HIGH ENERGY ASTROPHYSICS

Compulsary Home Exercises. Problem set 3. Turn in Exercises by Friday, October 4, 2024

Problems

3.1: Show that the Alfven radius is

$$R_A \approx \left(\frac{B_0^2 R_{NS}^6}{2\dot{M}\sqrt{2GM}} \right)^{2/7},$$

where B_0 is the surface magnetic field of the neutron star, R_{NS} is its radius, M is its mass, and $\dot{M}$ is the accretion rate.

3.2: A non-magnetized neutron star of mass $M = 1.4\mathrm{M}_\odot$ and radius $R = 13$ km is accreting matter at $10^{-10}\mathrm{M}_\odot/\mathrm{yr}$ via an accretion disk. How long does it take to spin up the star by the accreting matter from the initially large period to 3 ms? How much mass do you need to accrete to spin the star to such a period? Assume a constant moment of inertia $I = 10^{45}$ g cm^2.

3.3: The period of the X-ray pulsar Cen X-3 has changed from 1971 to 1975 from 4.844 to 4.837 seconds. Estimate the magnetic field of the pulsar if its average luminosity is $L \approx 2 \times 10^{37}$ erg s^{-1}.

3.4: A small spot of area S at the magnetic pole of the neutron star radiates as a black body (i.e. radiation intensity is the same in all direction). Assume the frequency integrated intensity is I_0. Derive first the formula for the frequency-integrated flux F observed from such a spot by an observer at distance D from the star as a function of inclination i (angle between the direction to the observer and the rotational axis), the angle between the rotational and magnetic pole θ, and the phase of the pulsar. Assume flat space-time (i.e. no photon bending) and neglect gravitational redshift. Compute then the amplitude of pulsation

$$A = \frac{F_{\max} - F_{\min}}{F_{\max} + F_{\min}},$$

where $F_{\max}$ and $F_{\min}$ are the maximum and minimum of the flux, respectively. Consider the case when the spot is visible all the time. How the result changes when gravitational bending is accounted for? Use approximate Beloborodov's formula for the light bending.

3.5 (2 bonus points): Computer exercise. Swift J1749.4$-$2807 is an accreting millisecond pulsar ($\nu = 518$ Hz) discovered in 2010. It shows X-ray eclipses. From variations of the pulsar frequency the following quantities have been measured (Altamirano et al. 2011): the orbital period $P_{\mathrm{orb}} = 31740.73$ s, projected semimajor axis $a_x \sin i = 1.89953$ light seconds, the pulsar mass function was measured $f_x = M_n^3 \sin^3 i/(M_n + M_x)^2 = 0.0545278 M_\odot$. The duration of the eclipse was determined to be 2172 seconds (Markwardt & Strohmayer 2010). Determine the inclination of the orbit i, the mass M_n and the radius R_n of the companion star. Assume neutron star mass of $M_x = 1.5 M_\odot$. Use the Faulkner formula for the size of the Roche lobe:

$$R_n \approx R_L = 0.459a \left(\frac{q}{1+q} \right)^{1/3},$$

where $q = M_n/M_x$ is the mass ratio and a is the binary separation.

Hint: use the results of exercise 2.4, where we showed that the inclination of the binary orbit i and the half-angle θ_e of the eclipse are related by

$$\left(\frac{R_n}{a} \right)^2 = \cos^2 i + \sin^2 i \sin^2 \theta_e.$$