HIGH ENERGY ASTROPHYSICS
Compulsary Home Exercises. Problem set 6.
Turn in Exercises by Friday, November 8, 2024.

Problems

6.1: The X-ray spectrum of an accreting black hole GX 339–4 is shown in Fig. 1. Estimate the photon spectral index Γ of the Comptonized component (shown with dashed line) in the standard X-ray band 2–10 keV. Estimate the electron temperature that is needed to produce the observed spectrum by Comptonization. Compute the X-ray luminosity of the object, assuming the distance of 5 kpc.

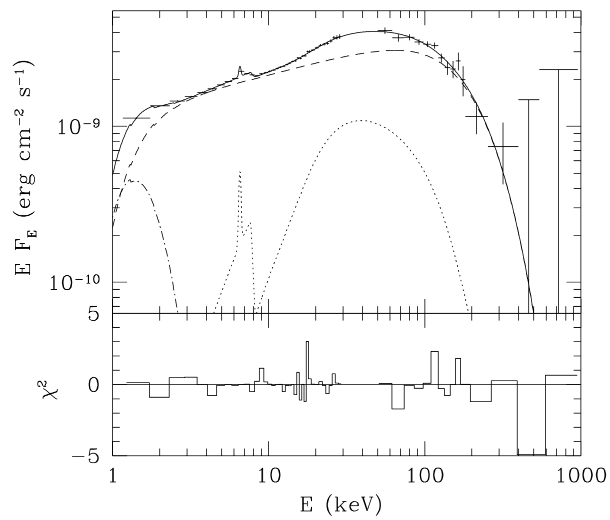

Figure 1: Broad-band spectrum of GX 339–4 as observed by *Ginga* and OSSE/*CGRO* in 1991 (from Zdziarski et al. 1998).

6.2: Consider photon gas with the intensity given by the Planck (blackbody) distribution of temperature of $kT_{\rm BB} = 0.33$ keV. The photons are penetrating into a hot medium with electron temperature $kT_{\rm e} = 100$ keV and are being Compton up-scattered. Compute how many scatterings are needed for a typical photon to achieve the final energy $E_{\rm f} = 100$ keV.

6.3: Consider an accretion disc illuminated by an isotropic X-ray source located 30 km above the centre of the disc. The disc has a hole in the centre with radius of 100 km, but otherwise is flat and extends to infinity. Assuming flat space, calculate the reflection factor $R = \Omega/2\pi$ from such a disc, where Ω is the solid angle occupied by the disc as viewed from the X-ray source.

6.4: Consider a light curve with the counts per bin $s_k,\ k = 1, ..., N$. Show the relation

$$\text{rms}^2 \equiv \frac{\overline{s^2} - \overline{s}^2}{\overline{s}^2} = \Delta f \sum_{j>0} P(f_j), \tag{1}$$

where $P(f_j) = 2|S_j|^2/(R^2 T)$, $R = \overline{s}N/T$ - mean count rate per second, $f_j = j/T$, $\Delta f = 1/T$,

$$S_j = \sum_{k=0}^{N-1} s_k e^{2\pi i jk/N}, \quad j = -N/2, ..., N/2 - 1, \tag{2}$$

is the discrete Fourier transform of the count rate and

$$\overline{s} = \frac{1}{N}\sum_{k=1}^{N} s_k, \quad \overline{s^2} = \frac{1}{N}\sum_{k=1}^{N} s_k^2. \tag{3}$$

6.5: Proove a relation between the discrete autocorrelation function and the power-density spectrum:

$$A_p = \frac{1}{N}\sum_{j=-N/2}^{N/2-1} |S_j|^2 e^{-2\pi i jp/N} \tag{4}$$

using the formal definition

$$A_p \equiv \sum_k s_k s_{k-p} \tag{5}$$

and the orthogonality condition

$$\sum_{j=-N/2}^{N/2-1} e^{2\pi i j(m-n)/N} = N\delta_{mn}. \tag{6}$$

6.6: Consider the shot noise model with the shot profile at soft energies described by

$$g_s(t) = e^{-t/\tau_s}, t \geq 0. \tag{7}$$

(a) Compute the PDS of the light curve.
(b) Let the hard photons have a similar shot profile with time-constant $\tau_h = \gamma\tau_s$. Assume that the start time of the shots in both energies coincide. Compute the phase and time lags, $\Delta\phi(f)$ and $\Delta t(f)$. Are they positive or negative? Explain.
(c) Compute the low ($f \ll 1/2\pi\tau_s$) and the high ($f \gg 1/2\pi\tau_s$) frequency limits for $\Delta\phi(f)$ and $\Delta t(f)$.