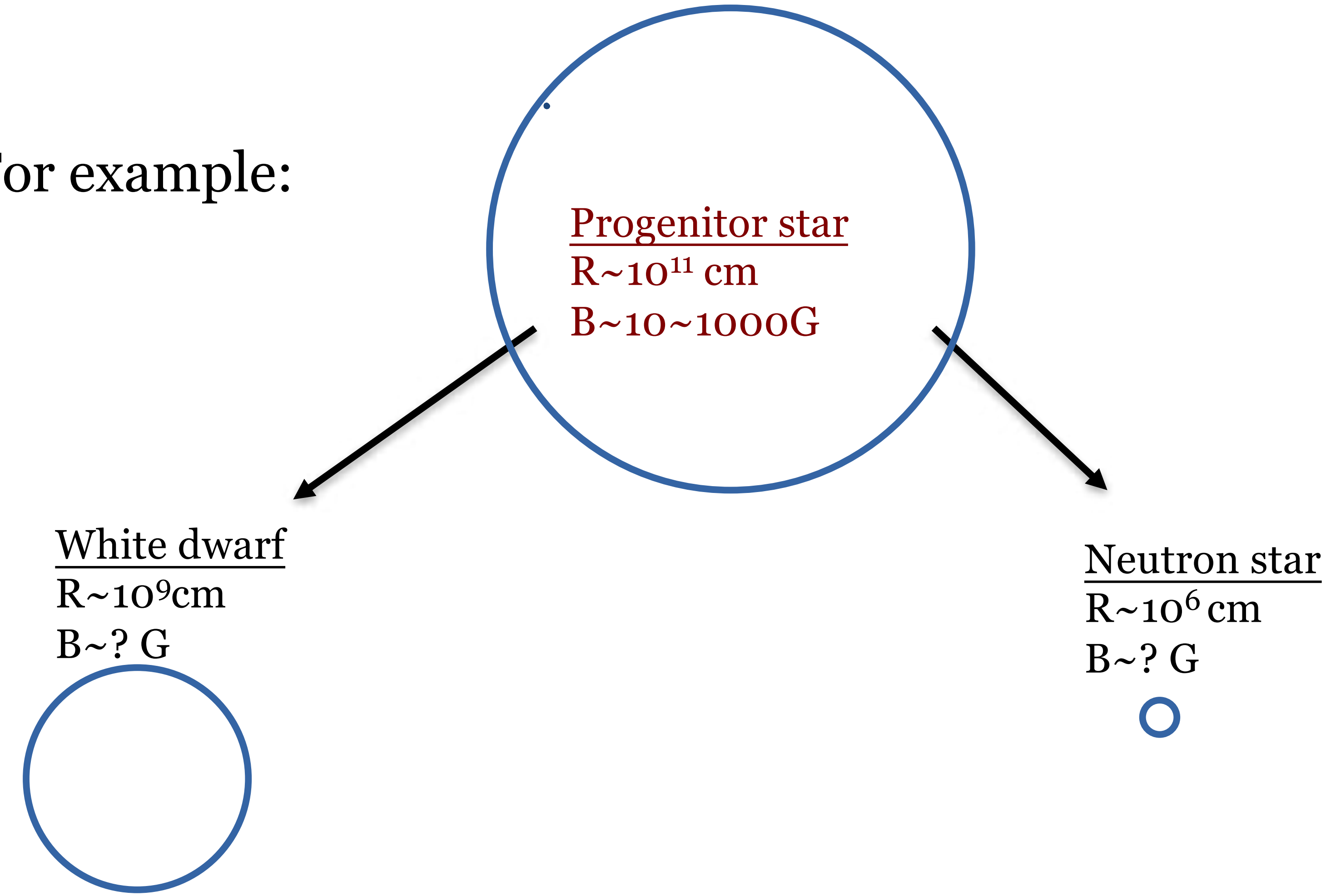
For example:



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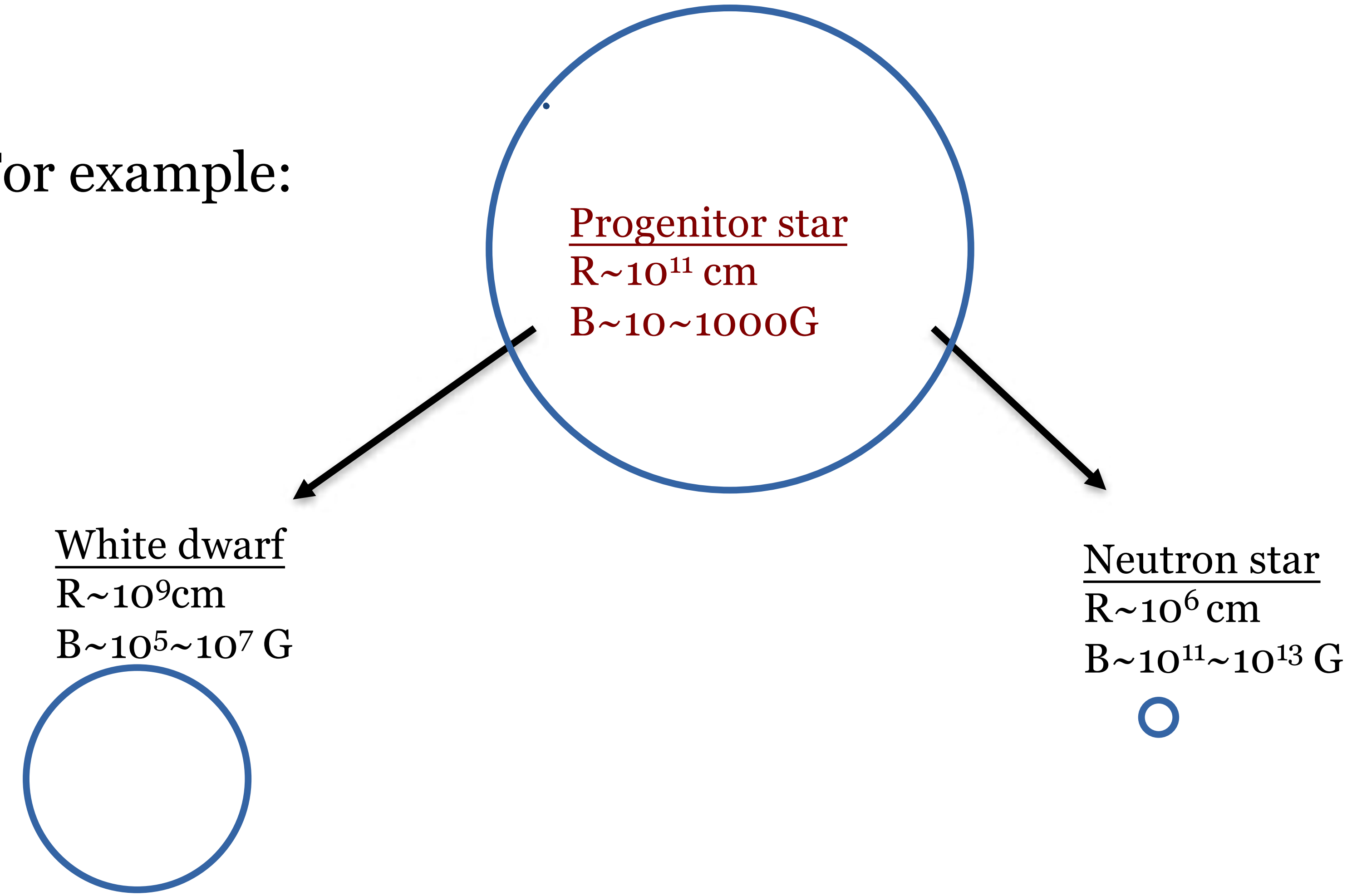
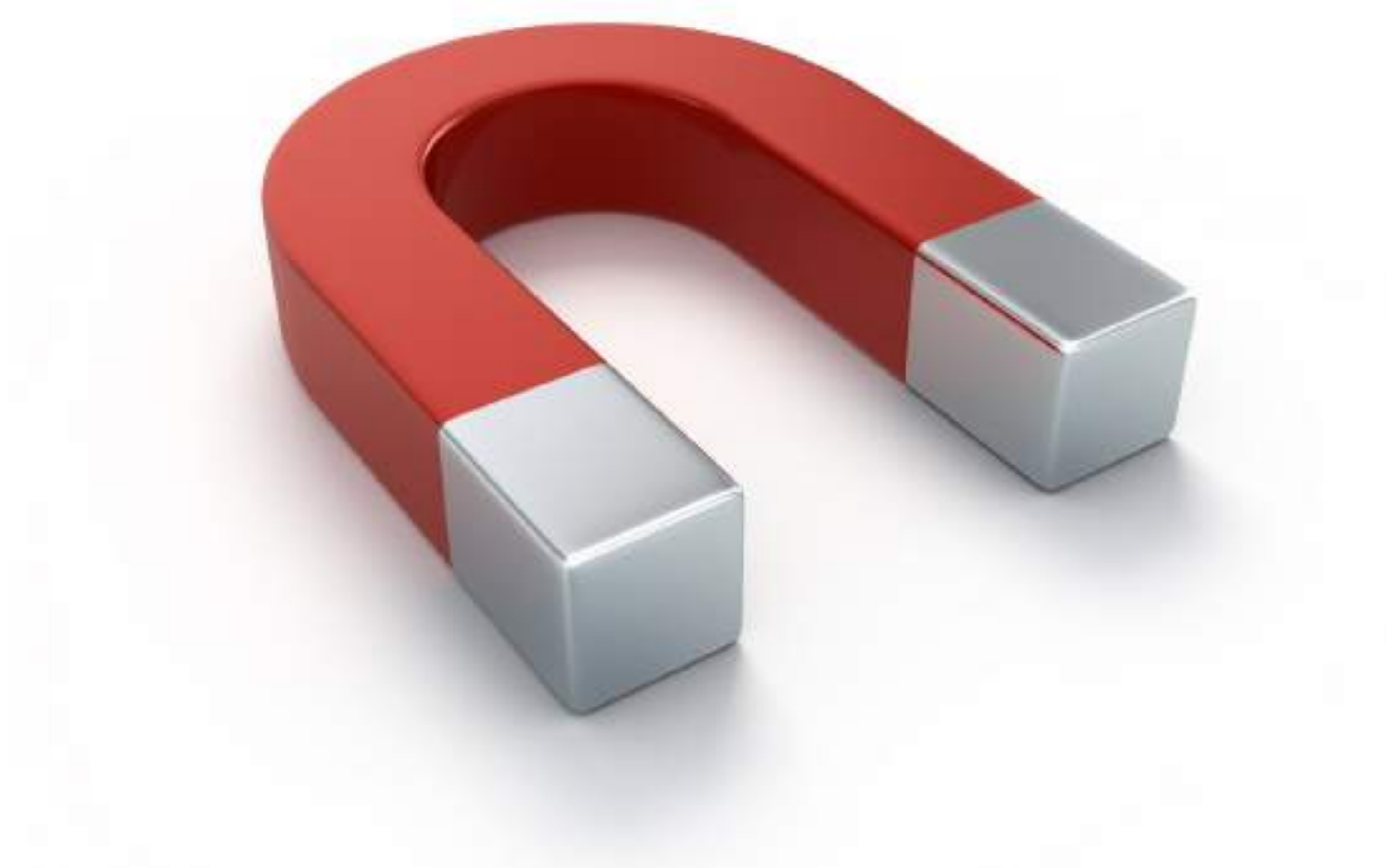
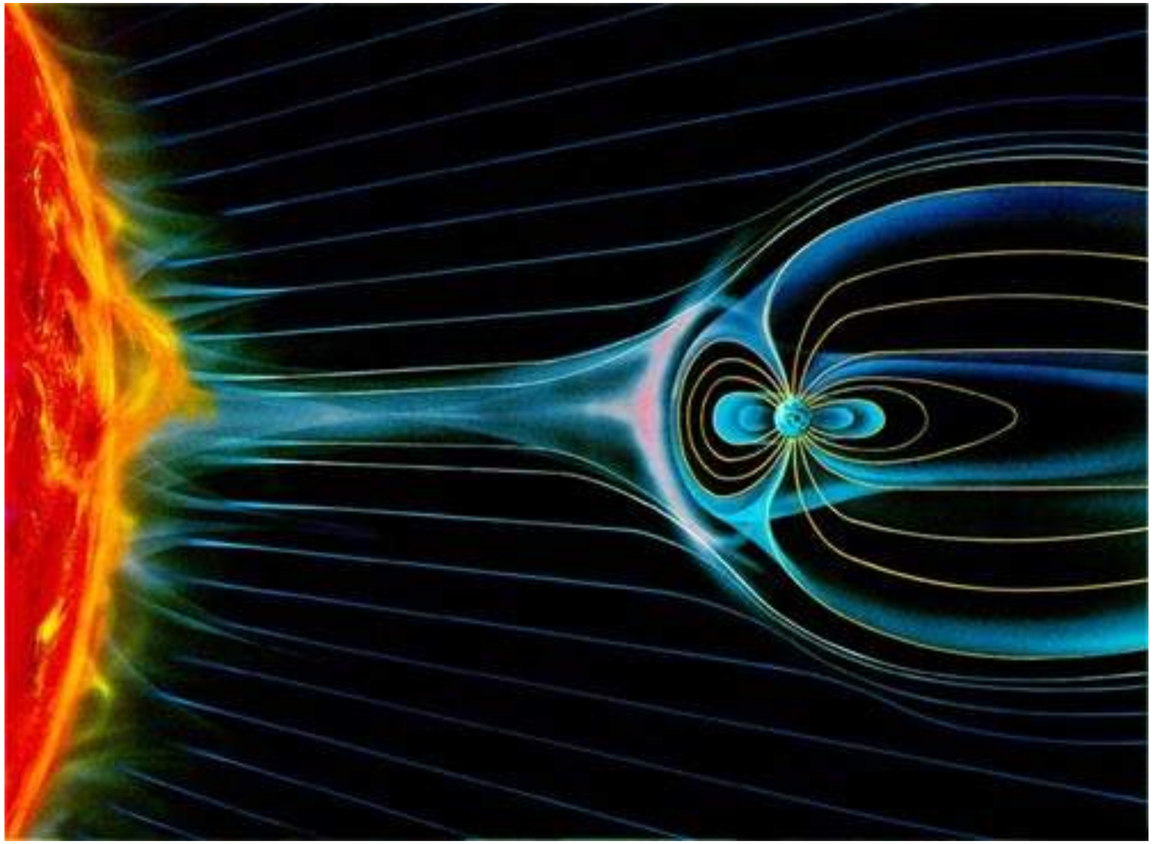


The strongest magnets in the Universe



Magnetic field strengths

Earth **0.5 G**



Magnetic field strengths

Earth	0.5 G
Magnet on fridge	**50 G**

Magnetic field strengths

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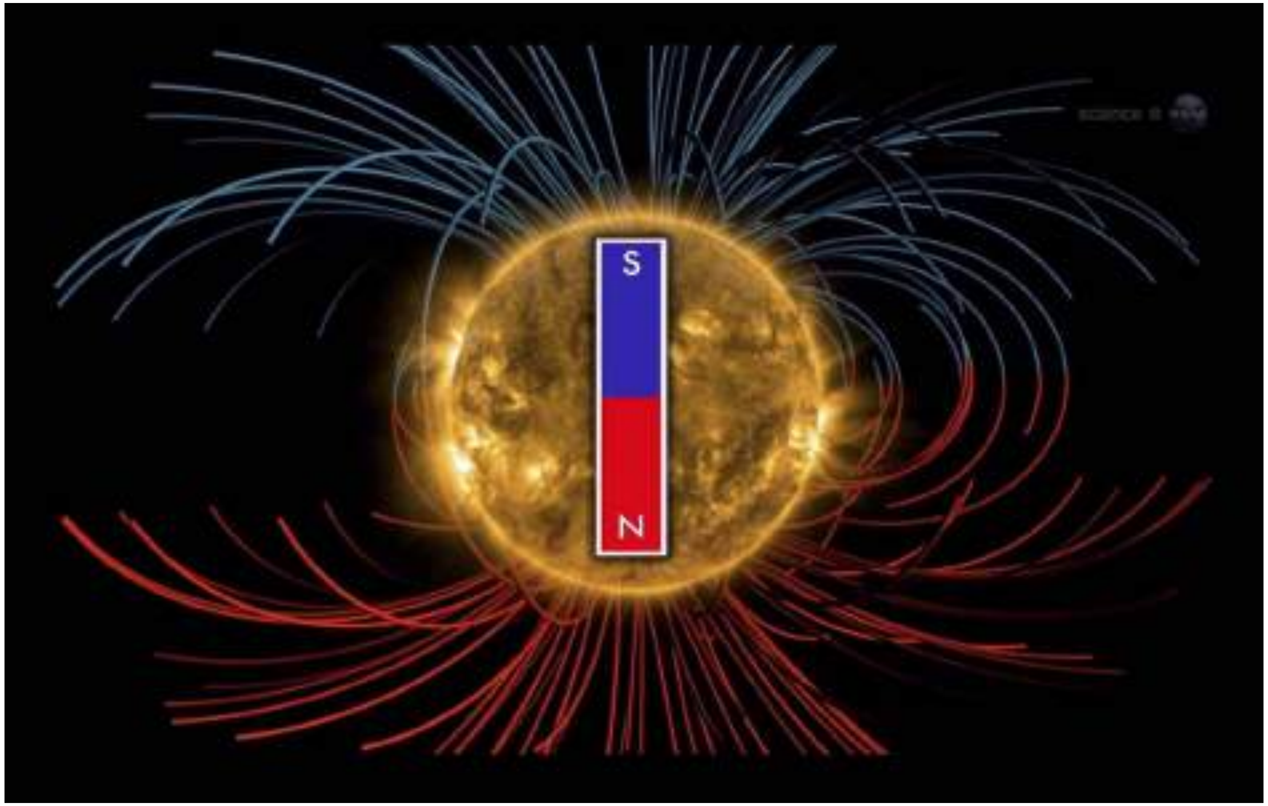
Magnet on fridge	50 G
Stars	**10-1000 G**



Magnetic field strengths

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Field for the levitating frogs	**10^5 G**

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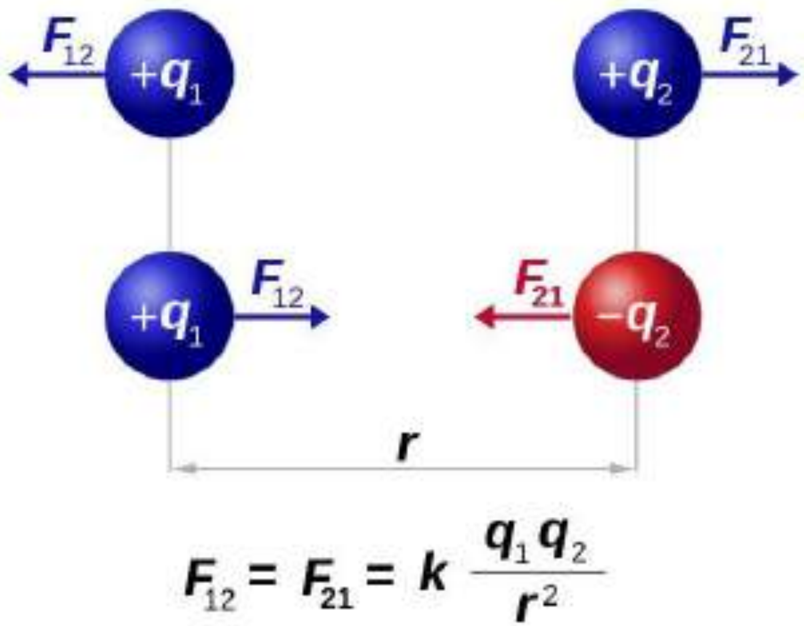
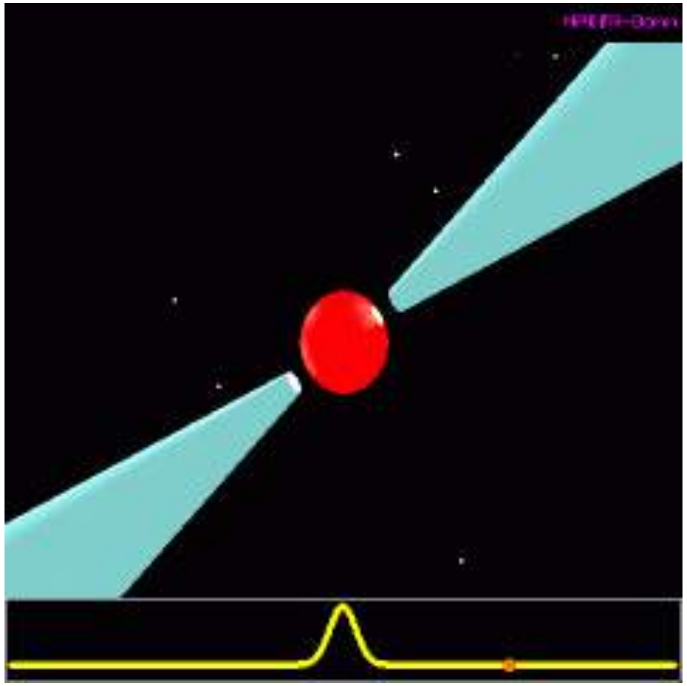
E_{cycl} comparable to $E_{Coulomb}$	**10^9 G**

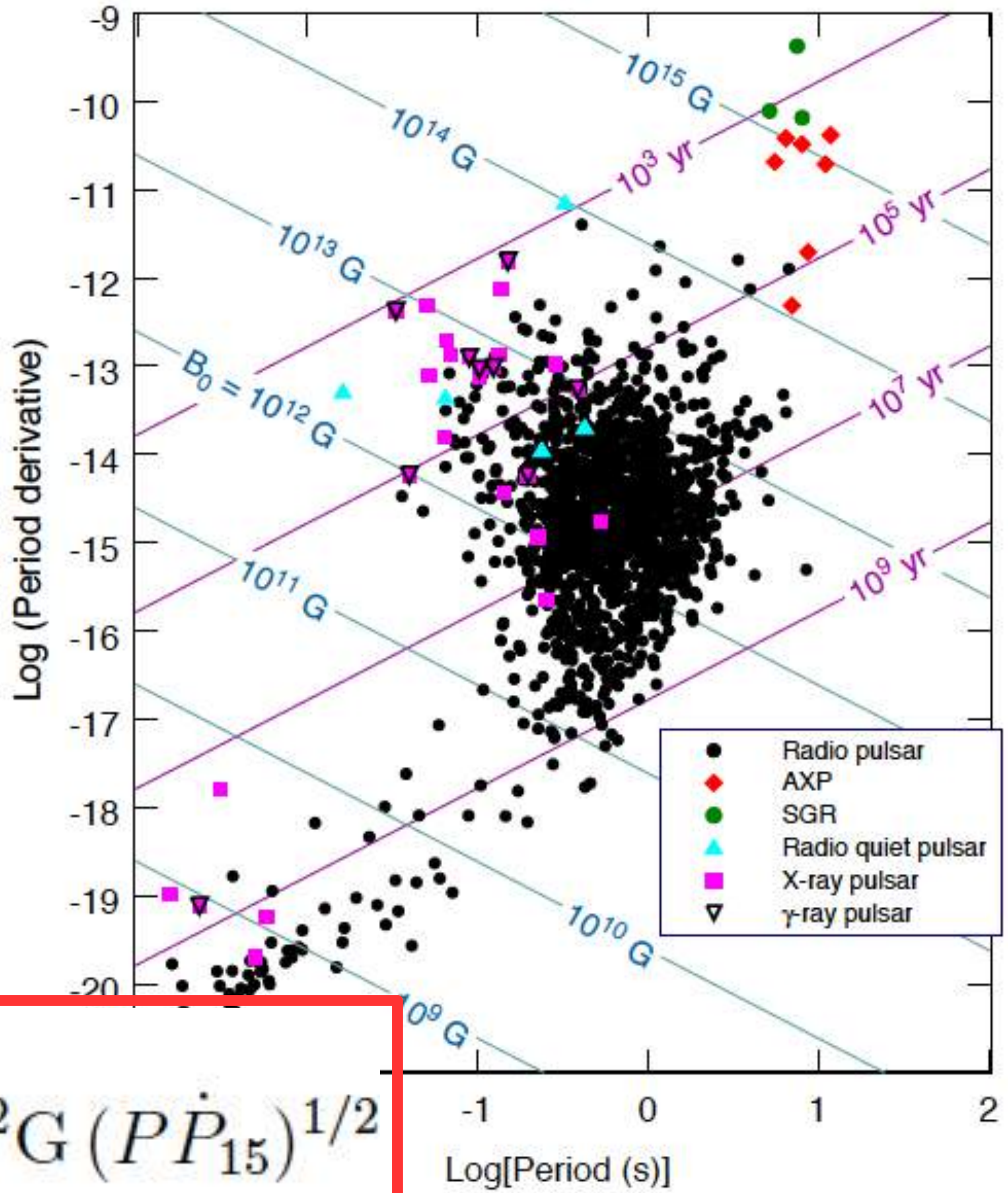


Magnetic field strengths

Earth	0.5 G
Magnet on fridge	50 G
Stars	10-1000 G
Field for the levitating frogs	10^5 G
Strongest steady field in lab	10^6 G
E_{cycl} comparable to E_{Coulomb}	10^9 G
Young neutron stars	**$10^{12\text{-}15}$ G**

NS magnetic field





$$R \simeq 10^6 \ \mathrm{cm}$$

$$I \simeq 10^{45} \ \mathrm{g\ cm^2}$$

$$\dot{P}_{15} \equiv \dot{P}/(10^{-15}\,\mathrm{s\,s^{-1}})$$

$$B_s = \left(\frac{3Ic^3P\dot{P}}{2\pi^2R^6}\right)^{1/2} \simeq 2\times10^{12}\mathrm{G}\,(P\dot{P}_{15})^{1/2}$$

NS magnetic field

Strong magnetic fields:

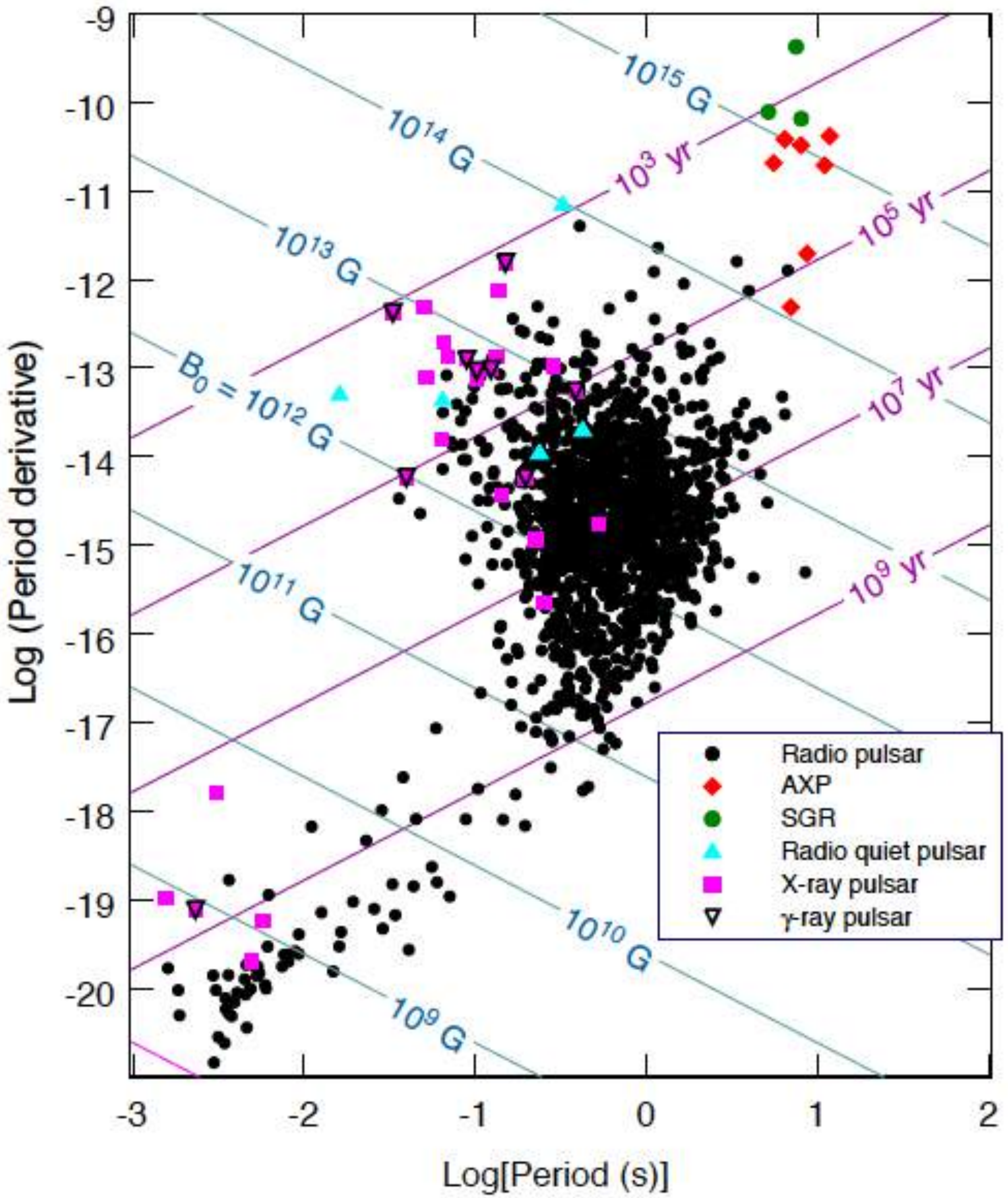
Cyclotron energy is comparable with the typical Coulomb energy at

$$B_0 = 2.3505 \times 10^9 \, \mathrm{G}$$

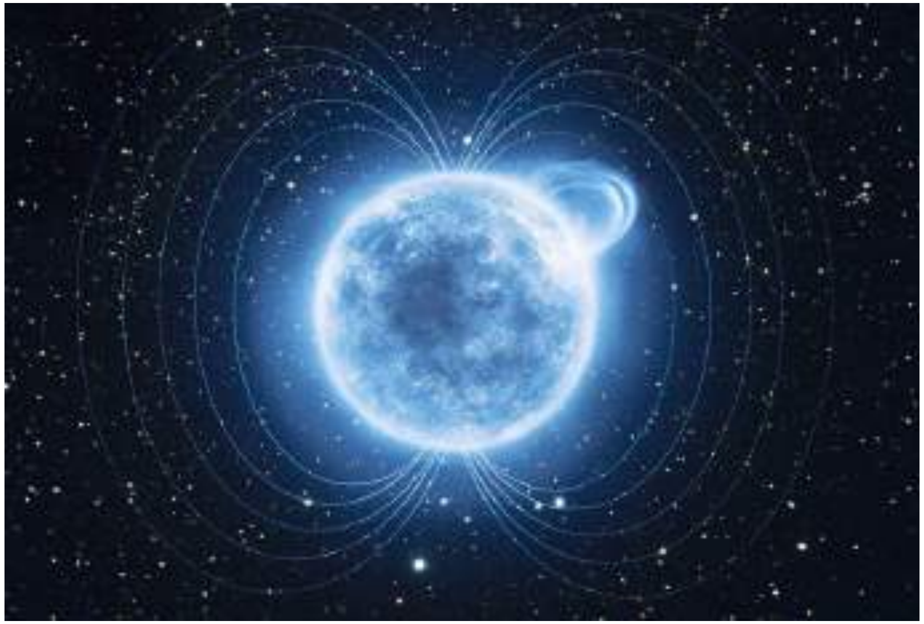
Theoretical upper limit for NSs:

$$\frac{E_{\rm mag}}{E_{\rm grav}} \sim \frac{B_{\rm in}^2 R^3/6}{GM^2/R} \lesssim 1$$

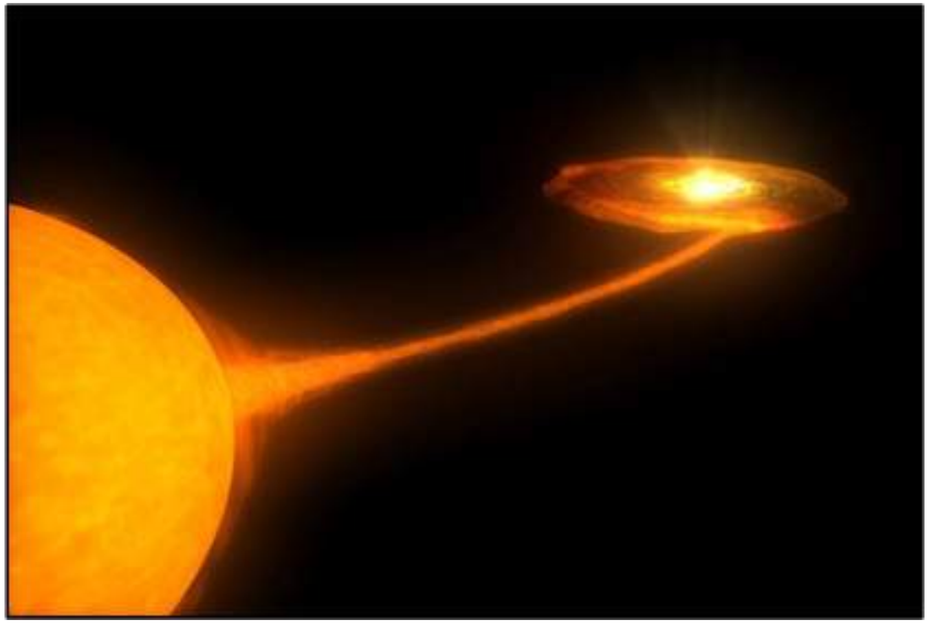
$$B_{\rm in} \lesssim 10^{18} \, \mathrm{G}$$



What if NS has a companion?

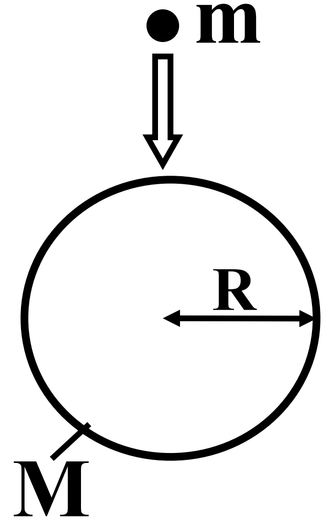


90% of all neutron stars are isolated. The main source of energy is rotation.



If neutron star has a close companion, it will be powered by accretion.

Accretion is the most efficient way for energy production in the Universe



$$E_{acc} = G\frac{Mm}{R}$$

Compare this to nuclear fusion:

H => He releases ~ 0.007 mc^2 - **<u>20x smaller</u>**

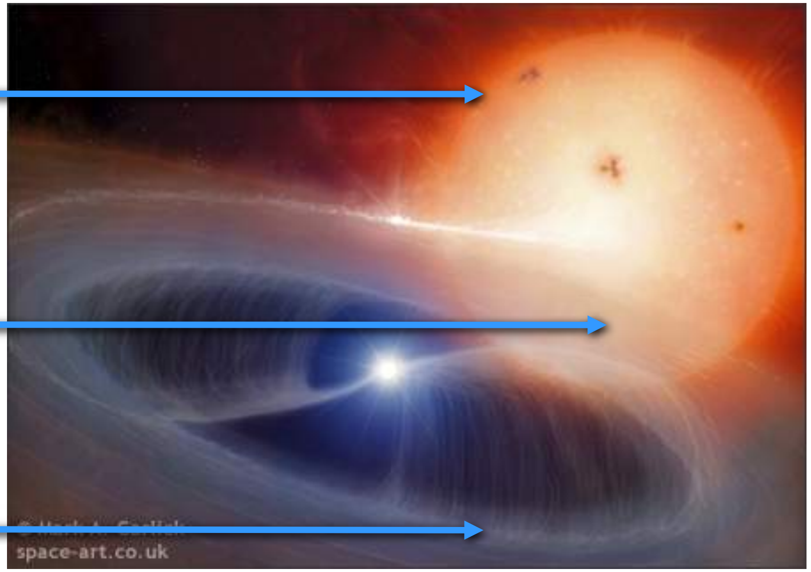
X-ray pulsar

Rotating neutron star in binary systems

High mass companions

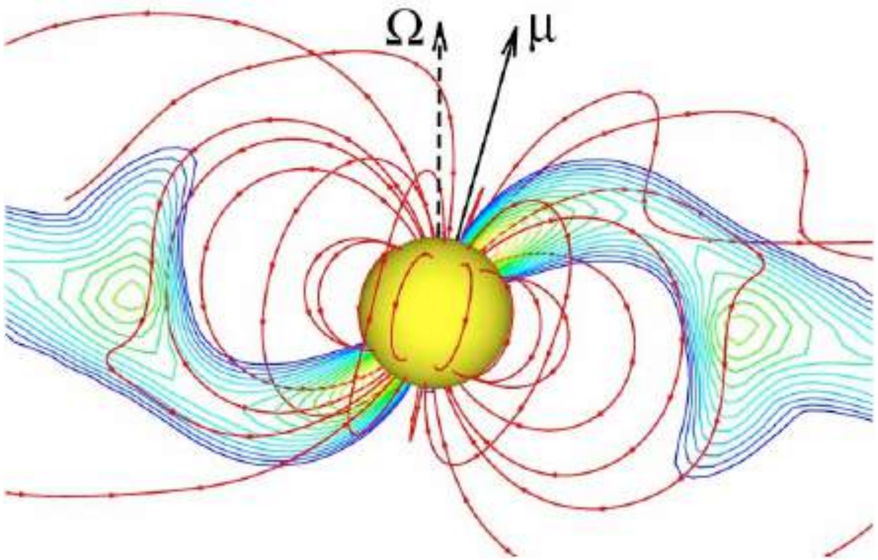
Mass capture from wind/disk

High B-fields



Disruption of disk by B-field

Matter channeled onto star

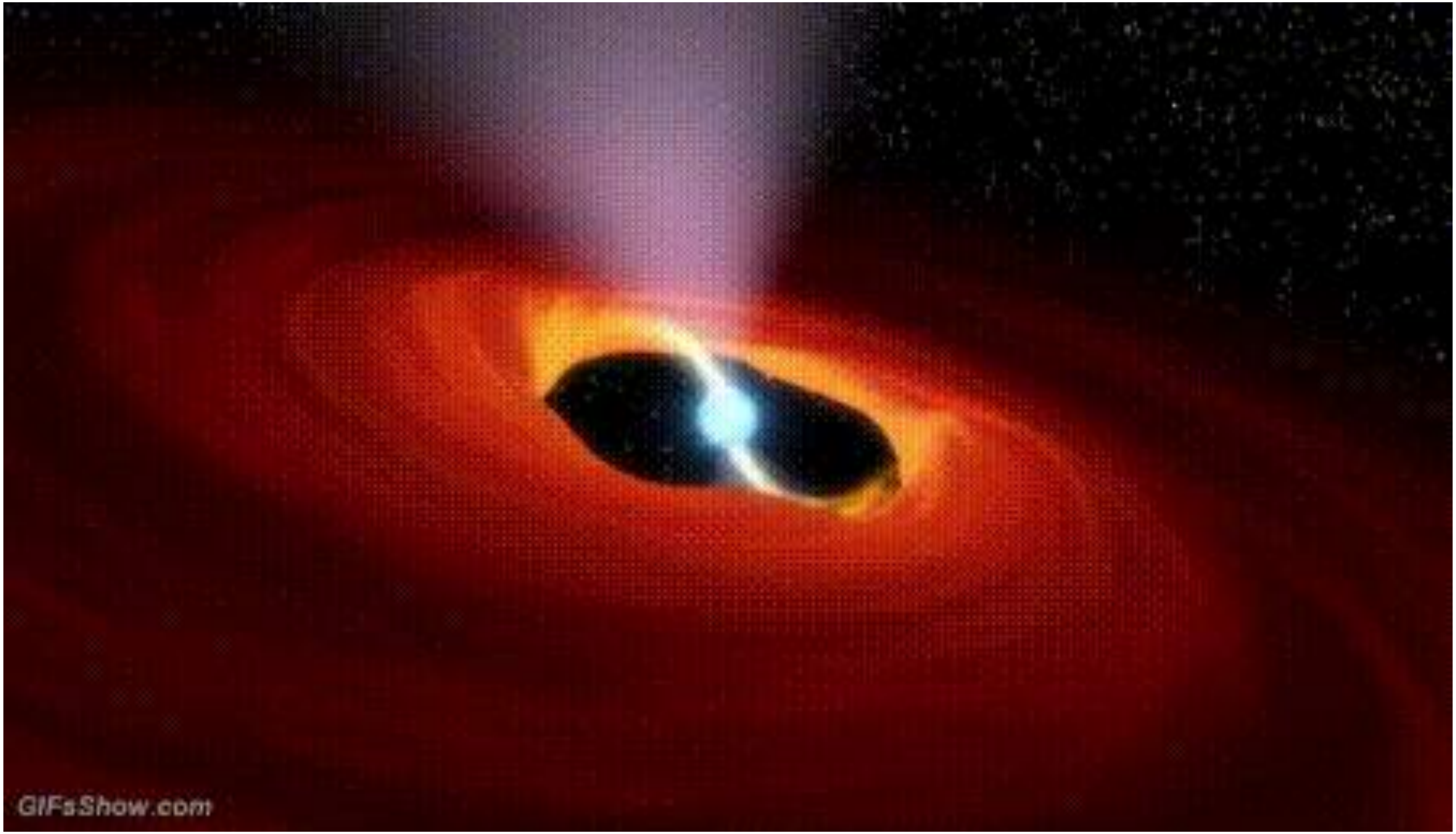


Neutron star parameters:

M_{NS}~1.4 M_{sun}
R_{NS} ~10 km (10^6 sm)
P_{spin} ~1 – 10^3 s
B_{NS} ~$10^{11\text{-}13}$ G

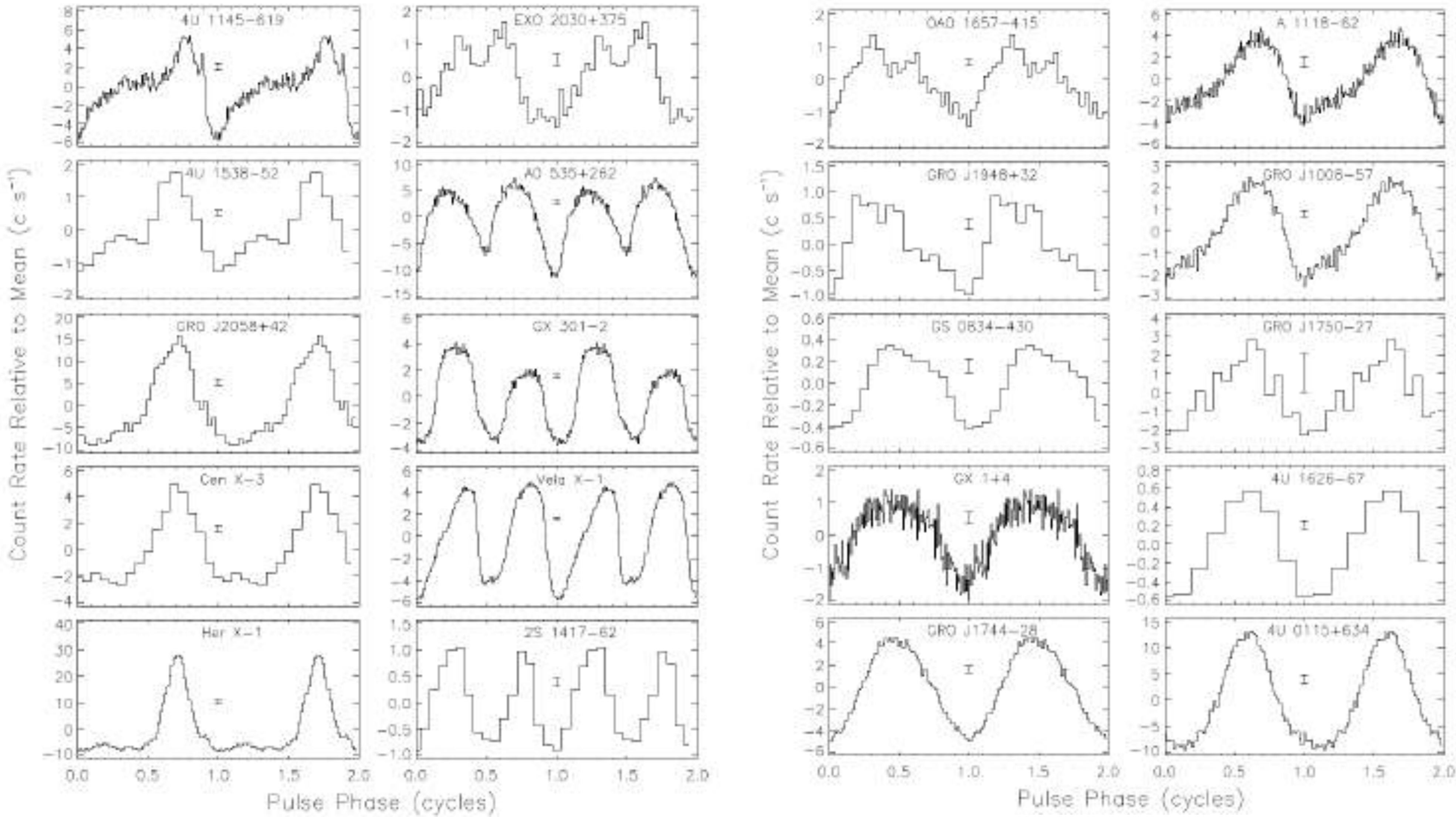
X-ray pulsar

Rotating neutron star in binary systems

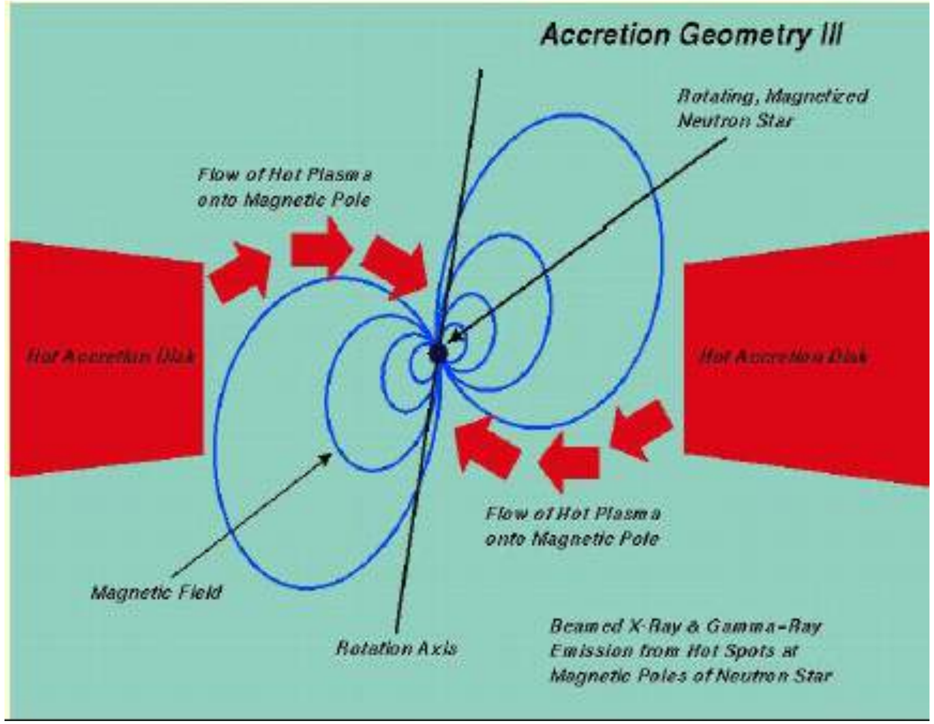


X-ray pulsars

- **Pulse periods from ~1 s to ~1000 s.**



X-ray (?) pulsars



Stefan-Boltzmann law:

- $L=\sigma T^4 S$
- $\sigma=5.67\ 10^{-5}$ erg s^{-1} cm^{-2} K^{-4}

(Stefan-Boltzmann constant)

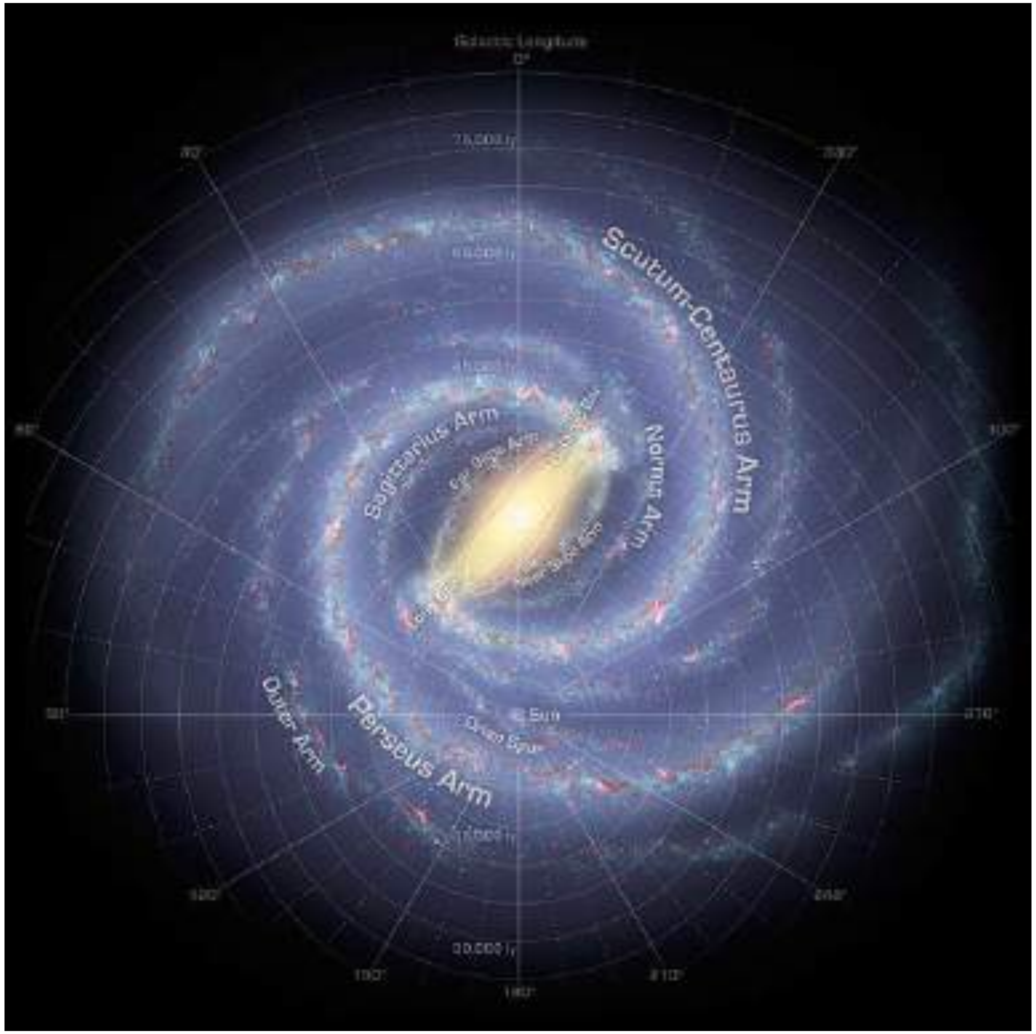
- Radius of the polar cap ~100 m

$S \sim 10^8\ cm^2$

- Observed $L \sim 10^{37}$ erg s^{-1}
- $T=(L/\sigma S)^{1/4} \sim 10^8$ K
- 1 eV = 11605 K ~ 10^4 K
- T ~ 10 keV

Emission is not a black body!

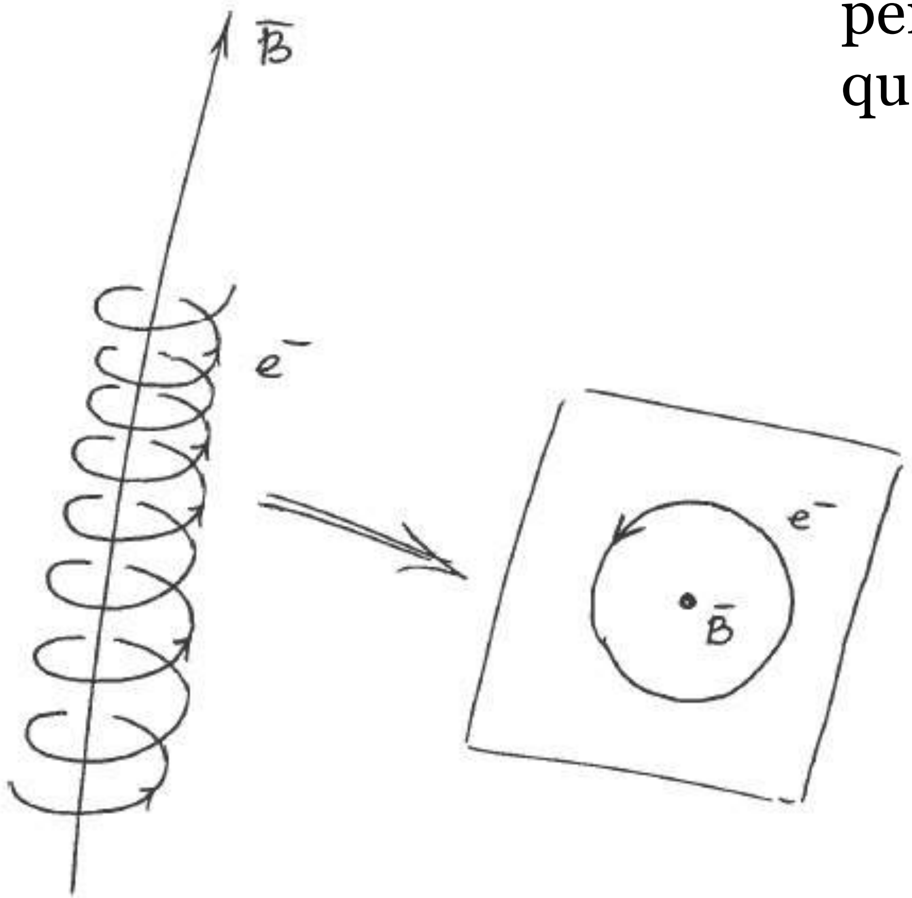
X-ray pulsars



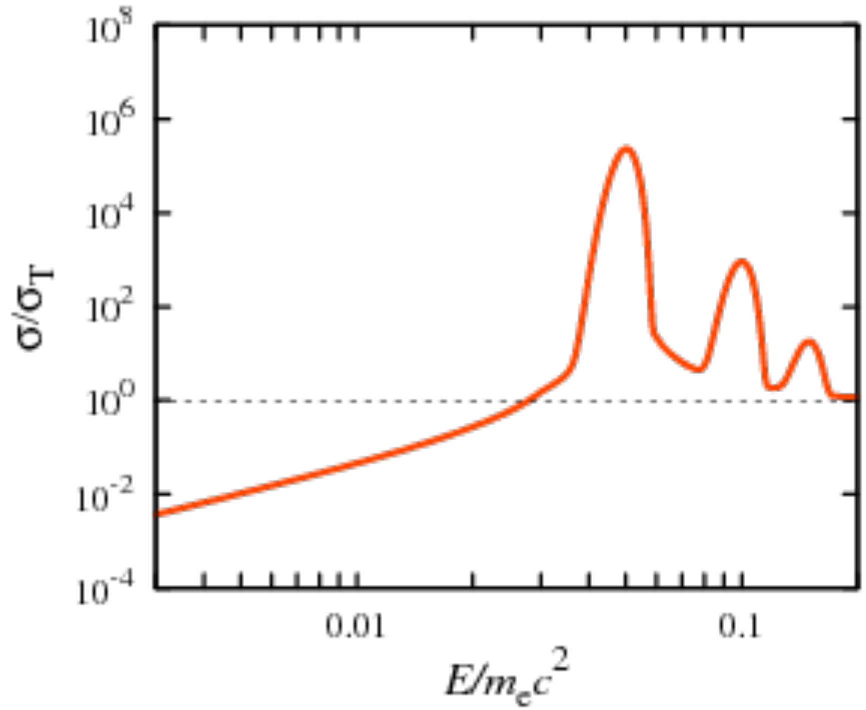
- Currently about 100 X-ray pulsars are known.
- Usually in massive binary systems with O-B optical companions. Only a few are known in low-mass X-ray binaries (e.g., Her X-1).
- Orbital periods from ~1 day to ~1 year.

Ultra strong magnetic field

The motion of the electron perpendicularly to the B field is quantized in the Landau levels



Scattering cross section

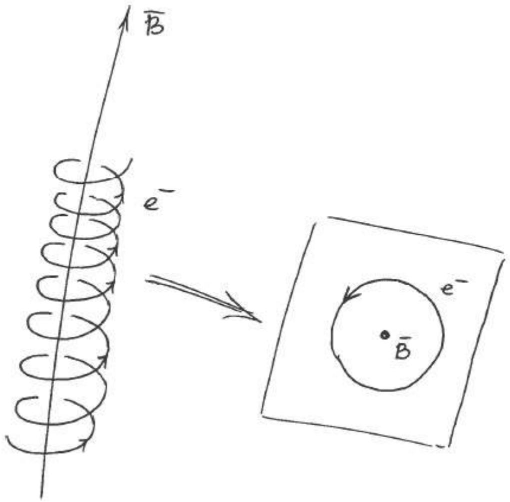


Magnetic field of the neutron star

Lorentz force =

centripetal force

$$\frac{|q|}{c} V B = \frac{m V^2}{r}$$



$$r = \frac{c m V}{|q| B}$$

$$\frac{2\pi r}{V} = \frac{2\pi}{\omega} \qquad \omega = \frac{V}{r}$$

Cyclotron frequency

$$\omega_c = \frac{eB}{m_e c}$$

Cyclotron energy, keV

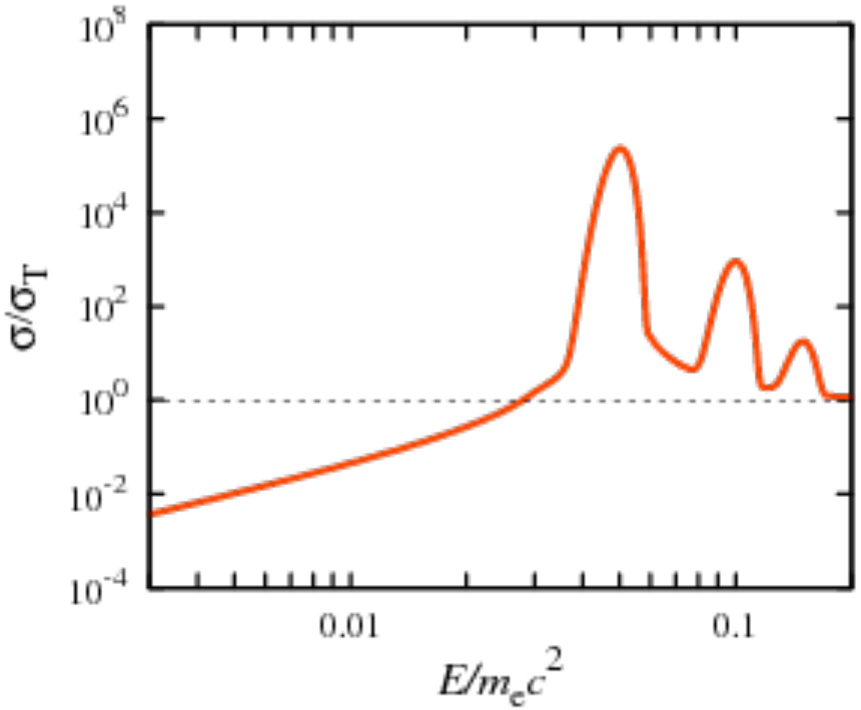
$$E_{\rm cyc} = \hbar\omega_c \qquad \hbar\,\omega_n = n\,\hbar\,\omega_c$$

Magnetic field of NS, 10^{12} Gauss

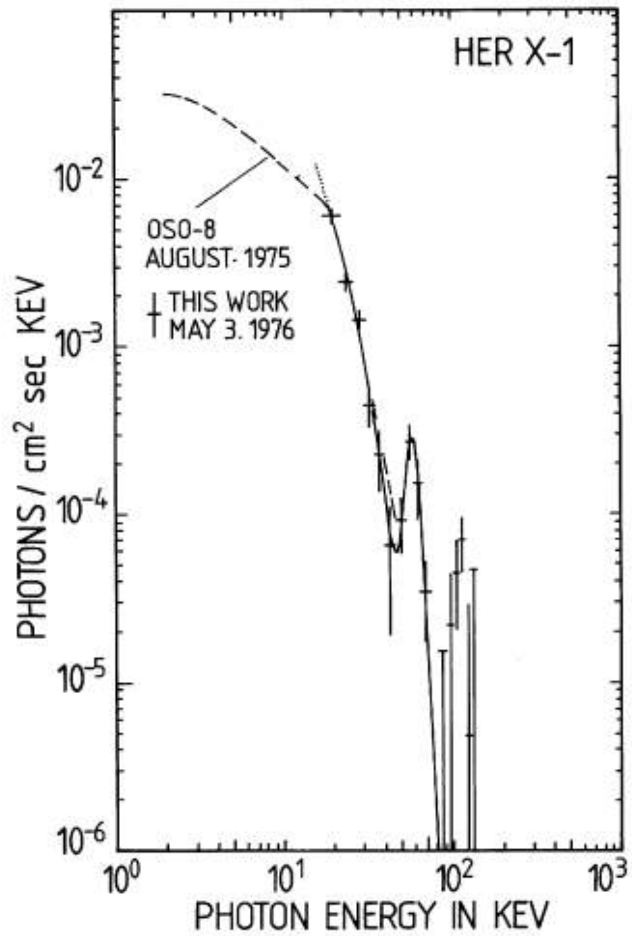
$$B_{12} = \frac{E_{\rm cyc}}{11.2}$$

Cyclotron absorption line

Scattering cross section



Truemper et al. (1978)

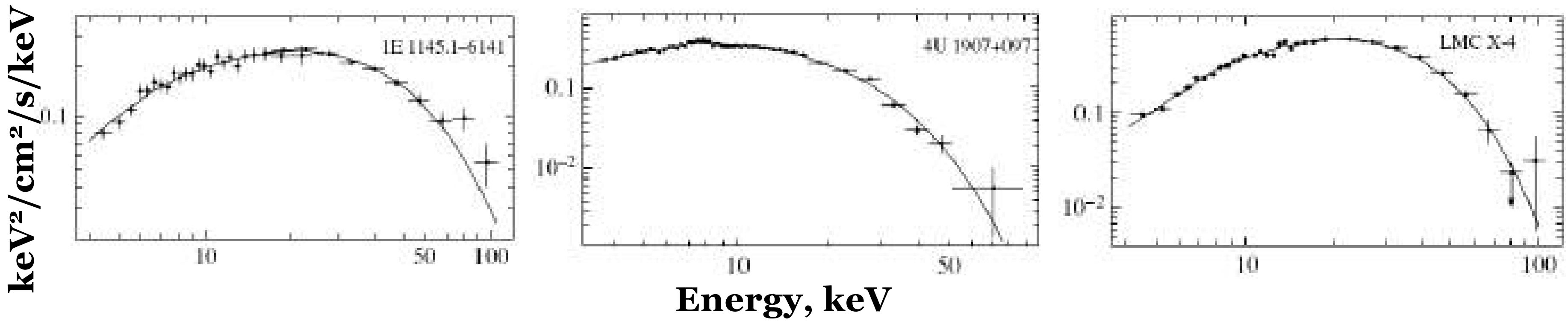


Cyclotron lines directly probe the magnetic fields of neutron stars!

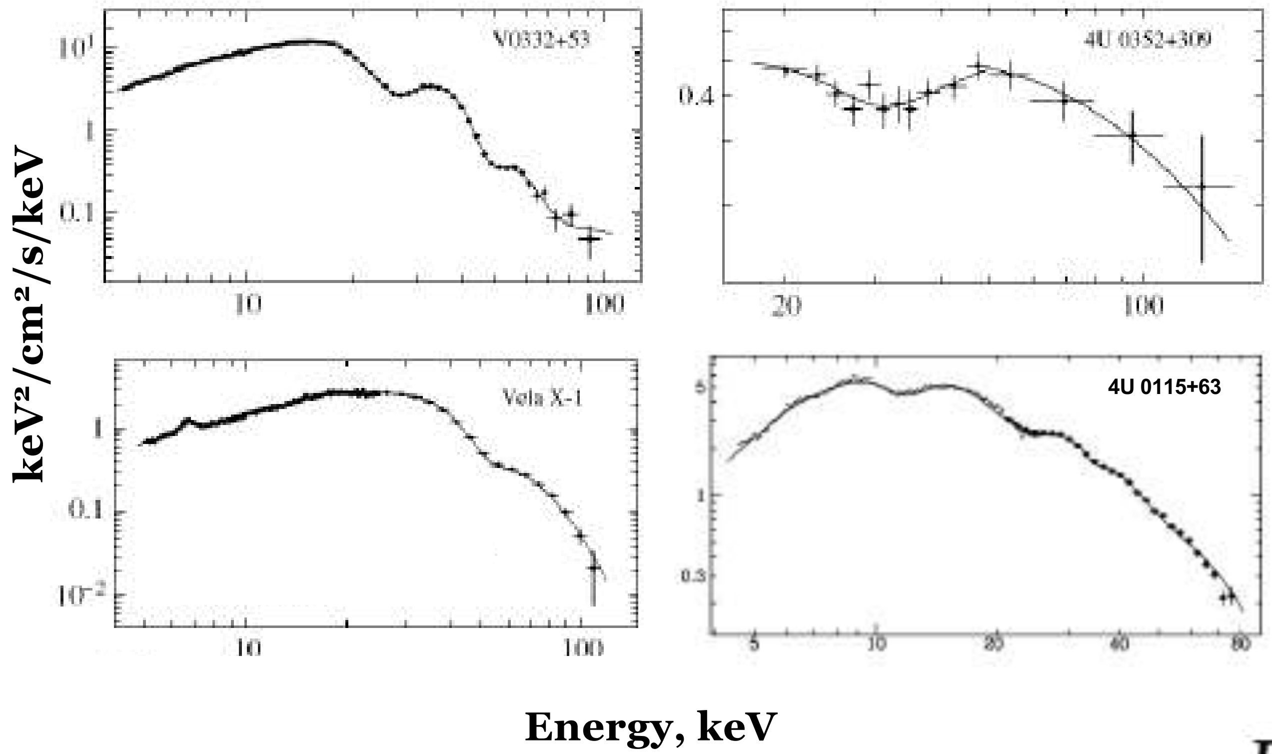
$$B_{12} = \frac{E_{\rm cyc}}{11.2}$$

E_{cyc} in units of keV, B_{12} in units of 10^{12} G

Spectra of X-ray pulsars



With cyclotron absorption line

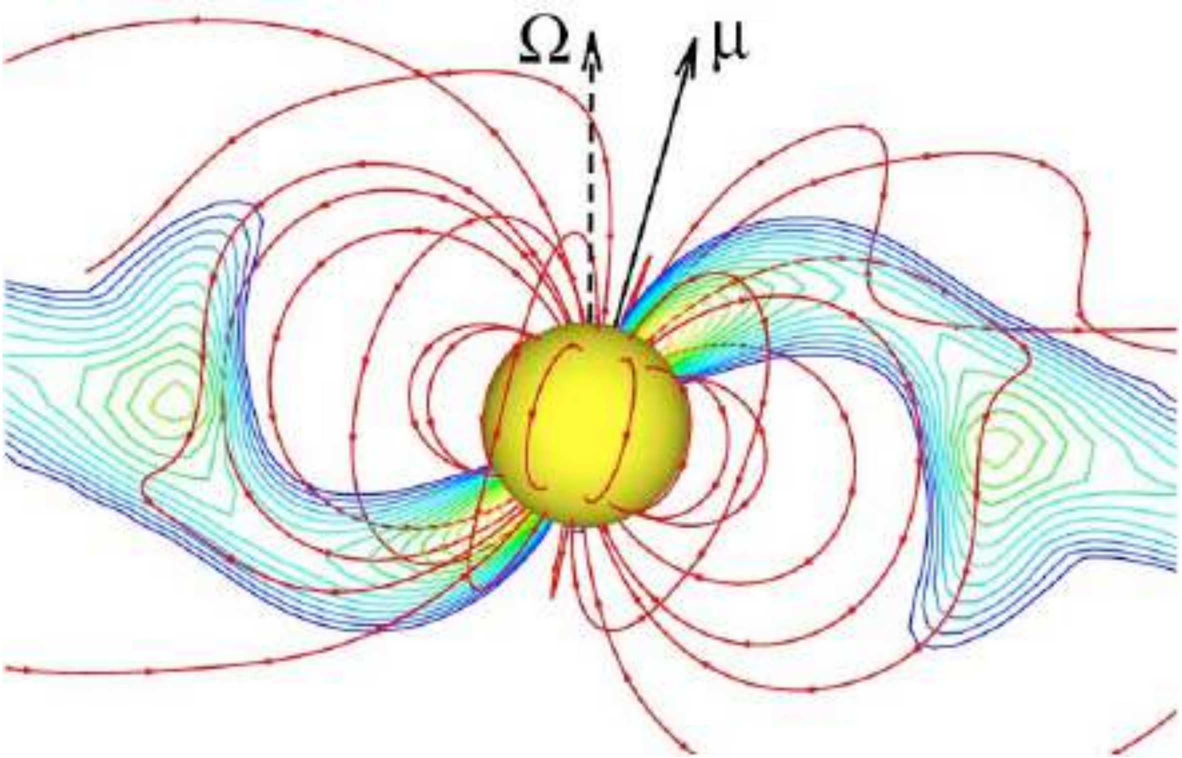


Name	*Ecyc, keV*
4U 0115+63	~11, 21, 34, 45
V 0332+53	~28, 55
A 0535+26	~45, 100
Hercules X-1	~38
Vela X-1	~22, 56
GX 301-2	~50
GX 1+4	~34
Cen X-3	~31
OAO 1657-415	~78
EXO 2030+375	~63

$$E_{cyc} = 11.6\ B_{12}\ keV$$

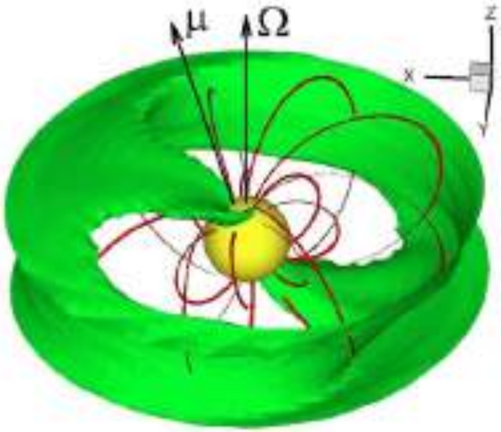
Filippova et al. (2005)

X-ray pulsars

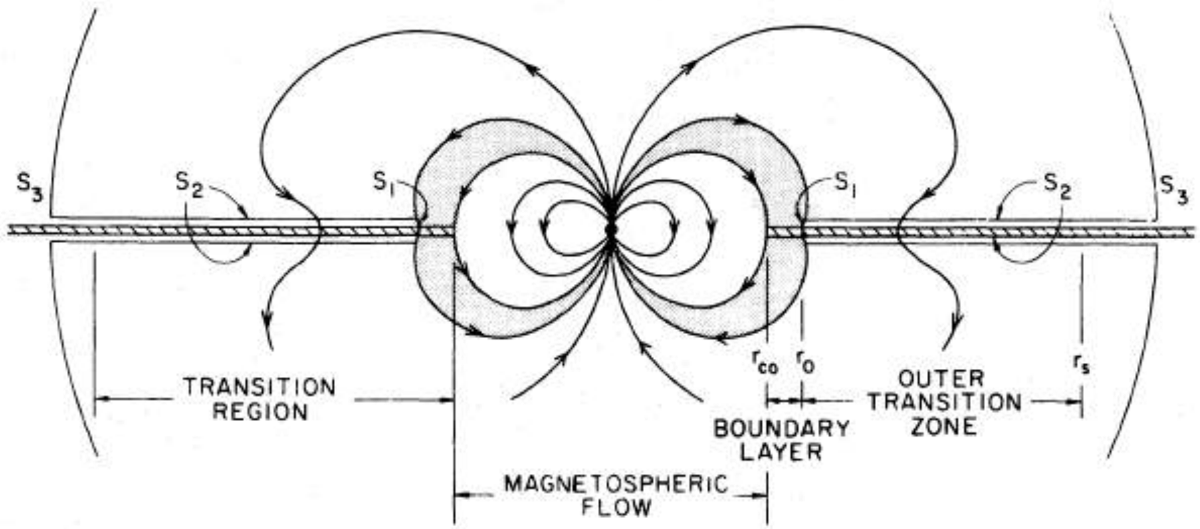


There are two main characteristic radii related to the accretion onto the magnetized NS:

- Magnetospheric radius
- Corotation radius



Romanova et al. 2009



Magnetospheric radius (Alfven radius)

Magnetic pressure equals the matter pressure

$$\frac{B^2}{8\pi} = \rho V^2$$

Free fall velocity

$$\frac{GMm}{r} = \frac{1}{2}mV^2 \qquad V = \sqrt{\frac{2GM}{r}}$$

Accretion rate; continuity equation

$$\dot{M} = 4\pi r^2 \rho V \qquad \rho = \frac{\dot{M}}{4\pi r^2 V}$$

Magnetic field strength in the dipole configuration

$$B = B_0 \left(\frac{R_{ns}}{r}\right)^3$$

(/2) *for dipole magnetic field*

$$\frac{B_0^2 R_{ns}^6}{r^6 8\pi} = \frac{\dot{M}}{4\pi r^2}\sqrt{\frac{2GM}{r}}$$

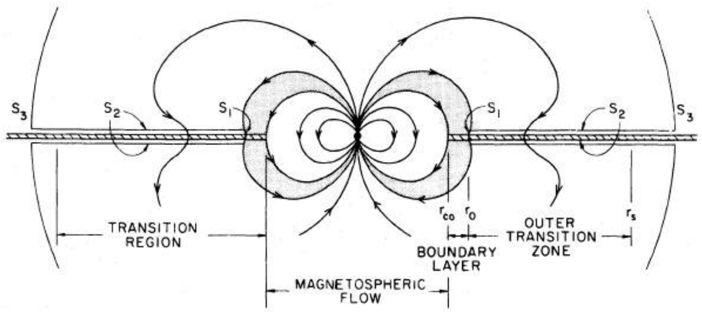
Magnetospheric radius (Alfven radius)

$$\frac{B_0^2 R_{ns}^6}{r^6 8\pi} = \frac{\dot{M}}{4\pi r^2}\sqrt{\frac{2GM}{r}}$$

$$\frac{B_0^4 R_{ns}^{12}}{r^8} = \frac{8GM\dot{M}^2}{r}$$

Magnetospheric radius

$$r_A = \left(\frac{B_0^4 R_{ns}^{12}}{8GM\dot{M}^2}\right)^{1/7}$$



$$r_m = \xi r_A \qquad \xi \sim 0.5$$

$$R_m \simeq 1.8 \times 10^8 \xi B_{12}^{4/7} \dot{M}_{17}^{-2/7} M_{\odot}^{-1/7} R_6^{12/7} cm$$

$$R_{\rm m} \simeq 2.5 \times 10^8\, \xi\, M_{1.4}^{1/7}\, R_6^{10/7}\, B_{12}^{4/7}\, L_{37}^{-2/7} \quad {\rm cm}$$

Corotation radius

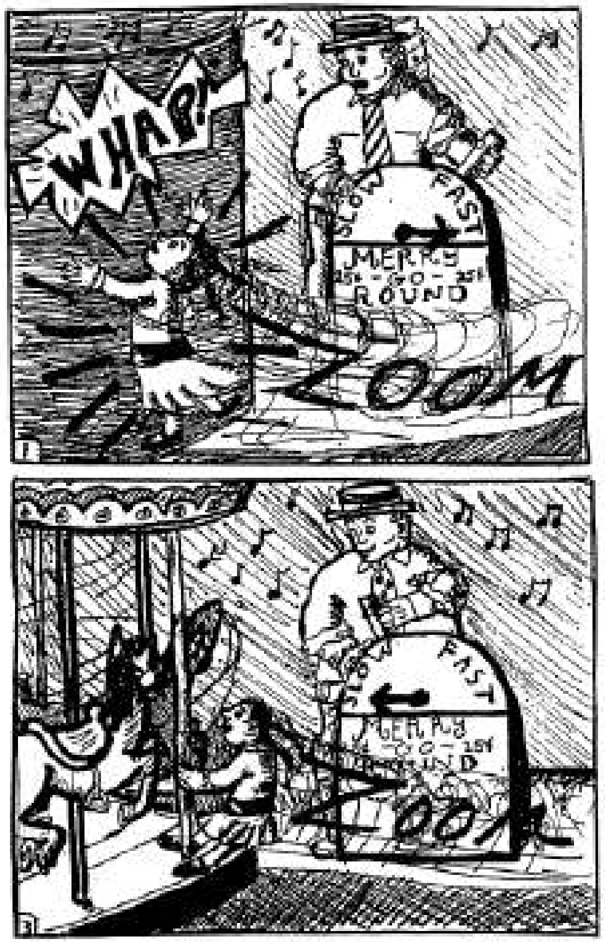
Radius where velocity of the magnetic field lines is equal to the Keplerian one

$$\omega R = \sqrt{\frac{GM}{R}}$$

$$R_c = \left(\frac{GM}{\omega^2}\right)^{1/3}$$

$$R_{\rm c} = \left(\frac{GMP^2}{4\pi^2}\right)^{1/3} \simeq 1.68 \times 10^8 \, M_{1.4}^{1/3} \, P^{2/3} \quad \rm cm$$

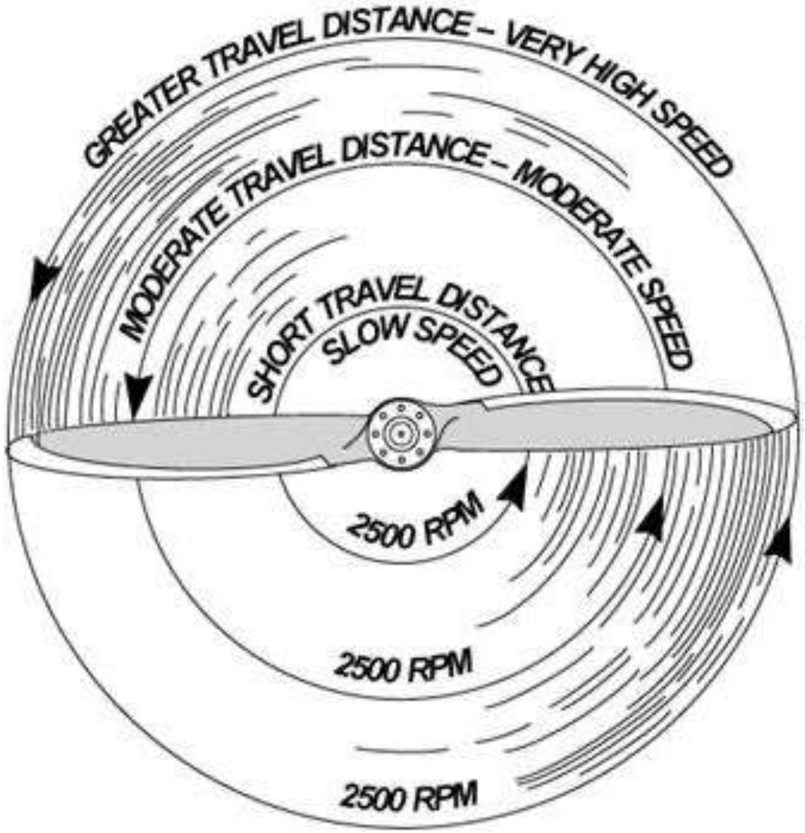
“Propeller” effect



Patterson, 1994

$R_m < R_c$ - accretion is possible

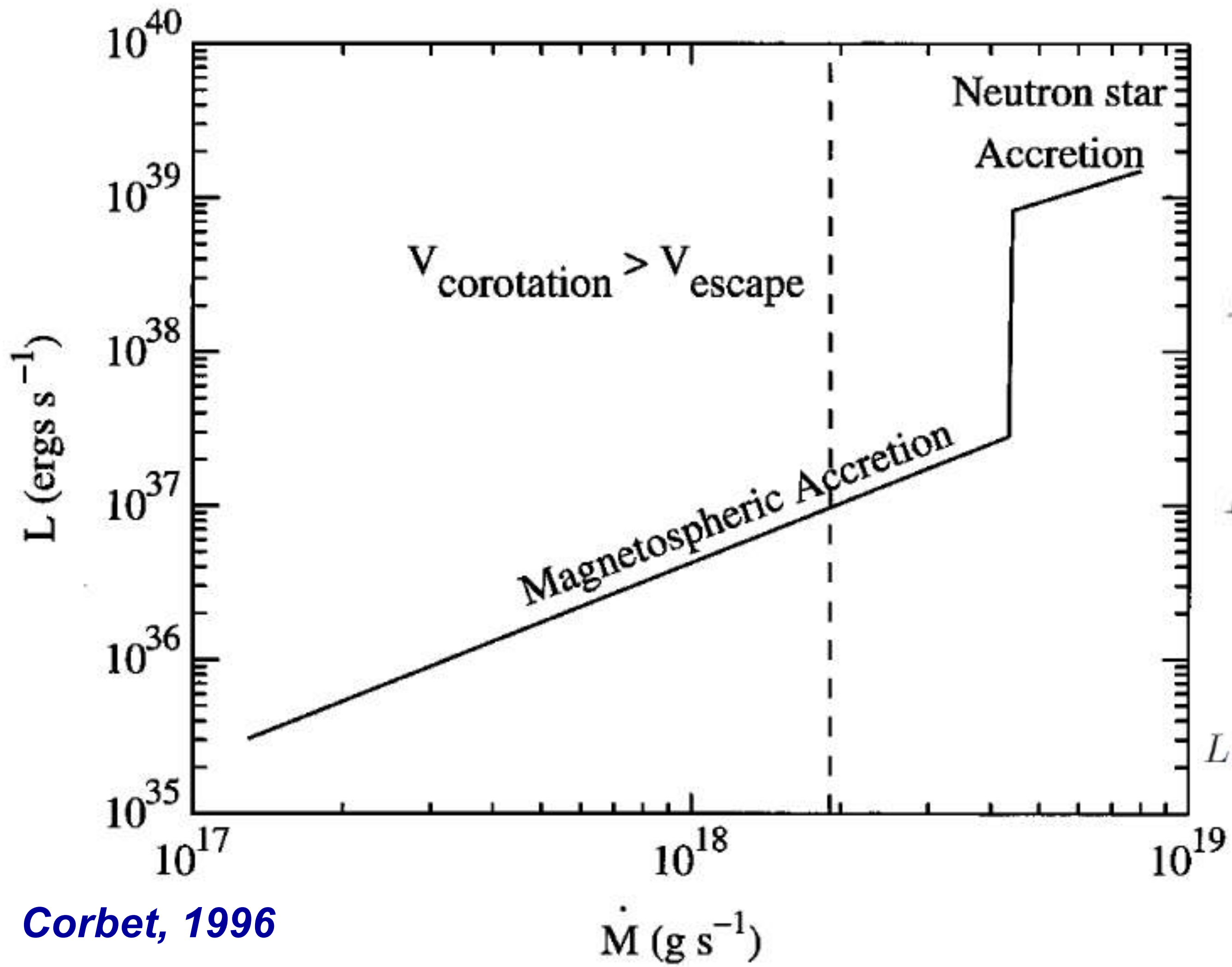
$R_m > R_c$ - accretion is prohibited due to centrifugal barrier



“Propeller effect”

Illarionov & Sunyaev, 1975

“Propeller” effect



Corbet, 1996

$$R_c = R_m$$

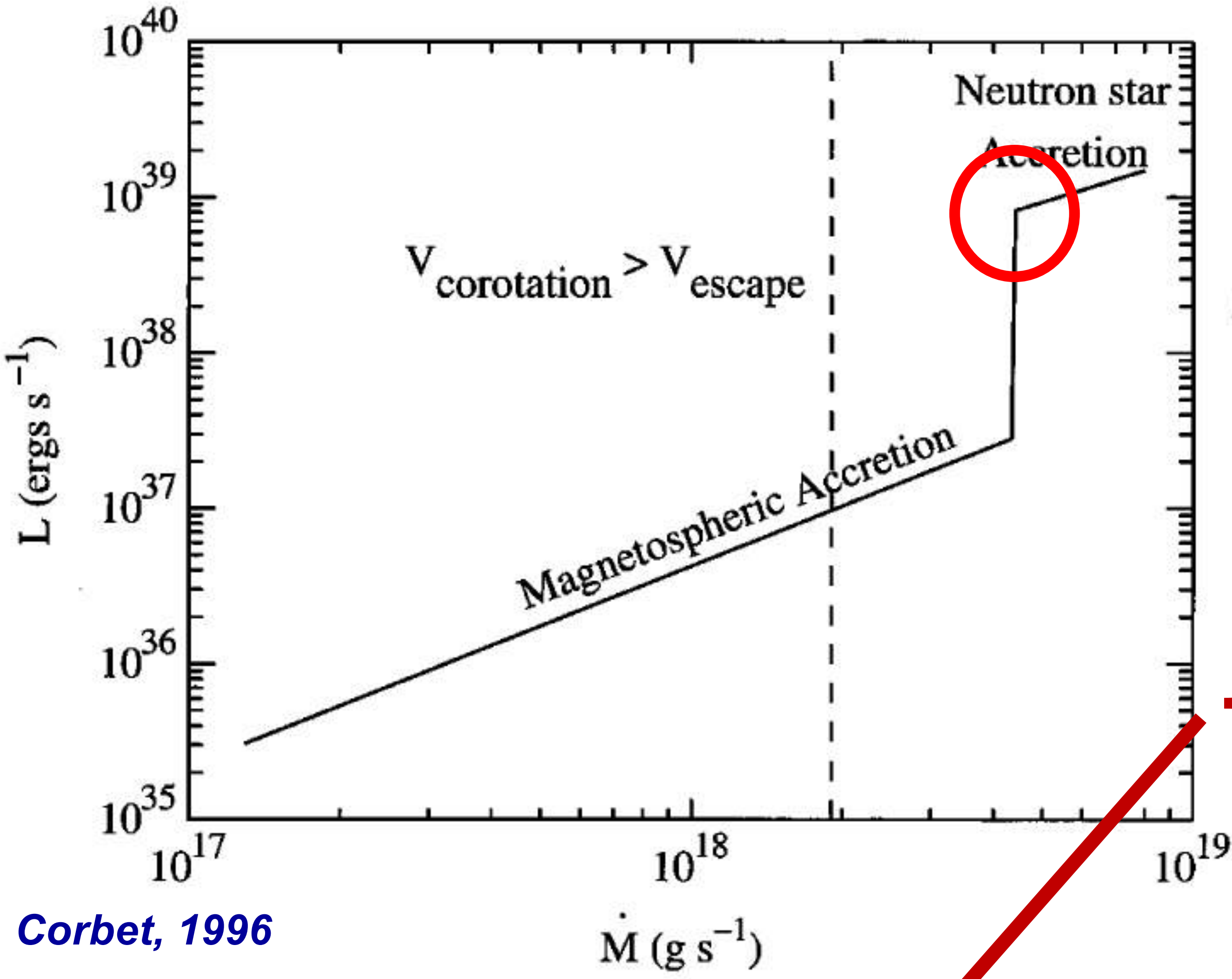
$$L_{lim}(R) \simeq \frac{GM\dot{M}_{lim}}{R}$$

$$L_{lim}(R_c) = \frac{GM\dot{M}_{lim}}{R_c}$$

$$L_{lim}(R_c) = \frac{GM\dot{M}_{lim}}{R_c} = L_{lim}(R)\frac{R}{R_c}$$

$$r_A = \left(\frac{B_0^4 R_{ns}^{12}}{8GM\dot{M}^2}\right)^{1/7}$$

“Propeller” effect



Corbet, 1996

$$R_c = R_m$$

$$L_{\rm lim}(R) \simeq \frac{GM\dot{M}_{\rm lim}}{R}$$

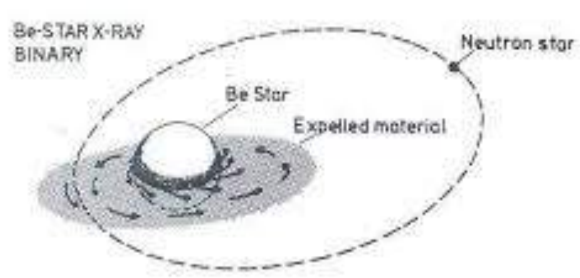
$$r_{co} = \left(\frac{GM}{\Omega^2}\right)^{1/3}$$

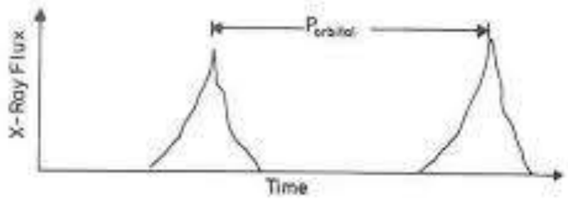
$$r_A = \left(\frac{B_0^4 R_{ns}^{12}}{8GM\dot{M}^2}\right)^{1/7}$$

$$r_m = \xi r_A$$

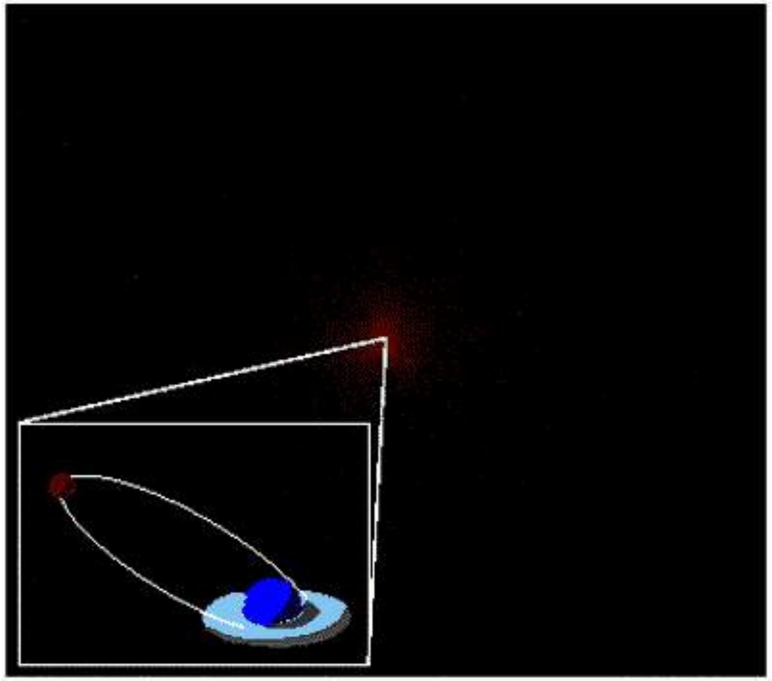
$$L_{\rm lim}(R) \simeq \frac{GM\dot{M}_{\rm lim}}{R} \simeq 4\times10^{37}\xi^{7/2}B_{12}^2P^{-7/3}M_{1.4}^{-2/3}R_6^5\ {\rm erg\ s^{-1}}$$

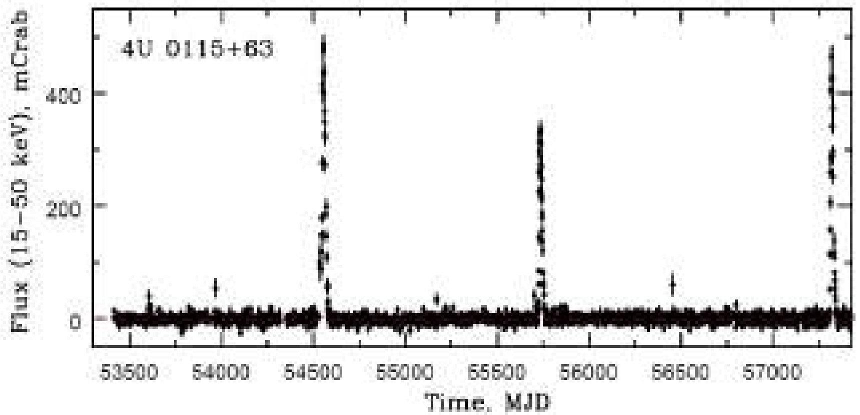
Transient X-ray pulsars

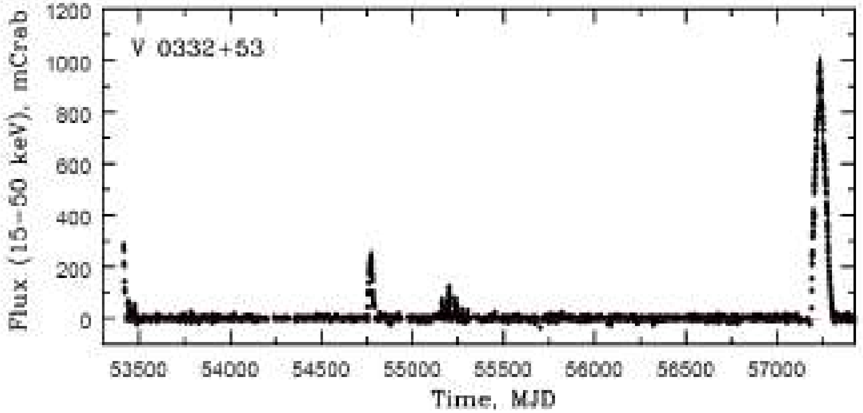




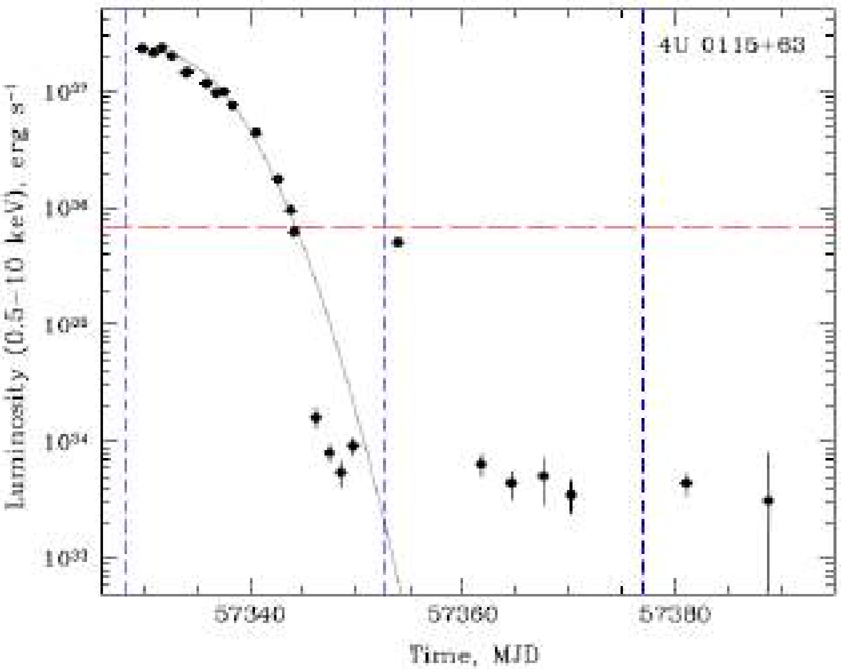
Neutron star in binary system with Be optical companion demonstrates periodic episodes of accretion due to interaction with decretion disc of the companion.

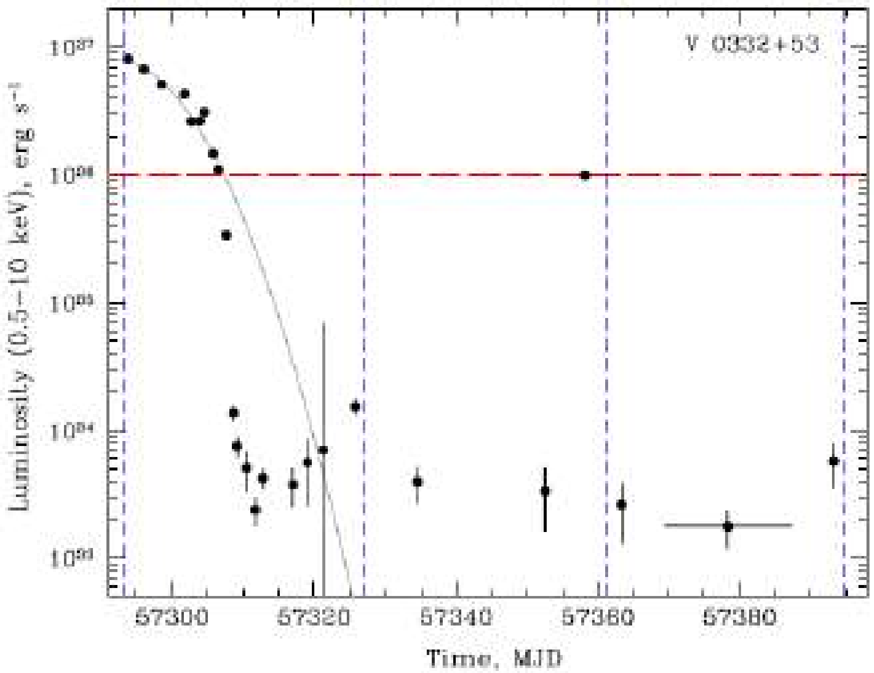


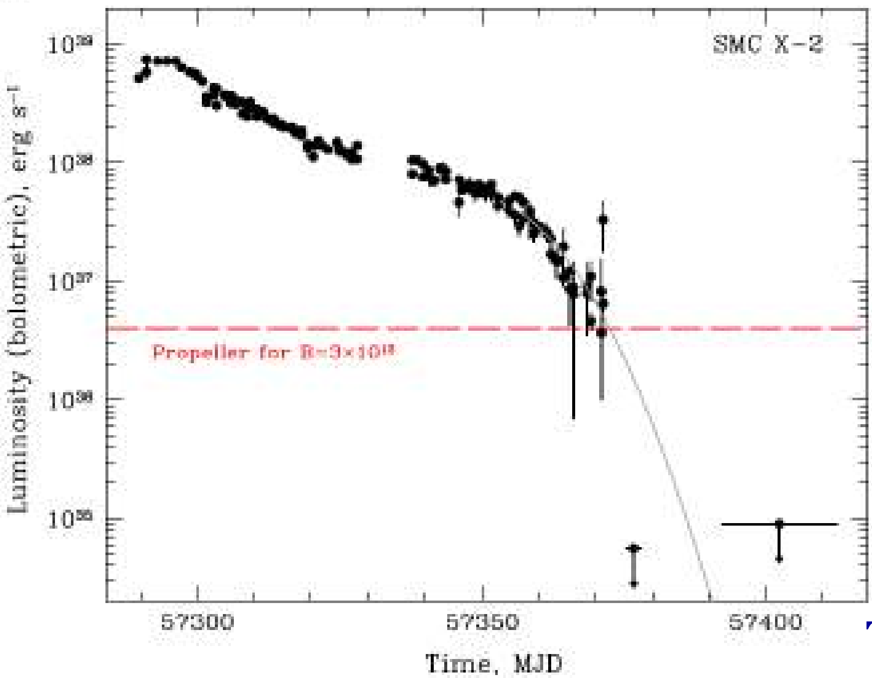




Temporal analysis

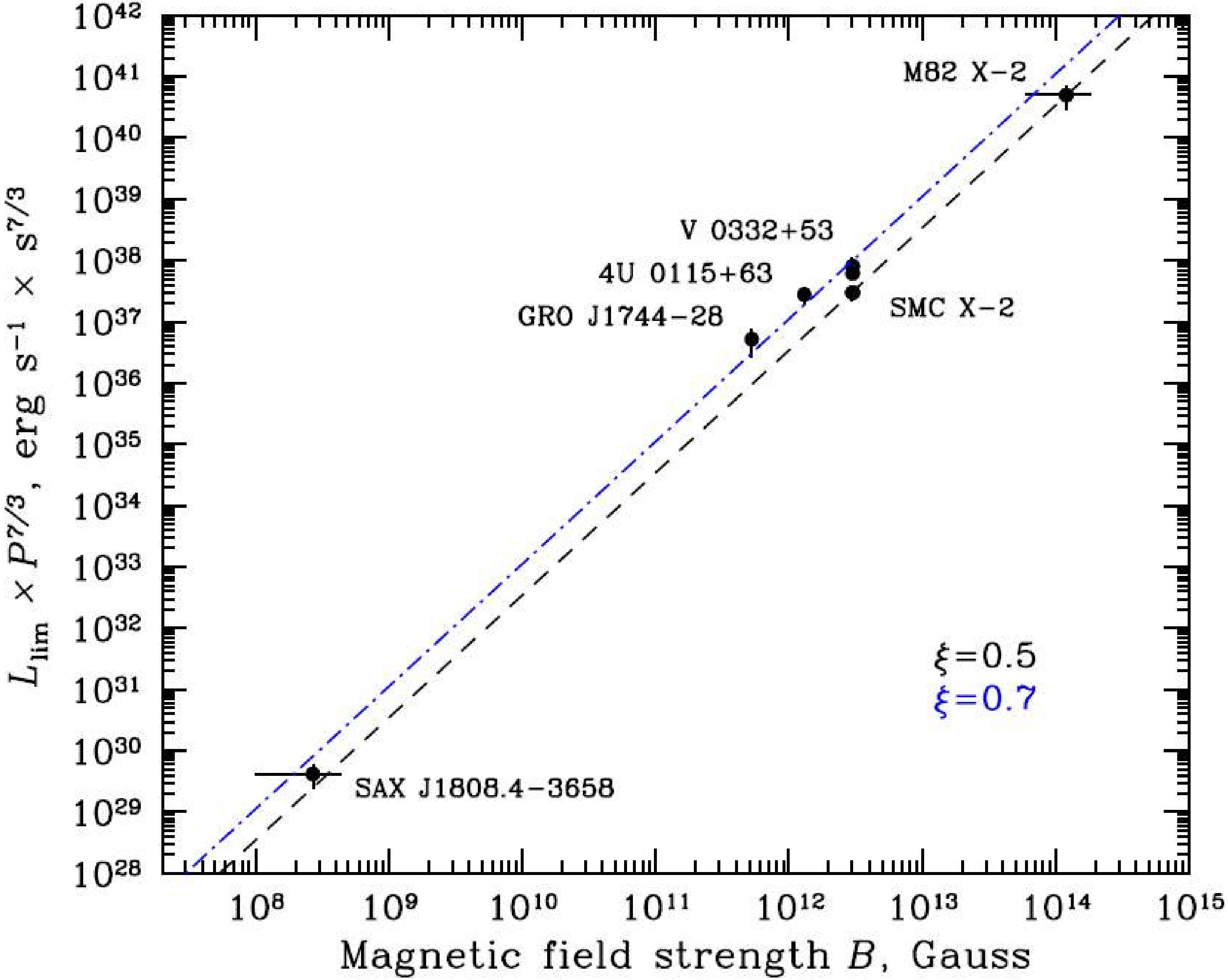






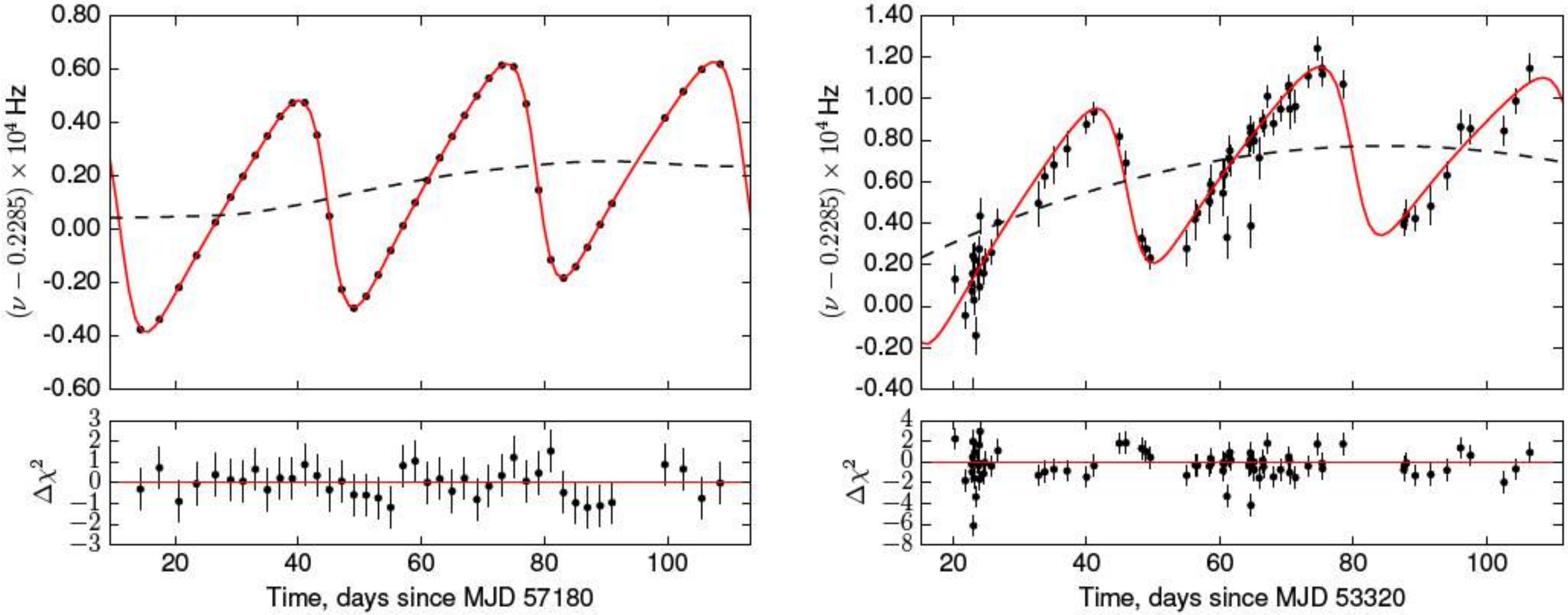
Tsygankov et al. (2016)

"Propeller" effect



$$L_{\rm lim}(R) \simeq \frac{GM\dot{M}_{\rm lim}}{R} \simeq 4\times10^{37}\xi^{7/2}B_{12}^{2}P^{-7/3}M_{1.4}^{-2/3}R_6^5\,{\rm erg\ s^{-1}}$$

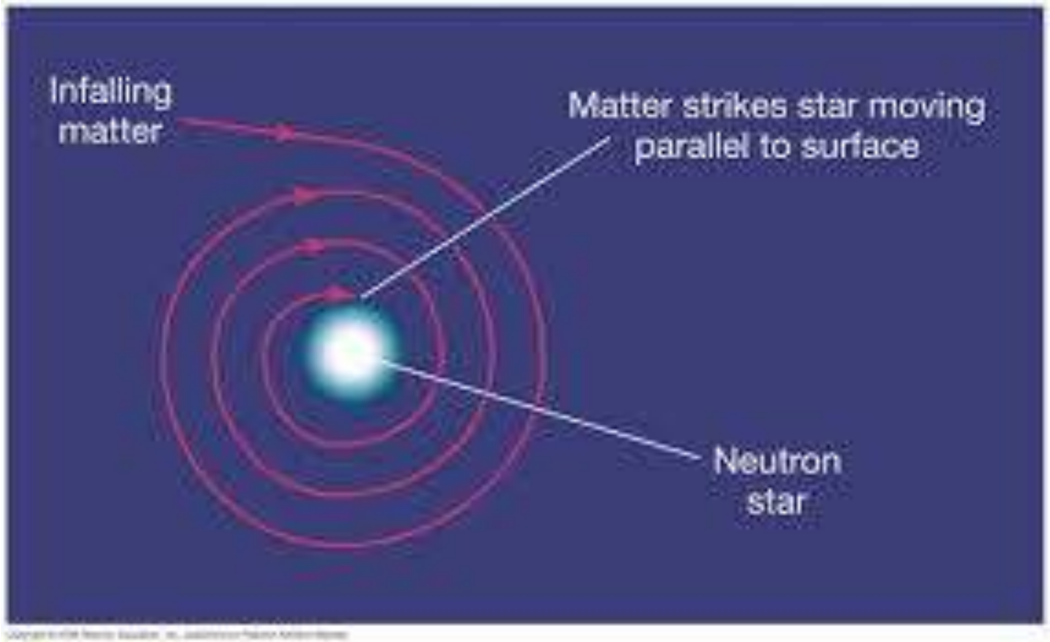
Pulse period evolution

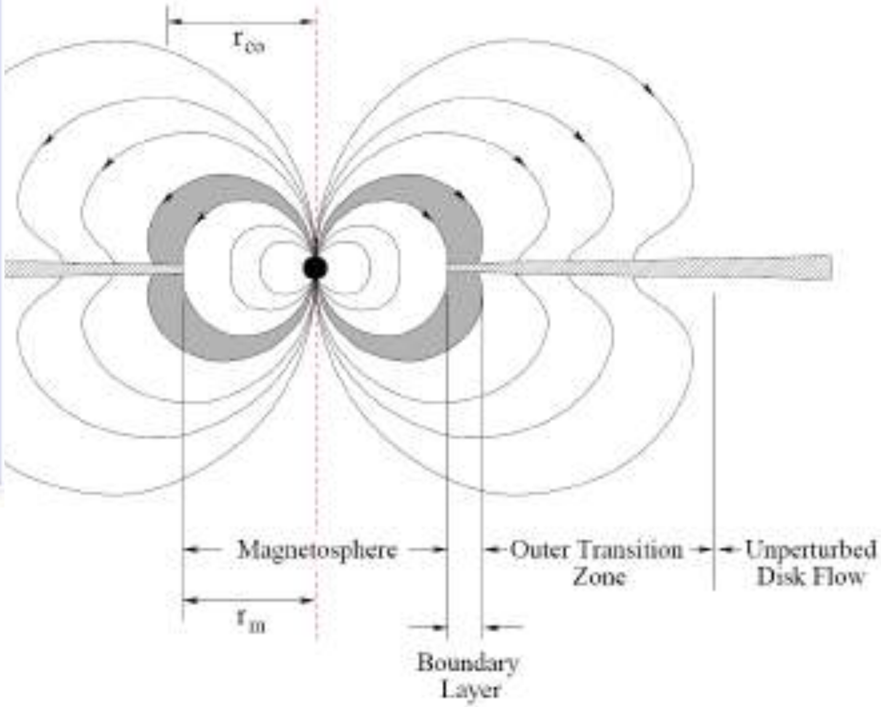


Pulse period of the transient X-ray pulsar V 0332+53 during bright outbursts in 2015 and 2004.

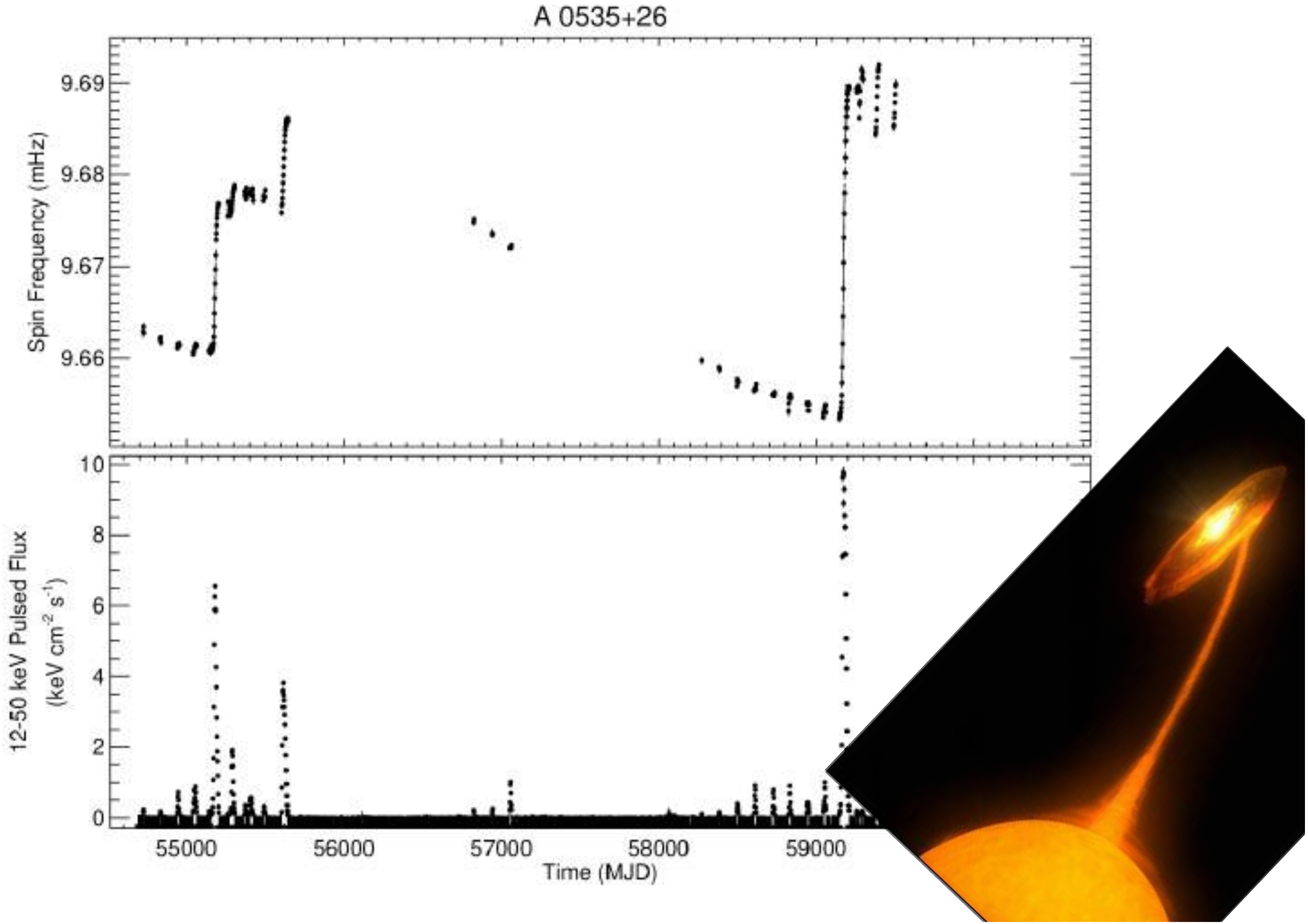
Pulse period evolution

Matter moving in the accretion disk carries a significant angular momentum. When matter interacts with the NS magnetic field, this angular momentum accelerates rotation of the star.

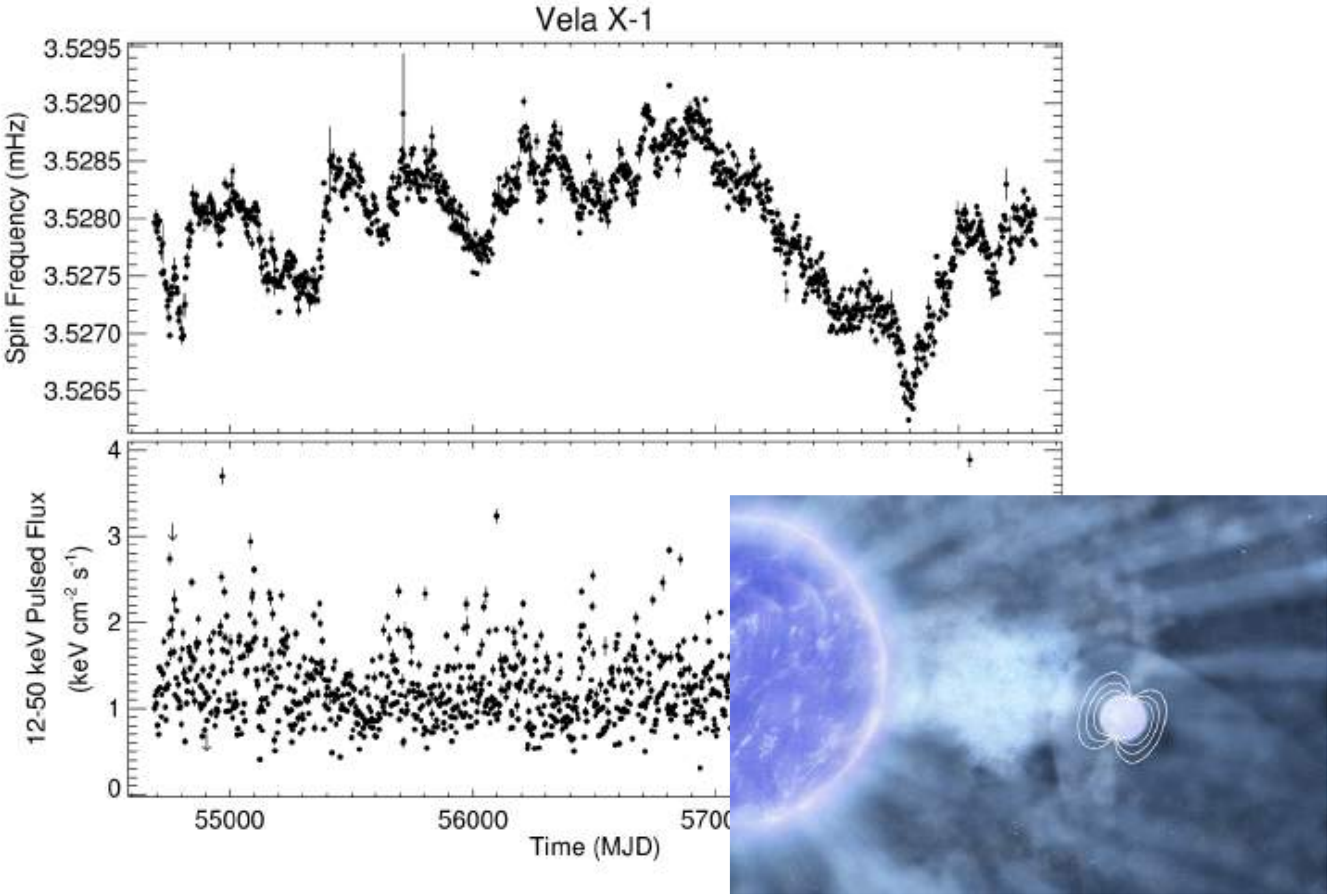




Pulse period evolution



Pulse period evolution



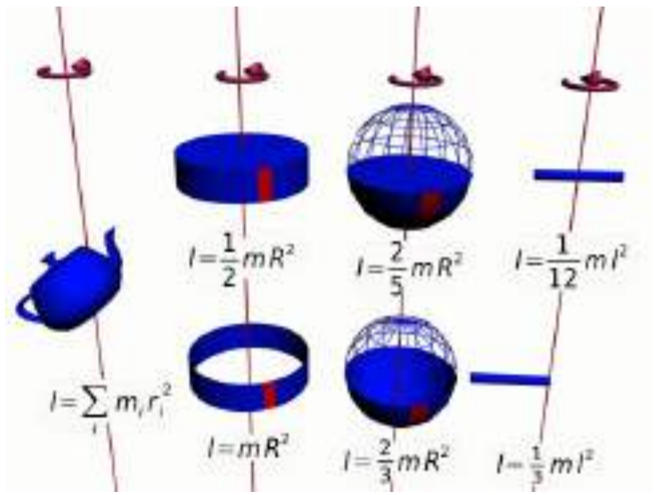
Pulse period evolution

Angular moment of matter in the accretion disc

$$L = I \times \omega = r^2 \times m \times \omega$$

Specific angular momentum

$$l = \frac{L}{m} = r \times V \qquad \omega = \frac{V}{r}$$



$$l(r) = \sqrt{GMr} \qquad V_{\rm K} = \sqrt{\frac{GM}{r}}$$

I – moment of inertia

Torque (moment of force)

$$N_{\rm acc} = \frac{{\rm d}L}{{\rm d}t} = \frac{{\rm d}I\omega}{{\rm d}t} = I\frac{{\rm d}\omega}{{\rm d}t}$$

Pulse period evolution

$$\omega = \frac{2\pi}{P} \qquad N_{\rm acc} = \frac{{\rm d}L}{{\rm d}t} = \frac{{\rm d}I\omega}{{\rm d}t} = I\frac{{\rm d}\omega}{{\rm d}t}$$

$$N_{\rm acc} = I\frac{{\rm d}\omega}{{\rm d}t} = -2\pi I\frac{\dot{P}}{P^2}$$

$$N_{\rm acc} \approx \dot{M} l(r_{\rm m}) \approx \dot{M}\sqrt{GMr_{\rm m}}$$

$$-\dot{P} = \frac{N_{\rm acc}P^2}{2\pi I} \approx \frac{\dot{M}\sqrt{GMr_{\rm m}}P^2}{2\pi I}$$

$$R_{\rm m} \simeq 2.5\times 10^8\, \xi\, M_{1.4}^{1/7}\, R_6^{10/7}\, B_{12}^{4/7}\, L_{37}^{-2/7} \quad \text{cm}$$

Pulse period evolution

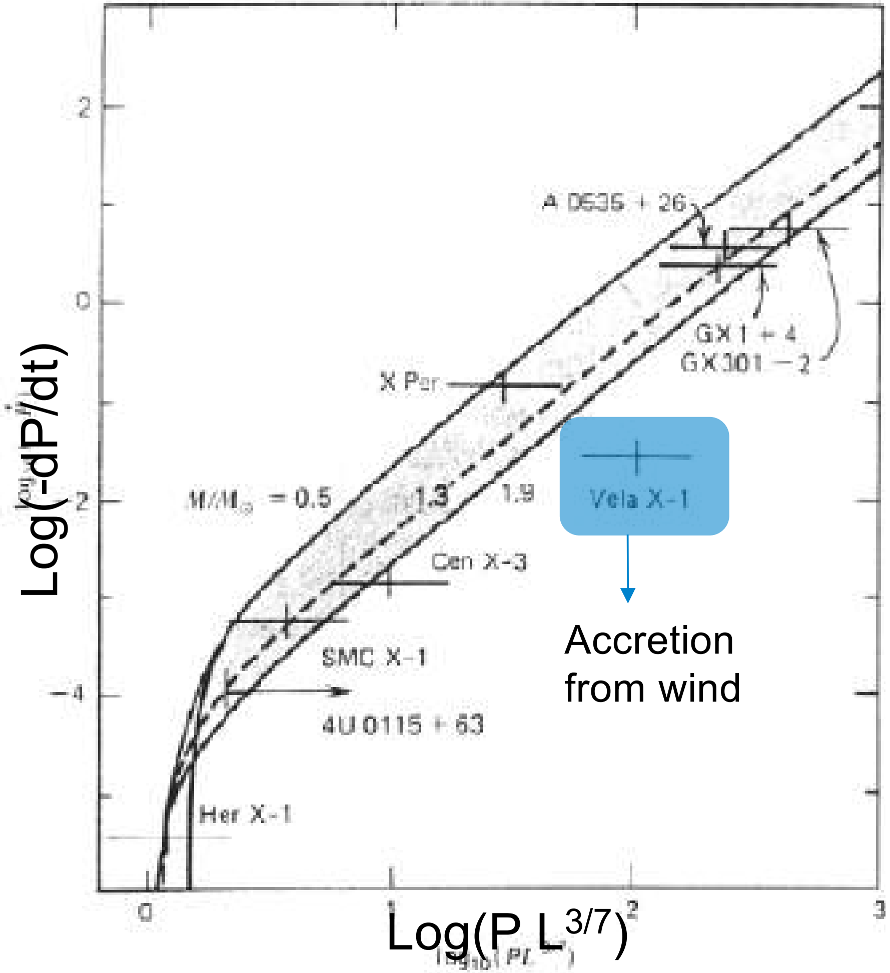
Prediction of the Ghosh and Lamb (1979) theory is a relation between the spin-up rate, the period, luminosity and magnetic moment:

$$-\dot{P} = 5.0 \times 10^{-5} \mu_{30}{}^{2/7} n(\omega_s) S_1(M) (PL_{37}{}^{3/7})^2 \text{ s yr}^{-1}$$

$\mu = B_0 R^3/2$ – magnetic moment

$$-\dot{P} = f(PL^{3/7})$$

for given mass M and magnetic moment μ.



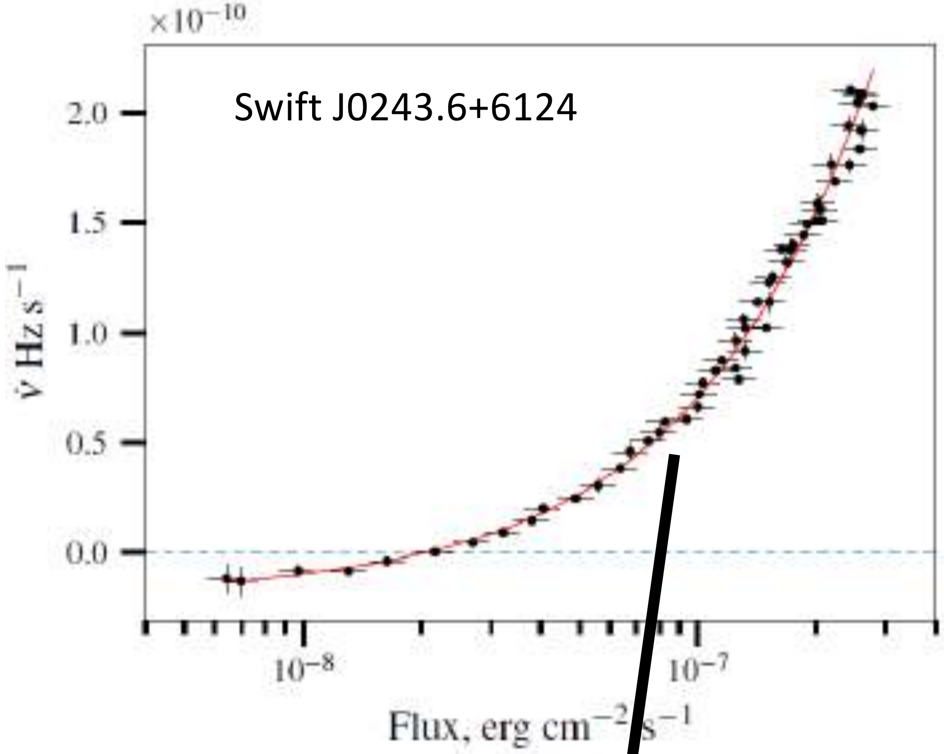
But if μ is unknown?

Shapiro, Teukolsky (1983)

Pulse period evolution

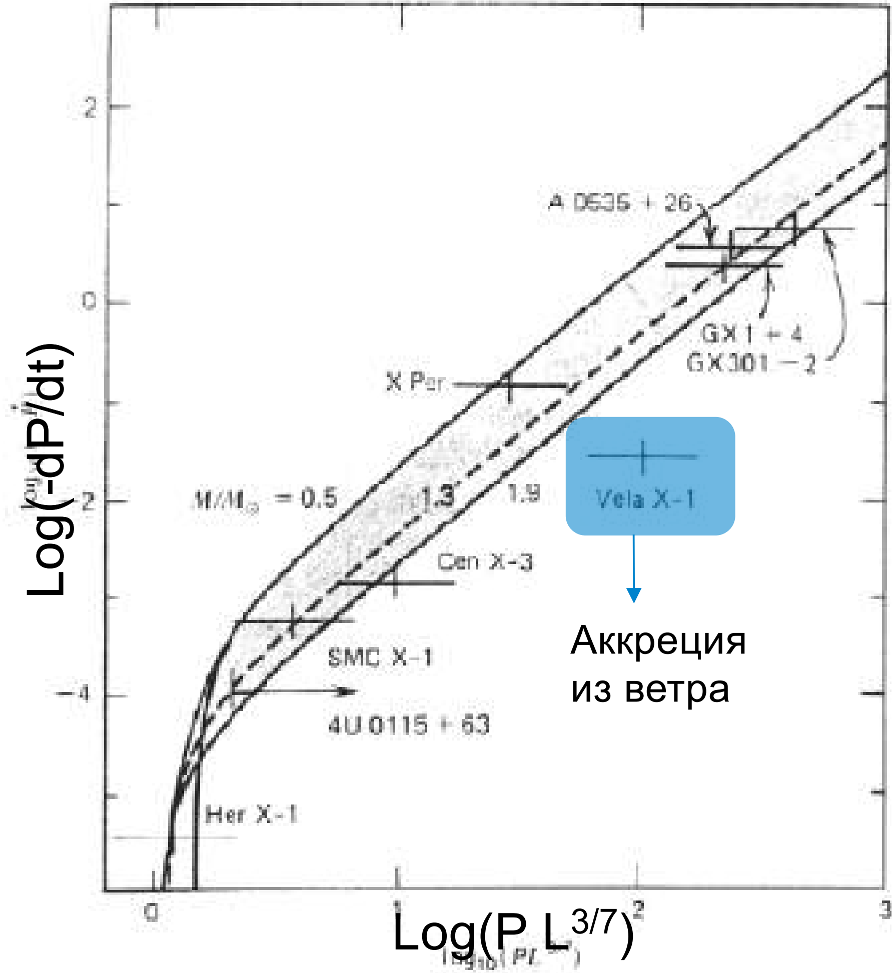
Prediction of the Ghosh and Lamb (1979) theory is a relation between the spin-up rate, the period, luminosity and magnetic moment:

$$-\dot{P} = 5.0 \times 10^{-5} \mu_{30}{}^{2/7} n(\omega_s) S_1(M) (P L_{37}{}^{3/7})^2 \text{ s yr}^{-1}$$



Doroshenko et al. (2018)

B~10^{13} G, d~7 kpc



Shapiro, Teukolsky (1983)

Spin-up time

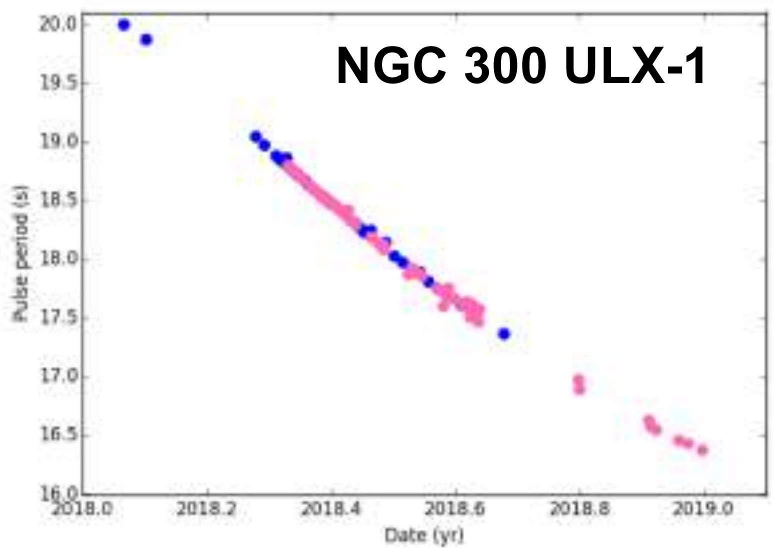
$$-\dot{P} = \frac{N_{\rm acc} P^2}{2\pi I} \approx \frac{\dot{M}\sqrt{GMr_{\rm m}}P^2}{2\pi I}$$

$$\frac{|\dot{P}|}{P} \propto P L^{6/7}$$

Spin-up time ($\ll$ pulsar age)

$$t_s = \frac{P}{|\dot{P}|} = 2^{17/14}\pi I P^{-1}\dot{M}^{-6/7}\mu^{-2/7}(GM)^{-3/7} =$$

$$2\times10^5 \text{ yr}\left(\frac{10^{-10} M_{sun}\text{yr}^{-1}}{\dot{M}}\right)\left(\frac{1\text{ s}}{P}\right)^{4/3}\left(\frac{R_{co}}{R_m}\right)^{1/2}$$



Note that about a Gy is needed to reach a ms period. For ULX pulsars (with $\dot{M}\sim10^{-6} M_\odot$), the spin-up time is $\lesssim$100 yr.

Spin equilibrium

If magnetospheric radius is smaller than corotational radius, then the accreting matter rotates faster than the star, which is spun-up as a result (and vice versa). The spin equilibrium is achieved when $R_m \approx R_{co}$:

$$L = \eta \dot{M} c^2, \text{define } m = M / M_o, \ \dot{m} = \frac{\eta \dot{M} c^2}{L_{Edd}} = \frac{\eta \dot{M} c^2}{1.3\, 10^{38} m}$$

$$1.5\, 10^8 B_{12}^{4/7} R_6^{12/7} \dot{m}^{-2/7} m^{-3/7} = 1.5\, 10^8 P_s^{2/3} m^{1/3}$$

$$P_s = B_{12}^{6/7} R_6^{18/7} \dot{m}^{-3/7} m^{-8/7} \text{s}$$

For $\dot{m} = 1, R_6 = 1, m = 1.4$

$$P_{s,\min} = 0.7\,\text{s}\ B_{12}^{6/7} = 1.8\,\text{ms}\ B_9^{6/7}$$

- ”spin-up” line

$$P_{s,\min} = 1.8\,\mathrm{ms}\; B_9^{6/7}$$

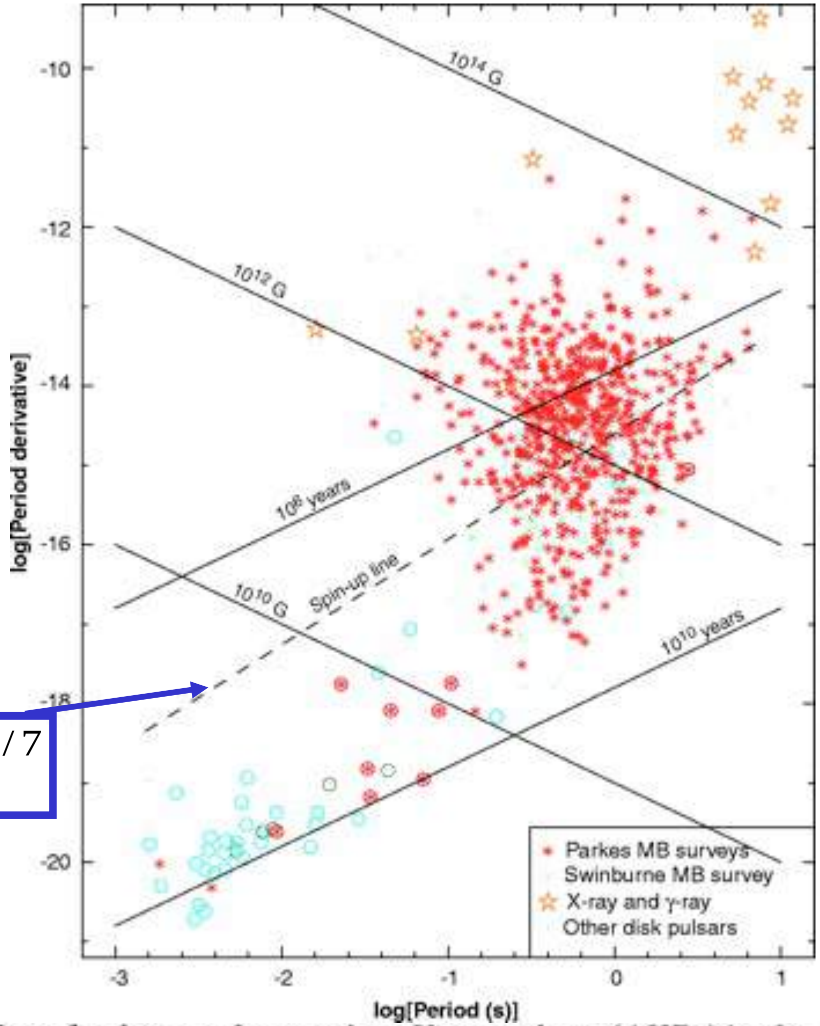
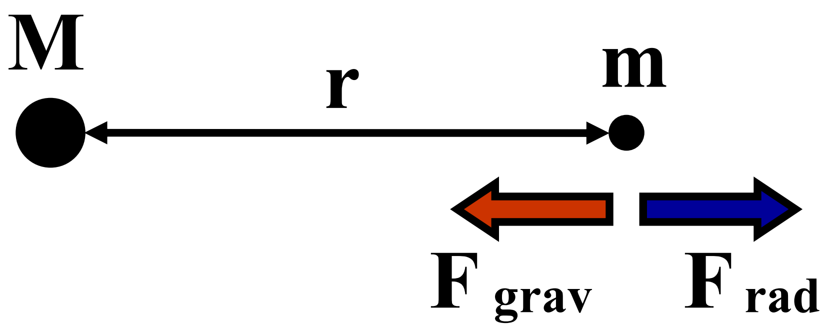


Figure 5: Distribution of pulsars and anomalous X-ray pulsars (AXPs) in the $P - \dot{P}$ plane. Binary systems are indicated by a circle around the point. Lines of constant pulsar characteristic age, $\tau_c = P/(2\dot{P})$ and surface dipole magnetic field strength, $B_s \propto (P\dot{P})^{1/2}$ are indicated. The spin-up line, representing the minimum period attainable by accretion from a binary companion, is also shown.

$$B \approx \sqrt{3c^3 IP\dot{P} / 2\pi^2 R^6}\,\mathrm{Gauss} \approx 6\times10^{19}\mathrm{G}\sqrt{P\dot{P}}$$

Eddington luminosity (Eddington limit)



Outgoing photons from *M* scatter off accreting material (electrons and protons).

$$P = \frac{1}{c}F = \frac{1}{c}\frac{L}{4\pi r^2}$$

where r is the distance to the source, F is the radiation flux and L is the source luminosity.

$$F_{grav} = \frac{GM\left(m_p + m_e\right)}{r^2}$$

$$F_{\mathrm{rad}} = P\sigma_T = \frac{\sigma_T}{c}\frac{L}{4\pi r^2}$$

Electrostatic forces between e^- and p bind them so they act as a pair, so $m \approx m_\mathrm{p}$

$\sigma_T = \frac{8\pi}{3} r_e^2 \approx 0.665\times10^{-24}\mathrm{cm}^2$ - Thomson cross-section

Eddington luminosity (Eddington limit)

radiation pressure = gravitational pull

At this point accretion stops, effectively imposing a 'limit' on the luminosity of a given body.

$$\frac{L\sigma_T}{4\pi r^2 c} = G\frac{Mm}{r^2}$$

So the Eddington luminosity (for pure H plasma) is:

$$L_{\rm Edd} = \frac{4\pi G M m_{\rm p} c}{\sigma_{\rm T}} \approx 1.3 \times 10^{38} \frac{M}{M_\odot} \ \ \text{erg s}^{-1}$$

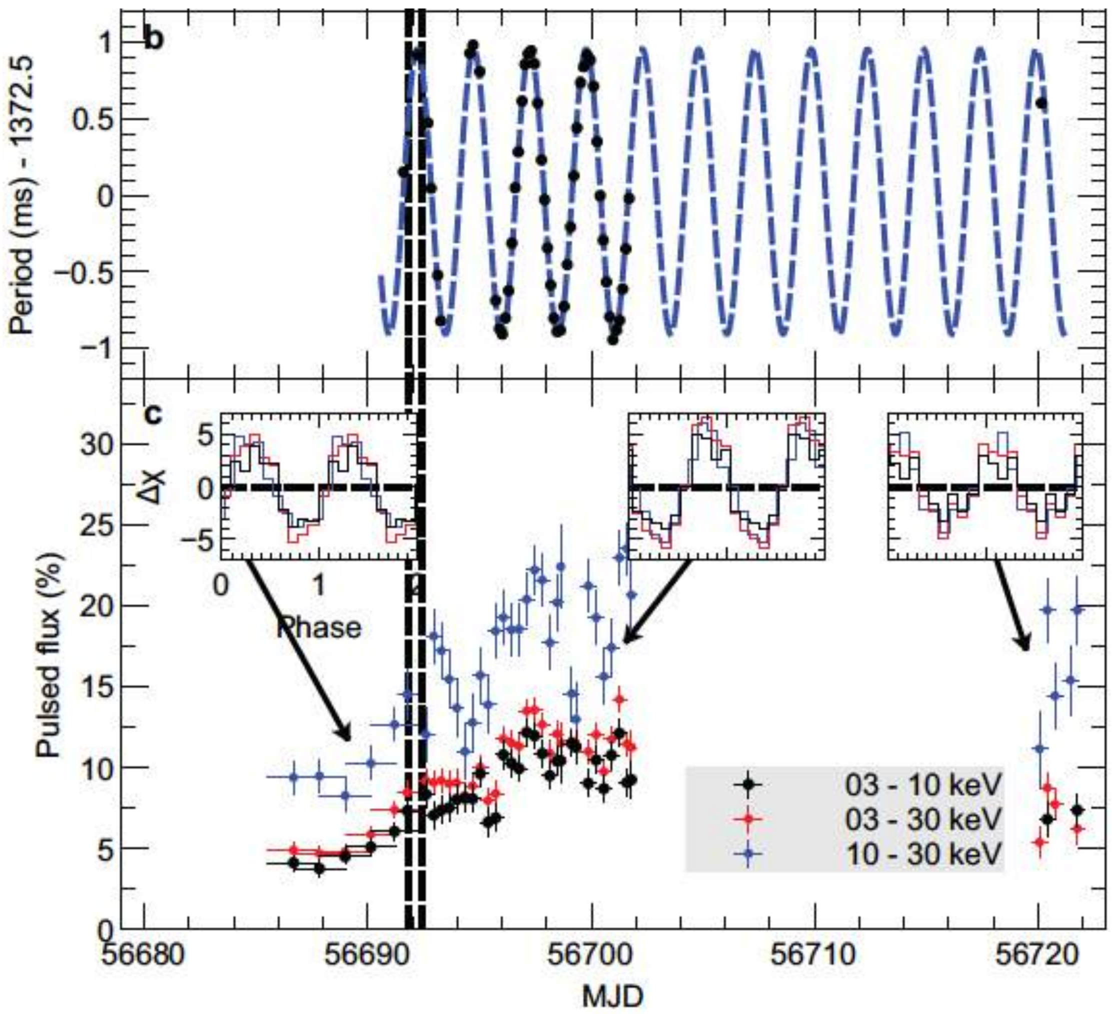
Ultraluminous X-ray sources (ULXs)

- Brightest extra-nuclear X-ray sources, with L_X>10^{39} erg s^{-1}
- Now around 500 candidates are known (Walton et al. 2011)

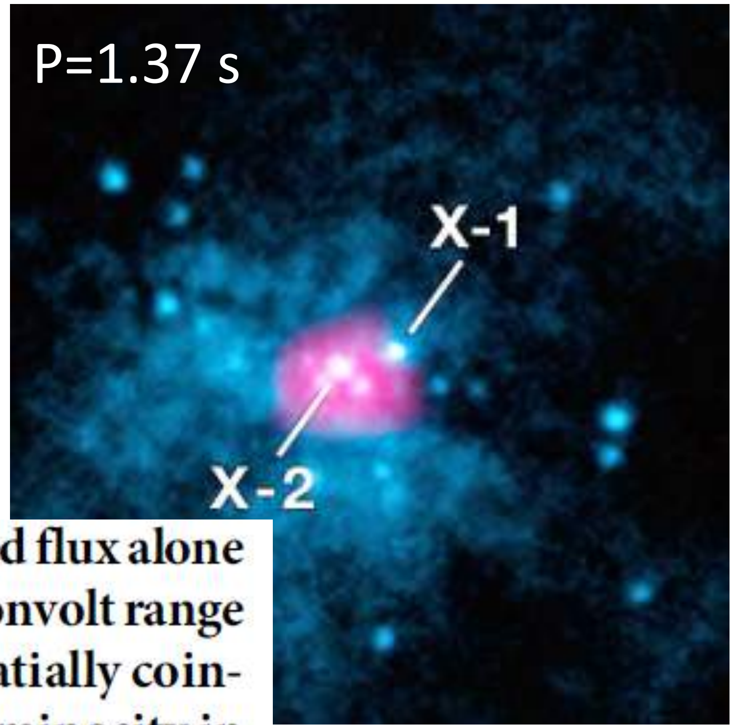
10^{39} erg s^{-1} is ~ Eddington limit for a 10 M_{sun} black hole

→ **ULX**: either (1) a new class of intermediate-mass black holes with M ~ 10^2 – 10^4 M_{sun} (IMBHs; e.g. Colbert & Mushotzky 1999) or (2) stellar mass black holes that exceed the Eddington limit, i.e. accrete in the most extreme environments (e.g. King 2001, Poutanen et al. 2007).

Discovery of pulsations from ULX in M82 galaxy



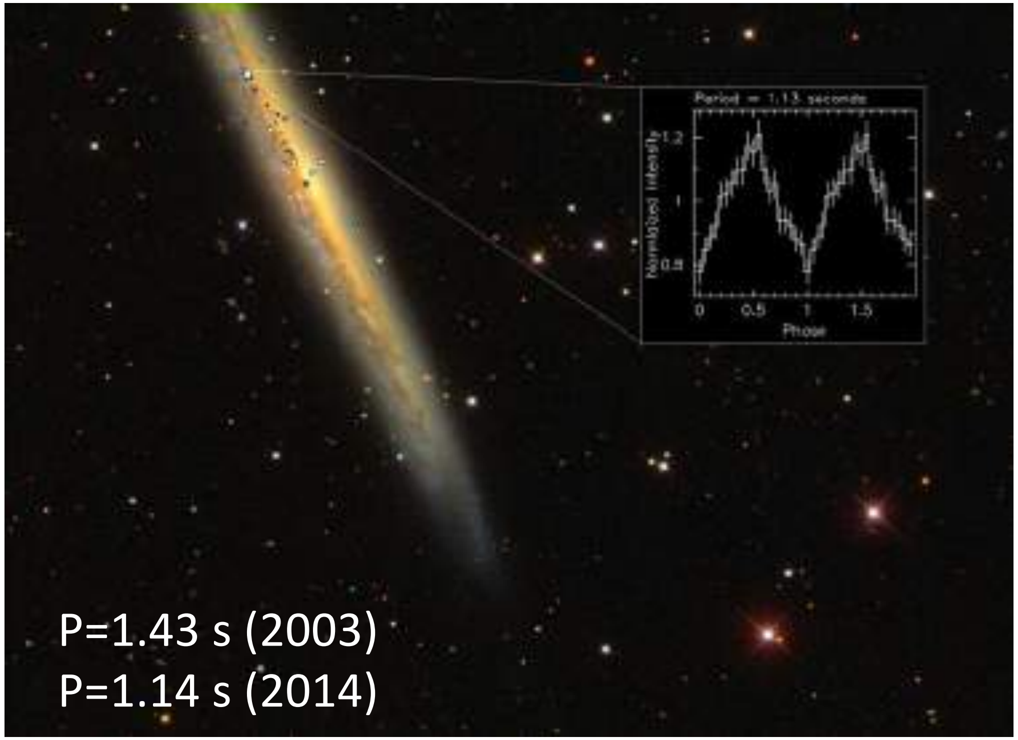
L~100 L_{Edd} for 1.4 M_{sun} mass object!

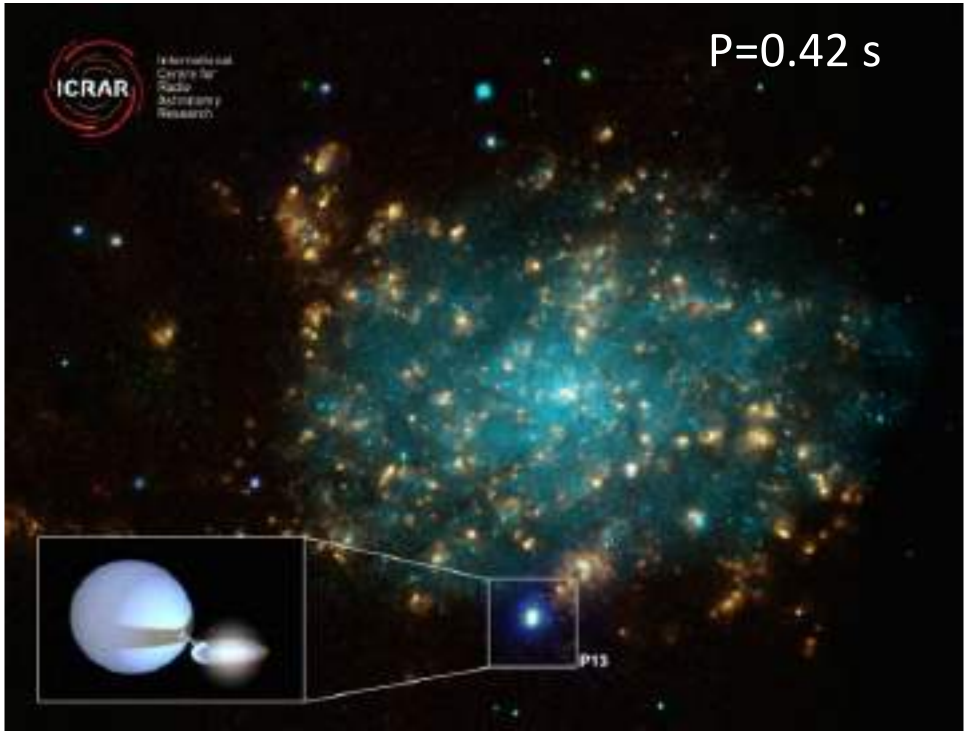


and the modulation arises from its binary orbit. The pulsed flux alone corresponds to an X-ray luminosity in the 3–30 kiloelectronvolt range of 4.9×10^{39} ergs per second. The pulsating source is spatially coincident with a variable source[4] that can reach an X-ray luminosity in the 0.3–10 kiloelectronvolt range of 1.8×10^{40} ergs per second[1]. This

Bachetti+ (2014)

More pulsating ULXs:
NGC7793 P13 and ULX-1 NGC5907





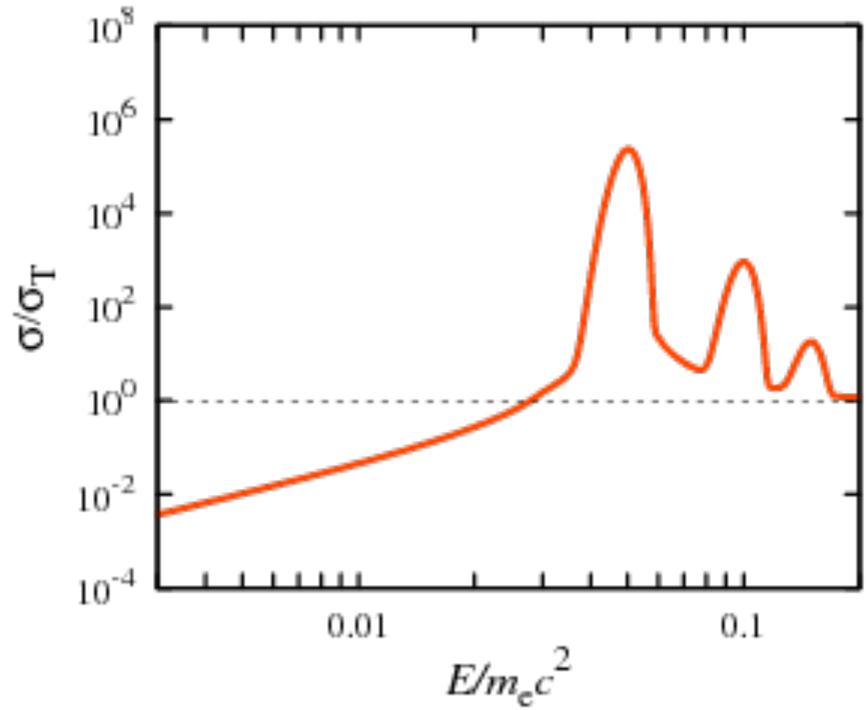
$L \sim 10^{41}$ erg s^{-1}

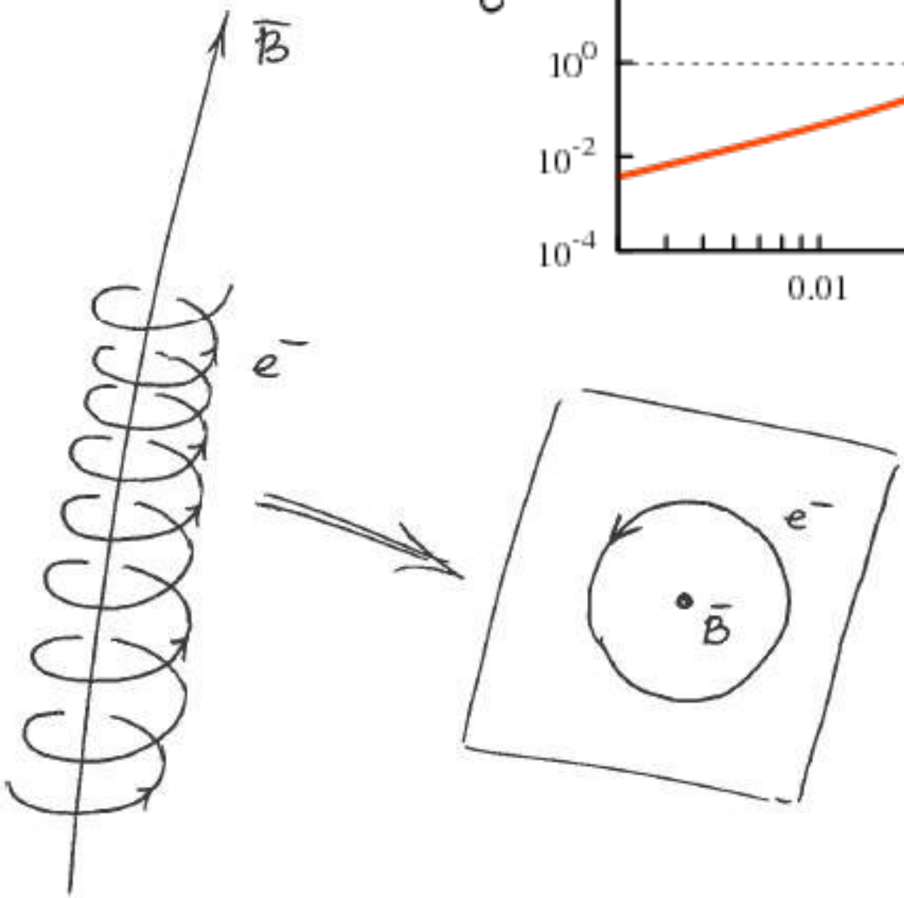
$L \sim 5x10^{39}$ erg s^{-1}

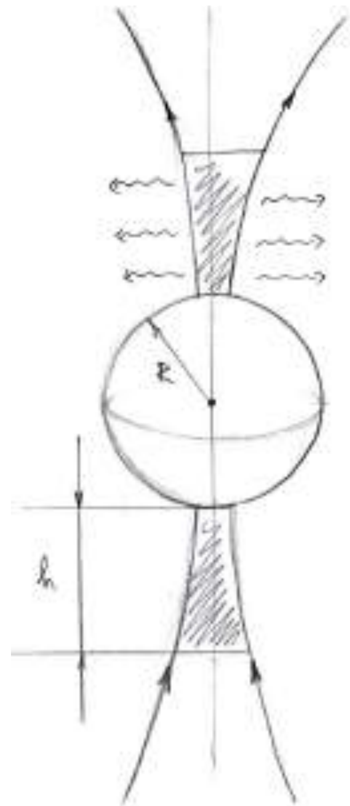
Israel+, MNRAS 2017; Fuerst+, ApJ, 2017,
Israel+, Science, 2017

Maximal luminosity of X-ray pulsar

$$\frac{\sigma_\perp}{\sigma_\mathrm{T}} \approx \left(\frac{E_\gamma}{E_\mathrm{cyc}}\right)^2$$



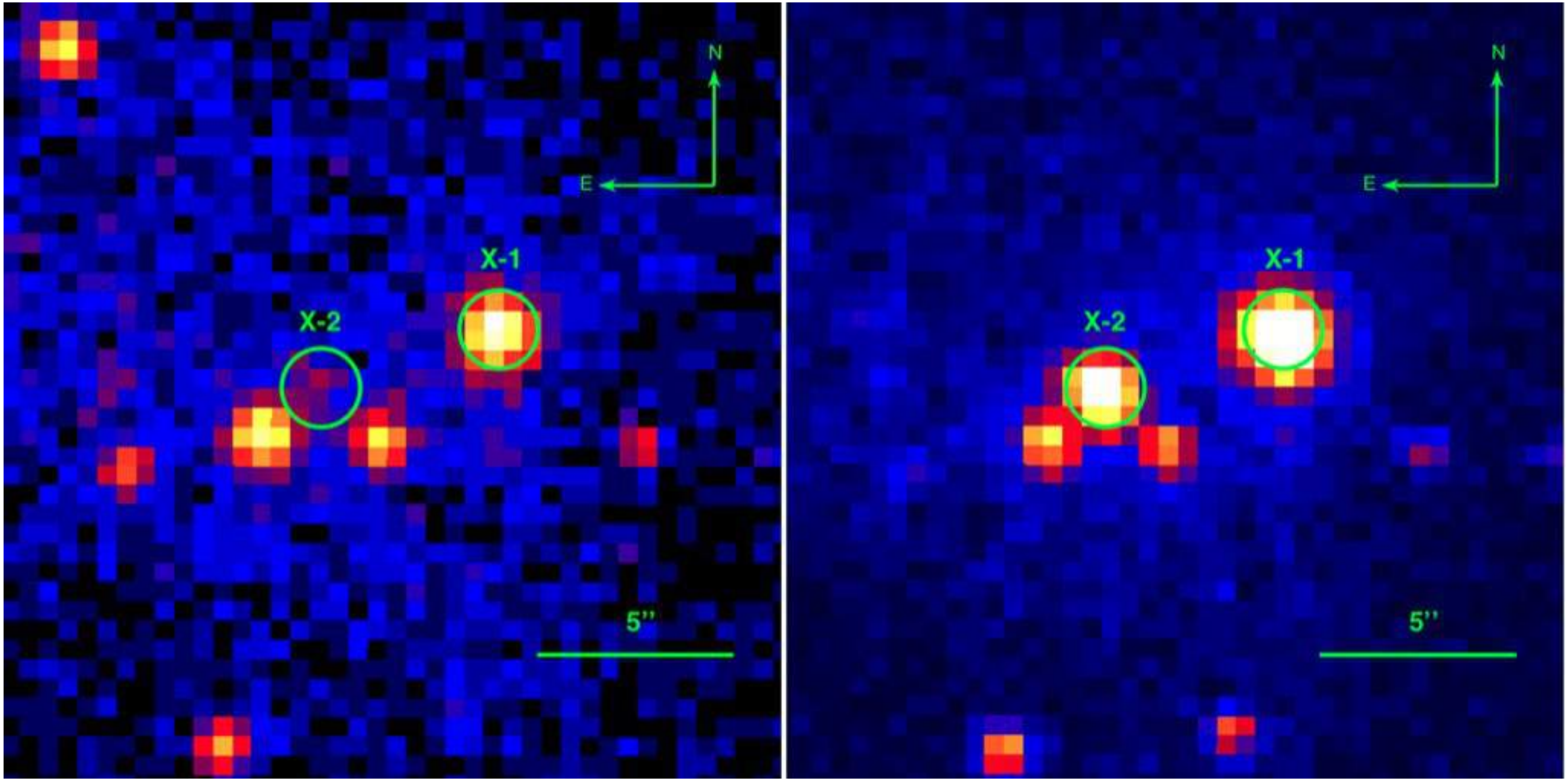




$$L_\mathrm{Edd} = \frac{4\pi G M m_\mathrm{p} c}{\sigma_\mathrm{T}}$$

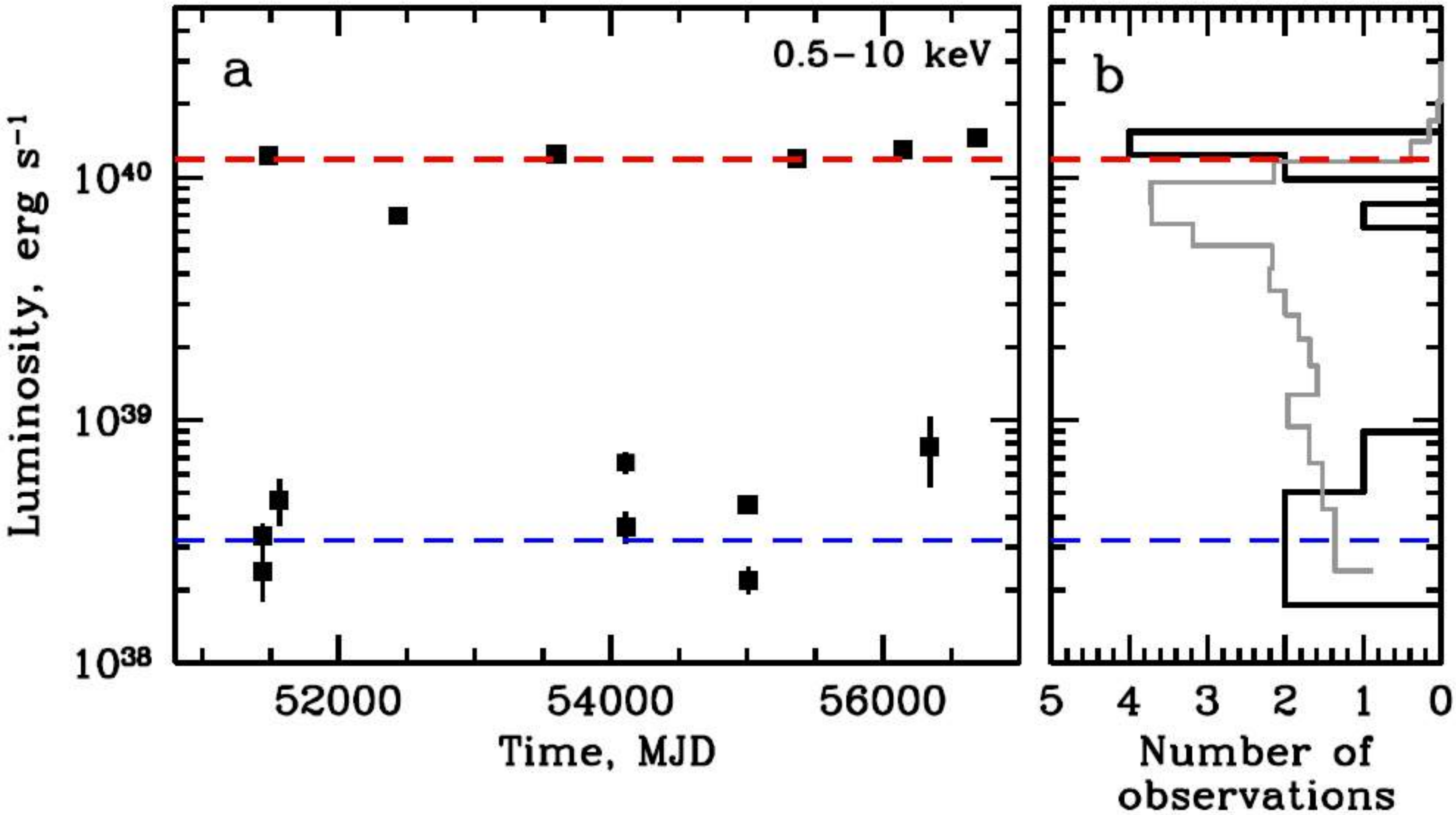
$$\approx 1.3 \times 10^{38} \frac{M}{M_\odot} \ \ \mathrm{erg\ s^{-1}}$$

Galaxy M82 as seen by Chandra



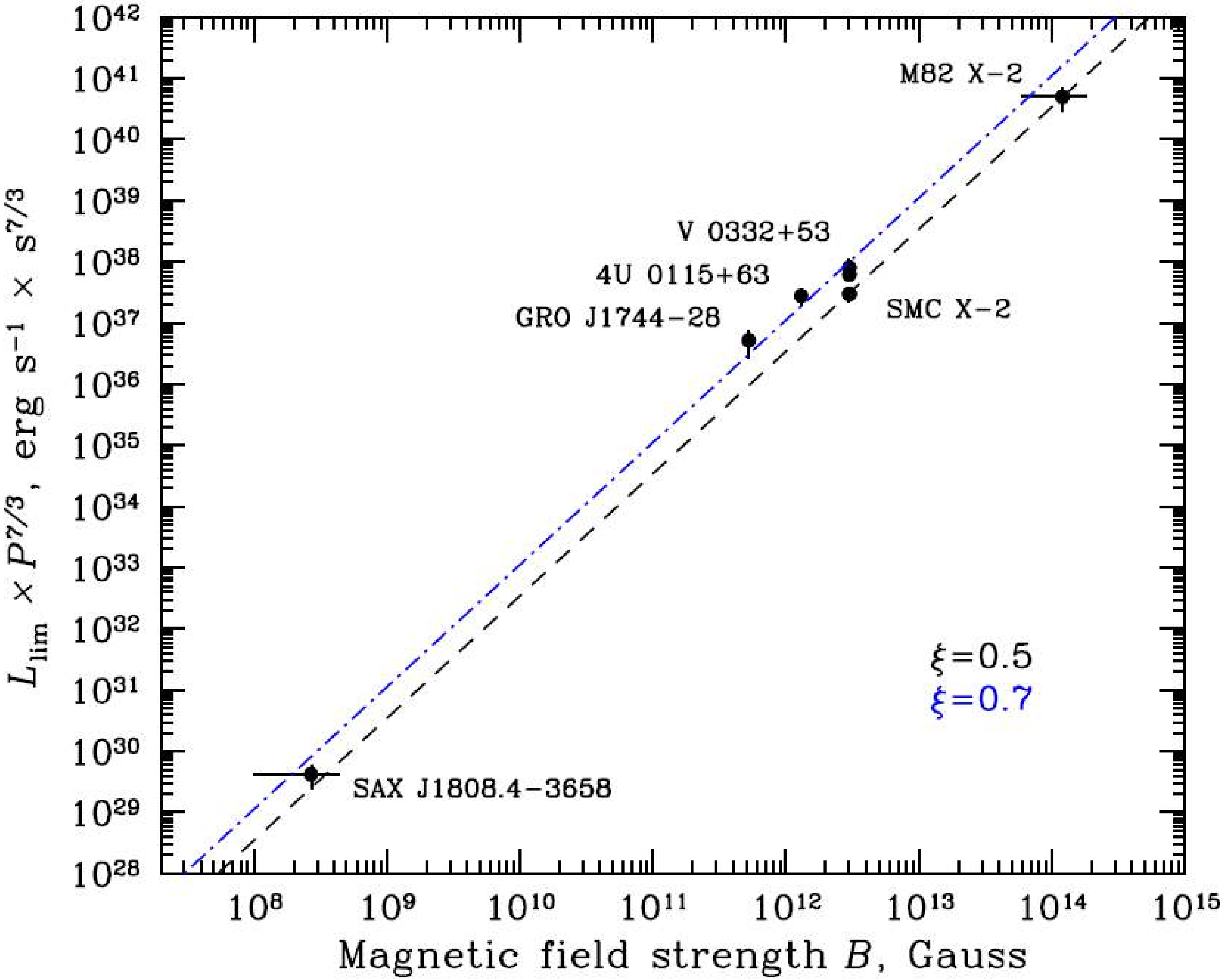
Tsygankov et al. (2016)

Luminosity distribution in M82 X-2



Tsygankov et al. (2016)

Propeller effect in ULX



P=1.37 s

ξ = 0.5

L_{lim}=2.0 x 10^{40} erg s^{-1}

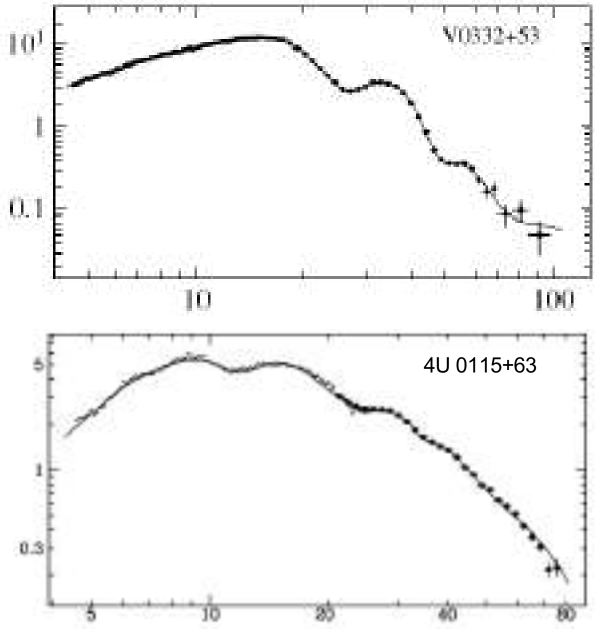
B~1.1 x 10^{14} G

$$L_{\rm lim}(R) \simeq \frac{GM\dot{M}_{\rm lim}}{R} \simeq 4\times10^{37}\xi^{7/2}B_{12}^2P^{-7/3}M_{1.4}^{-2/3}R_6^5 \,{\rm erg\ s^{-1}}$$

Tsygankov et al. (2016)

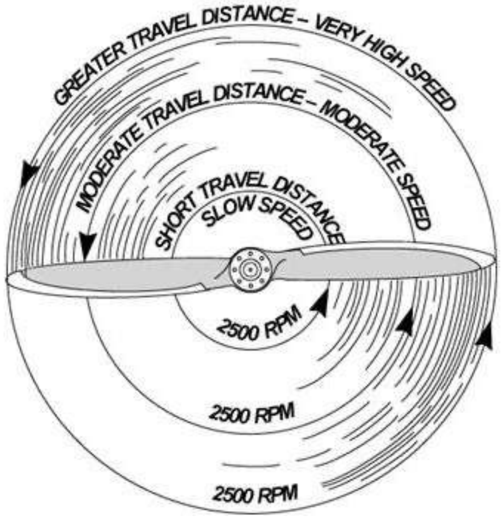
Neutron stars: the strongest and the brightest magnets in the Universe

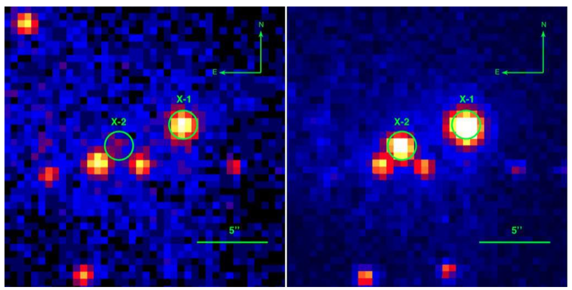
Cyclotron absorption lines



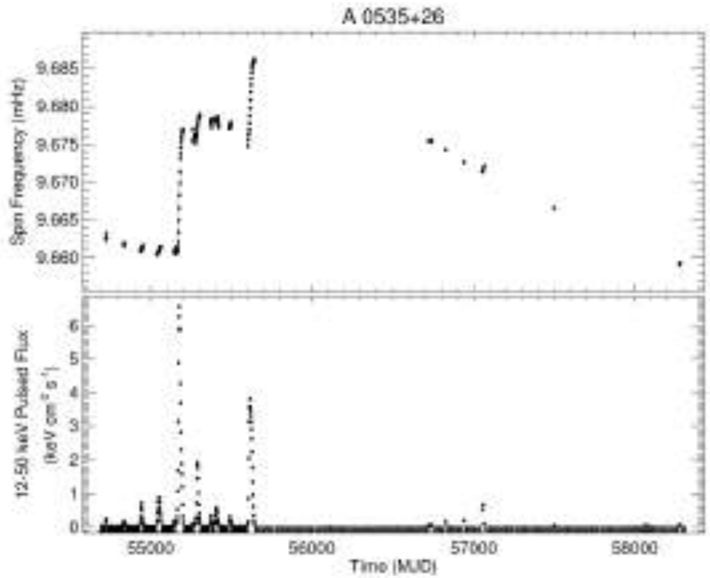
$$B_{12} = \frac{E_{\rm cyc}}{11.2}$$

"Propeller" effect





Pulse period evolution



$$-\dot{P} = \frac{N_{\rm acc} P^2}{2\pi I} \approx \frac{\dot{M}\sqrt{GMr_{\rm m}}P^2}{2\pi I}$$