HIGH ENERGY ASTROPHYSICS

Compulsary Home Exercises. Problem set 4. Solutions.

Problems

4.1: Consider a LMXRB that shows X-ray bursts. Estimate the interval between bursts if the accretion persistent uminosity is 1% of $L_{\rm Edd}$. Assume a neutron star mass of $1.4M_\odot$ and radius 10 km. Assume the efficiency of nuclear burning during the burst of 0.7% and you may also assume that the burst is a 5-second 'spike' at $L_{\rm Edd}$. Estimate (using Stefan-Boltzmann law) the maximum effective temperature (in keV) reached during the burst.

Solution: The Eddington limit for a neutron star of mass $1.4M_\odot$ is about

$$L_{\rm Edd} = 1.3\times 10^{38}\,\frac{M_{\rm NS}}{M_\odot} \approx 1.8\times 10^{38}\ {\rm erg\,s^{-1}}.$$

The mass "burned" during the time $\tau = 5$ s can be obtained from the total energy emitted during the burst:

$$E = L_{\rm Edd}\tau = \eta_{\rm nucl} M c^2,$$

where $\eta_{\rm nucl} = 0.007$ is the efficiency of nuclear burning. Thus we get

$$M = \frac{L_{\rm Edd}\tau}{\eta_{\rm nucl}c^2}.$$

This mass has to be accreted during time T between the bursts:

$$M = \dot{M}T.$$

The accretion luminosity is related to the accretion rate as

$$L_{\rm acc} = 0.01 L_{\rm Edd} = \eta_{\rm acc}\dot{M}c^2,$$

where $\eta_{\rm acc} = GM_{\rm NS}/Rc^2 \approx 0.2$ is the accretion efficiency. Thus we get

$$T = \frac{M}{\dot{M}} = \frac{L_{\rm Edd}\tau}{\eta_{\rm nucl}c^2}\frac{\eta_{\rm acc}c^2}{L_{\rm acc}} = 100\tau\frac{\eta_{\rm acc}}{\eta_{\rm nucl}} = 14300\,{\rm s} \approx 4\,{\rm hr}.$$

Stefan-Boltzmann law states:

$$\sigma_{SB}T_{\rm eff}^4 = \frac{L_{\rm Edd}}{4\pi R_{\rm NS}^2}.$$

Thus we get

$$T_{\rm eff} = \left(\frac{L_{\rm Edd}}{\sigma_{SB}4\pi R_{\rm NS}^2}\right)^{1/4} = \left(\frac{1.8\times 10^{38}}{5.67\times 10^{-5}\times 4\pi 10^{12}}\right)^{1/4} = 2.24\times 10^{7}\,{\rm K} = 1.93\,{\rm keV}.$$

4.2: Neutron star in an X-ray burster EXO 0740–676 rotates 552 times a second. Estimate what would be the observed physical width of the emission line due to the Doppler effect for an observer at an inclination $i = 0$, 60, 90 degrees to the rotational axis. Assume that the line energy in the star frame is 1 keV, the neutron star mass is $M = 1.5 M_\odot$ and the radius $R = 12$ km. Ignore light bending.

Solution: Consider spherical coordinates with the z-axis along the rotation axis of the star. Let the direction to the observer be $\vec{k} = (\sin i, 0, \cos i)$. The largest rotational velocity is reached at the equator (in units of speed of light):

$$\beta_{\rm eq} = \frac{v_{\rm eq}}{c} = 2\pi \frac{\nu}{\sqrt{1 - R_{\rm S}/R}} \frac{R}{c} = 0.175.$$

Here we assumed the neutron star radius $R = 12$ km, mass $M = 1.5 M_\odot$ (i.e. Schwarzschild radius $R_{\rm S} = 4.45$ km) and we have corrected the observed rotational frequency by the red-shift $1+z = 1/\sqrt{1 - R_{\rm S}/R} = 1.26$. The corresponding Lorentz factor $\gamma_{\rm eq} = 1/\sqrt{1 - \beta_{\rm eq}^2} = 1.032$. Velocity depends on the azimuth ϕ of a point at the equator:

$$\vec{\beta}(\phi) = \beta_{\rm eq}(-\sin\phi, \cos\phi, 0).$$

Ignoring light bending, the angle ξ velocity makes with the line-of-sight is

$$\cos\xi = \frac{\vec{\beta}(\phi)}{\beta_{\rm eq}} \cdot \vec{k} = -\sin i \sin\phi.$$

The equatorial Doppler factor

$$\mathcal{D}_{\rm eq}(i,\phi) = \frac{1}{\gamma_{\rm eq}(1 - \beta_{\rm eq}\cos\xi)} = \frac{1}{\gamma_{\rm eq}(1 + \beta_{\rm eq}\sin i \sin\phi)}$$

reaches extrema at $\sin\phi = \pm 1$:

$$\mathcal{D}_{\rm eq,min} = 0.843, \quad \mathcal{D}_{\rm eq,max} = 1.144, \quad \text{for } i = 60^\circ,$$

$$\mathcal{D}_{\rm eq,min} = 0.826, \quad \mathcal{D}_{\rm eq,max} = 1.177, \quad \text{for } i = 90^\circ.$$

For zero inclination, we cannot just use the same formula, because the biggest effects would come from variation of γ from 1 at the pole to $\gamma_{\rm eq}$ at the equator. Thus

$$\mathcal{D}_{\rm min} = 1/\gamma_{\rm eq} = 0.97, \quad \mathcal{D}_{\rm max} = 1, \quad \text{for } i = 0^\circ.$$

The observed energy of the line is

$$E_{\rm obs} = \frac{E_{\rm emit}}{1+z}\mathcal{D} = \frac{1\,\text{keV}}{1+z}\mathcal{D},$$

and the width is

$$\Delta E_{\rm obs} = \frac{\mathcal{D}_{\rm max} - \mathcal{D}_{\rm min}}{1.26}.$$

Thus we get

$$\Delta E_{\rm obs}(0) = 0.024\,\text{keV}, \quad \Delta E_{\rm obs}(60^\circ) = 0.24\,\text{keV}, \quad \Delta E_{\rm obs}(90^\circ) = 0.28\,\text{keV}.$$

4.3: The observed Eddington flux corresponds to the Eddington luminosity reached at the neutron star surface:

$$F_{\mathrm{Edd,obs}} = \frac{L_{\mathrm{Edd,obs}}}{4\pi D^2} = \frac{GMc}{D^2 \kappa_e (1+z)},$$

where $\kappa_e = 0.2(1+X)$ cm^2 g^{-1} is the electron scattering opacity, X is the hydrogen mass fraction, D is the distance, M is the neutron star mass and z is the surface redshift. Derive the relation between the neutron star radius R and the compactness $u = R_S/R$ (here $R_S = 2GM/c^2$):

$$R = 14.138\,\mathrm{km}\ \frac{(1+X)D_{10}^2 F_{-7}}{u\sqrt{1-u}}, \qquad (1)$$

where $F_{-7} = F_{\mathrm{Edd,obs}}/10^{-7}$ erg cm^{-2} s^{-1}, $D_{10} = D/10$ kpc. What would be the neutron star radius and mass if the redshift is measured from the spectral lines $z = 0.26$? Assume $F_{\mathrm{Edd,obs}} = 6 \times 10^{-8}$ erg cm^{-2} s^{-1}, solar abundance $X = 0.73$, and distance $D = 5$ kpc.

Solution: The mass can be immediately obtained from the observed Eddington flux:

$$M = F_{\mathrm{Edd,obs}} \frac{D^2 \kappa_e (1+z)}{Gc} = 6\times 10^{-8} \frac{(5 \times 3.09 \times 10^{21})^2 0.2 \times 1.73 \times 1.26}{6.67 \times 10^{-8} \times 3 \times 10^{10}} = 3.12\times 10^{33}\ \mathrm{g} = 1.57 M_\odot.$$

We can now rewrite the definition of the Eddington flux as

$$F_{\mathrm{Edd,obs}} = \frac{R}{D^2 \kappa_e} \frac{GMc\sqrt{1 - R_S/R}}{R} = \frac{1}{0.2(1+X)D^2} \frac{c^3}{2} u\sqrt{1-u}\ R,$$

and now express the radius as

$$R = F_{\mathrm{Edd,obs}} 0.2(1+X) D^2 \frac{2}{c^3} \frac{1}{u\sqrt{1-u}}.$$

and substituting the numbers we get

$$R = 10^{-7} F_{-7}\, 0.2(1+X) D_{10}^2 (3.086\times 10^{22})^2 \frac{2}{(2.998 \times 10^{10})^3} \frac{1}{u\sqrt{1-u}} = 1.414\times 10^6\,\mathrm{cm} \frac{(1+X)D_{10}^2 F_{-7}}{u\sqrt{1-u}}.$$

If the redshift $z = 0.26$, then $u = 1 - 1/(1+z)^2 = 0.37$. The product $(1+X)D_{10}^2 F_{-7} = 0.26$. Thus we get

$$R = 48.1\,\mathrm{km}\ (1+X)D_{10}^2 F_{-7} = 12.5\,\mathrm{km}$$

and

$$\frac{M}{M_\odot} = \frac{Ruc^2}{2GM_\odot} = \frac{Ru}{2.95\,\mathrm{km}} = 1.57,$$

consistent with the one we already obtained above.

4.4: The spectrum of the photospheric radius expansion burst from 4U 1724–307 is well described by a black body. From the touchdown flux the observed Eddington limiting flux was determined as $F_{\rm Edd,obs} = 0.58 \times 10^{-7}$ erg cm^{-2} s^{-1}. The blackbody normalization in the cooling tail was $K = 220$ (km/10 kpc)^2. Estimate the neutron star mass and radius. Assume distance $D = 5.0$ kpc, solar abundance, the color correction factor in the tail $f_c = 1.4$.

Solution: From equation (1) of exercise 4.3 we get ($D_{10} = 0.5$, $F_{-7} = 0.58$, $X = 0.73$)

$$R = 14.138\,{\rm km}\ \frac{1.73 \times 0.5^2 \times 0.58}{u\sqrt{1-u}} = \frac{3.55\,{\rm km}}{u\sqrt{1-u}}. \qquad (2)$$

In the lecture note we show the relation between the radius at infinity, the actual neutron star radius and the black body radius $R_{\rm bb}$

$$R_\infty = \frac{R}{\sqrt{1-u}} = R_{\rm bb} f_c^2,$$

therefore the radius:

$$R = R_{\rm bb} f_c^2 \sqrt{1-u}.$$

Instead of $R_{\rm bb}$, we measure the normalization $K = (R_{\rm bb}/D)^2$ and we then get

$$R = \sqrt{K} D f_c^2 \sqrt{1-u}.$$

If we measure R in km, K in (km/10 kpc)^2, we have

$$R = D_{10}\sqrt{K} f_c^2 \sqrt{1-u}\,{\rm km} = 14.54\sqrt{1-u}\,{\rm km}. \qquad (3)$$

Dividing equations (2) and (3) we get a quadratic equation for u:

$$(1-u)u = 0.244 \Rightarrow u^2 - u + 0.244 = 0.$$

The solutions are

$$u = \frac{1}{2} \pm \sqrt{0.25 - 0.244} = 0.5 \pm 0.076.$$

The two solutions are $u_1 = 0.576$, $u_2 = 0.424$. Substituting those to equation (2) or (3), we get $R_1 = 9.47$ km and $R_2 = 11.04$ km. The corresponding masses

$$\frac{M}{M_\odot} = u\frac{R}{2.95\,{\rm km}}$$

and $M_1/M_\odot = 1.85$, $M_2/M_\odot = 1.59$. Both solutions are realistic.