HIGH ENERGY ASTROPHYSICS

Compulsary Home Exercises. Problem set 1. Turn in Exercises by Friday, September 20, 2024

1.1: Consider three model mass density profiles in a neutron star of radius R: (1) $\rho(r) = \rho_c$; (2) $\rho(r) = \rho_c\,[1 - (r/R)^2]$, where r is radial coordinate within the star and ρ_c is the central density. Take the canonical neutron star model with $M = 1.4\,M_\odot$ and $R = 10$ km. Find ρ_c (expressed in units of standard nuclear matter density $\rho_0 = 2.8 \times 10^{14}$ g cm^{-3}).

1.2: Calculate moment of inertia I of a neutron star of M and radius R for the two model density profiles (from problem 1.1, neglecting the effects of General Relativity). Evaluate I for a canonical neutron star with $M = 1.4\,M_\odot$ and $R = 10$ km.

1.3: Estimate at what density inside a neutron star neutrons become degenerate. Assume temperature of the neutron star of 10^9 K. At what density neutrons become relativistic?

1.4: Suppose a spherical body of a radius R_1 is spinning originally at a rate ν_1 Hz collapses to a body of radius R_2 conserving its mass M and angular momentum L. Express the ratio of the new and old spin rates ν_2/ν_1 and the new and old rotational energies E_2/E_1 in terms of the ratio R_2/R_1. You may assume the moment of inertia for a homogeneous sphere. By what factor would the star spin faster if it were to collapse from a radius typical of a white dwarf to the dimensions typical of a neutron star? By what factor would the rotational energy increase in such a collapse? Where ultimately does this energy come from?

1.5: A neutron star cannot spin with less than a certain period or it will start to shed mass from its equator due to centrifugal force. Consider a neutron star of mass M and radius R. Show that this critical period is

$$P_{\rm min} = K \left(\frac{1.4 M_\odot}{M}\right)^{1/2} \left(\frac{R}{10\rm km}\right)^{3/2} \text{ ms},$$

where K is a constant. Compute K using Newtonian gravity (neglect also any deformation of neutron star due to rotation). Calculations using general relativity give $K = 0.77$. Compute the limit on the radius of the neutron star which has a period $P = 1.4$ ms.

1.6: Crab pulsar has period of $P = 0.033$ s and period derivative $\dot{P} = 4.21 \times 10^{-13}$ s/s. Estimate the age of the pulsar. Compare it with the true age (SN in year 1054). Estimate the pulsar magnetic field using magnetic dipole radiation formula.