HIGH ENERGY ASTROPHYSICS

Compulsary Home Exercises. Problem set 7.

Turn in Exercises by Friday, November 15, 2024

Problems

7.1: Derive formulae for rotational velocity, angular velocity and specific angular momentum in pseudo-Newtonian potential. Show that $\mathrm{d}l/\mathrm{d}r = 0$ at $r = 3R_\mathrm{S}$. Here $R_\mathrm{S} = 2GM/c^2$ is the Schwarzschild radius.

7.2: Compute the radiative efficiency

$$\epsilon \equiv \frac{L}{\dot{M}c^2} = -\frac{v_\varphi^2(r_*)}{2c^2} - \frac{\phi(r_*)}{c^2}$$

of the accretion disc around a black hole for Newtonian $\phi = \phi_N$ and pseudo-Newtonian (Paczynski-Wiita) $\phi = \phi_{PN}$ potentials. Here $r_* = 3R_\mathrm{S}$. At what accretion rate $\dot{M}$ the black hole should accrete to produce Eddington luminosity? Compute the numerical value of this $\dot{M}$ (in g/s and $M_\odot$/year) for 10 $M_\odot$ and $10^8 M_\odot$ black holes.

What is the luminosity of a 10 $M_\odot$ black hole accreting at a rate $\dot{M} = L_\mathrm{Edd}/c^2$? What is the corresponding luminosity of a quasar of $10^8 M_\odot$?

7.3: Compute the radius (in units of R_S) where the effective temperature of standard accretion disc reaches the maximum for Newtonian and pseudo-Newtonian potentials. Compute the numerical value of the maximum temperature (in K and keV) for a 10 $M_\odot$ black hole accreting at a rate $\dot{M} = L_\mathrm{Edd}/c^2$. How the maximum temperature scales with the mass of the central object? What would be the corresponding temperature for a $1.4M_\odot$ neutron star and a $10^8 M_\odot$ quasar? How the maximum temperature scales with the accretion rate?

7.4: Show that the prescription of the viscous stress $t_{r\phi} = \alpha P$ is equivalent to the prescription

$$\nu = \frac{2}{3}\alpha c_s H.$$

7.5: (a) How long (in years) a stellar mass black hole (of say $M = 7M_\odot$) has to accrete matter at the Eddington limit (i.e. producing Eddington luminosity) in order to reach a luminosity $L = 10^{47}$ erg s^{-1}? To determine this, write down and solve a simple differential equation for how the mass changes with time due to accretion. Assume the radiative efficiency of 0.1. Compare the time scale you get to the Hubble time (the age of the Universe).

(b) If a galaxy with 10^{11} stars contains a dead quasar that grew as in a previous part (a) until reaching $10^8 M_\odot$, compare its total gravitational energy release to the energy release due to thermonuclear burning (in stars) during the time it took for the black hole to grow. You may take all stars to have $M = 1M_\odot$ (and $L = L_\odot$).

(c) A quasar has luminosity $L = 10^{47}$ erg s^{-1} and varies on the time-scale of a day. Deduce a mass and a radius for the emitting region using Eddington limit and the light crossing time arguments. How does the implied density compare with that of the Earth? What is the mass accretion rate assuming the radiative efficiency of 10%? How does the amount of mass accreted per second compare with the mass of the Earth?