HIGH ENERGY ASTROPHYSICS

Compulsary Home Exercises. Problem set 2. Turn in Exercises by Friday, September 27, 2024

Problems

2.1: Show that the accretion rate of the wind accreting object is

$$\dot{M} = \dot{M}_w \left(\frac{M_x}{M_n}\right)^2 \frac{(v/v_w)^4}{[1+(v/v_w)^2]^{3/2}},$$

where $\dot{M}_w$ is the mass loss rate, M_x and M_n are the masses of the X-ray and normal star, respectively, v is the orbital velocity of a compact object around a companion and v_w is the wind velocity.

2.2: (a) In class we derived the Eddington limiting luminosity assuming the accretion of pure ionized hydrogen. Show that more generally the Eddington limit can be written as

$$L_{\rm Edd} = \frac{4\pi G M c}{\kappa},$$

where κ is the *mass absorption coefficient.* It has units of cm^2 g^{-1} and is the absorption cross-section per unit mass, $\kappa = \sigma/m$, where σ is the cross-section, m is the particle mass.

(b) What is the Eddington limit for a plasma composed entirely of completely ionized helium? Compute the numerical coefficient x in $L_{\rm Edd} = (x \text{ erg s}^{-1})(M/M_\odot)$.

(c) What is the Eddington limit for a plasma composed entirely of electron-positron pairs? Compute the numerical coefficient x in $L_{\rm Edd} = (x \text{ erg s}^{-1})(M/M_\odot)$. Note that the positron will also now scatter photons. The small Eddington limit here is one of the reasons people believe some jets may have large number of electron-positron pairs.

(d) What is the (pure ionized hydrogen) Eddington limit for an $\sim 10 M_\odot$ black hole (like Cygnus X-1), an $\approx 4 \times 10^6 M_\odot$ black hole (like on our Galactic center), and an $\approx 10^9 M_\odot$ black hole (like in a luminous quasar)?

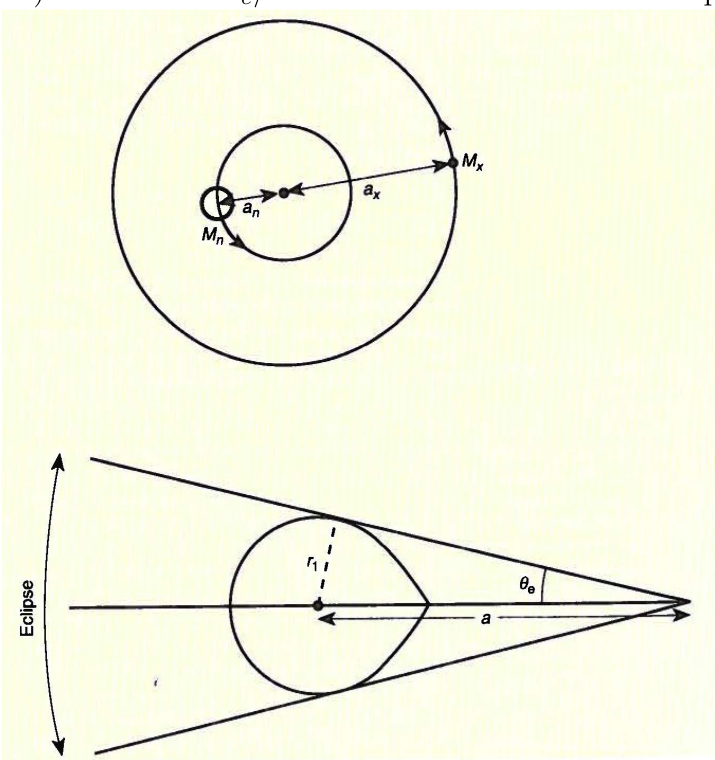
2.3: Calculate the terminal velocity, v (i.e. velocity at $r = \infty$), for an electron-positron pair under radiation and gravitational force alone, if it starts from rest at distance $r = 10 R_S$ from the black hole. ($R_S = 2GM/c^2$ is the Schwarzschild radius.)

Hint: Write down effective force (gravitational minus radiation force), and use the energy conservation equation. Use the relativistic formula for the electron (positron) energy.

2.4: Show that the inclination of the binary orbit i and the half-angle θ_e of the eclipse of the central compact object are related by

$$\left(\frac{R_n}{a}\right)^2 = \cos^2 i + \sin^2 i \sin^2 \theta_e,$$

where R_n is the radius of the companion star and a is the binary separation (see figure below). Note that $2\theta_e/2\pi$ is the fraction of the orbital period occupied by the eclipse.



2.5 (2 bonus points): XTE J1807−294 is an accreting millisecond pulsar. Using variations of the arrival time of the pulse to the observer the pulsar mass function was measured

$$f_x = \frac{M_n^3 \sin^3 i}{(M_x + M_n)^2} = 1.49 \times 10^{-7} M_\odot.$$

Solve for the mass of the companion as a function of the inclination $M_n(i)$ and plot that relation. Assume that the neutron star mass is $M_x = 1.4 M_\odot$. What is the minimum value for M_n? Obtain the upper limit on the mass with 90% confidence taking into account that the probability density for $\cos i$ is a flat function (i.e. independent of $\cos i$).