HIGH ENERGY ASTROPHYSICS

Compulsary Home Exercises. Problem set 8. Solutions.

Problems

8.1: The Imaging X-ray Polarimeter Explorer (IXPE) has observed the central part of the Milky Way (Marin et al. 2022), in particular a molecular cloud Sgr A, which is situated at a projected distance of 25 pc from Sgr A*. The detected X-rays are polarized, with the polarization degree $P = 31 \pm 11\%$ and the polarization angle perpendicular to the direction to Sgr A*. Polarization is likely produced as a result of single Thomson scattering of photons that originated from a short X-ray flare of Sgr A* some time ago. Estimate the time when this flare has occurred.
Hint: The polarization degree for Thomson scattering is $P = 100\% \times (1 - \mu^2)/(1 + \mu^2)$, where $\mu = \cos\theta$ and θ is the scattering angle.

Solution: From the observed polarization degree we can estimate the scattering angle (here P is measured in the range [0,1]):

$$\mu^2 = \frac{1-P}{1+P} \pm \frac{2\sigma_P}{(1+P)^2} = \frac{1-0.31}{1+0.31} \pm \frac{2\times 0.11}{(1+0.31)^2} = 0.53 \pm 0.13, \tag{1}$$

where σ_P is the uncertainty of P. Thus for μ we get two solutions

$$\mu = \pm 0.73 \pm 0.09. \tag{2}$$

The time delay is

$$\tau = (1-\mu)R/c, \tag{3}$$

where R is the distance of Sgr A cloud from Sgr A*. On the other hand, from the projected distance $R\sin\theta = R\sqrt{1-\mu^2} = 25$ pc, we get R for given μ. Thus we get for the time delay

$$\tau = \frac{1-\mu}{\sqrt{1-\mu^2}}\frac{25\,\mathrm{pc}}{c} = 81.5\,\mathrm{yr}\frac{1-\mu}{\sqrt{1-\mu^2}} = 32 \pm 6 \text{ yr}, \quad \text{or} \quad \tau = 207 \pm 40 \text{ yr}. \tag{4}$$

This gives you an estimate how long time ago the flare occurred. The shorter time can be rejected, because Japanise ASCA satellite has observed the center of our Galaxy about 30 years ago and Sgr A* was not bright. Thus the only solution is that the center of our Galaxy, Sgr A*, was bright about 200 year ago.

8.2: In Figure 2, the infrared SED of a distant quasar at $z = 5.34$ is shown. Assuming that the infrared bump at about 100 microns is produced by an optically and geometrically thick torus consisting of molecular gas and dust, estimate the parameters of the torus: its bolometric luminosity, size, and the range of temperatures in the reference frame of the quasar. What should the variability of the infrared component look like? Estimate the minimal possible variability time scale and the time lag with respect to the big blue bump emission component.
Hints: To estimate the luminosity, you can use any of the freely available cosmology calculators (note the cosmological parameters!) to convert the redshift to luminosity distance. Assume that the infrared emission is reprocessed ultraviolet emission.

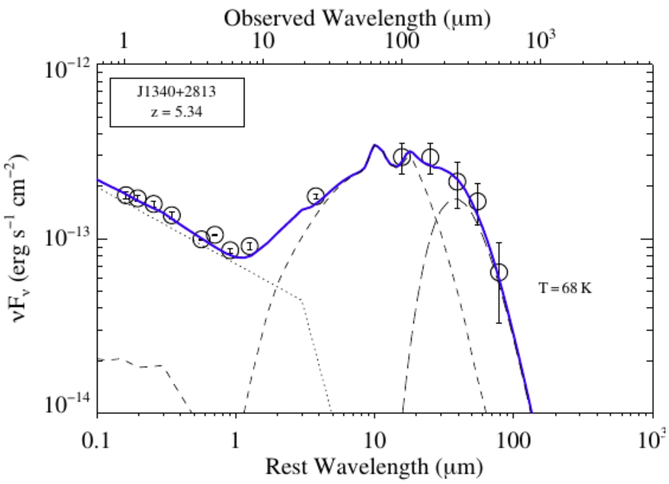

Figure 1: Spectral energy distribution of J1340+2813. Taken from Leipski et al. (2012).

Solution:

Wavelengths are already given in the rest frame of the quasar. Planck function (multiplied by frequency) $\nu B_\nu = \lambda B_\lambda$ has a maximum at $\nu_{\max} \simeq 4kT/h$ or $\lambda_{\max} = c/\nu_{\max} = hc/4kT$, that allows to estimate the temperature range of the dust as $T \sim hc/4k\lambda_{\max} \simeq 100-300$ K. Note that the emission of the dust is evidently broader than the colder thermal component with $T = 68$ K outlined in the plot, that suggests a spread in temperatures.

The flux in the original paper is the flux in the reference frame of the observer, related to the luminosity of the object as

$$F = \frac{L}{4\pi D_L^2}, \tag{5}$$

where the luminosity distance for the given redshift is $D_L \simeq 50$ Gpc. To convert approximately νF_ν to bolometric flux, note that the width of the Planck function is about an order of magnitude. More precisely, integration of the Planck function yields $\int_0^{+\infty} x^3/(e^x - 1)dx \simeq 6.5$, while the maximal value of $x^4/(e^x - 1)$ is about 5. Hence, the maximal value of the flux may be used as a proxy for the bolometric flux to an accuracy much better than the accuracy of our approximation of the SED by a Planck function.

For the luminosity distance of 50 Gpc, $4\pi D_L^2 \approx 3 \times 10^{59}$ cm^2. Taking the maximal flux $(\nu F_\nu)_{\max} = (2-3)\times 10^{-13} \mathrm{erg\,cm^{-2}\,s^{-1}}$, we arrive at the bolometric luminosity of

$$L \approx (\nu F_\nu)_{\max}\, 4\pi D_L^2 \approx (6-10)\times 10^{46} \mathrm{erg\,s^{-1}}. \tag{6}$$

Combining the luminosity with the temperature allows to estimate the effective radiating area of the torus as $A \simeq L/\sigma_{\rm SB}T^4$, where $\sigma_{\rm SB}$ is Stephan-Boltzmann's constant. The shape of the torus is still unknown. If its two dimensions are comparable to each other, we get the following estimate for its spatial size

$$R \simeq \sqrt{L/\sigma_{\rm SB}T^4} \simeq 10^{21}\,\mathrm{cm} \simeq 300\,\mathrm{pc}, \tag{7}$$

with the uncertainty of about half an order of magnitude. In terms of light travel time, this corresponds to several hundreds or thousands of years.

The obtained value of $\gtrsim 100$ pc is large compared to the dusty tori observed in low-z quasars and Seyfert galaxies, where dust temperatures may be significantly higher (up to $\sim$1500 K) and the spatial sizes are fractions of a parsec to parsecs. Reverberation mapping for dust emission is possible if the light travel time is about several years or smaller.

8.3: The apparent velocity (in units of c) of a blob moving with relativistic velocity v is

$$\beta_{\rm app} = \frac{\beta \sin\theta}{1 - \beta\cos\theta}, \tag{8}$$

where $\beta = v/c$ and θ is the angle the velocity makes to the line of sight. Compute the maximum possible apparent velocity for a given β. Show that there is minimum velocity $\beta = \beta_{\rm min} = \beta_{\rm app}/\sqrt{1+\beta_{\rm app}^2}$ required to produce the apparent velocity $\beta_{\rm app}$. Show that this minimum corresponds to the angle between the blob velocity and the line of sight $\tan\theta_{\rm min} = 1/\beta_{\rm app}$.

Solution:

Take a derivative of Eq. (8) and find extremum:

$$\frac{d\beta_{\rm app}}{d\theta} = \beta\frac{\cos\theta - \beta}{(1-\beta\cos\theta)^2}. \tag{9}$$

The derivative is zero when $\cos\theta = \beta$ and the maximum velocity is

$$\beta_{\rm app,max} = \frac{\beta\sqrt{1-\beta^2}}{1-\beta^2} = \frac{\beta}{\sqrt{1-\beta^2}} = \Gamma\beta, \tag{10}$$

where Γ is the corresponding Lorentz factor.

Now express β via $\beta_{\rm app}$:

$$\beta = \frac{\beta_{\rm app}}{\sin\theta + \beta_{\rm app}\cos\theta}. \tag{11}$$

The derivative

$$\frac{d\beta}{d\theta} = -\beta_{\rm app}\frac{\cos\theta - \beta_{\rm app}\sin\theta}{(\sin\theta + \beta_{\rm app}\cos\theta)^2} \tag{12}$$

reaches zero when $\tan\theta = \tan\theta_{\rm min} = 1/\beta_{\rm app}$ and it is easy to see that β reaches a minimum here. Noting that $\sin\theta_{\rm min} = 1/\sqrt{1+\beta_{\rm app}^2}$ and $\cos\theta_{\rm min} = \beta_{\rm app}/\sqrt{1+\beta_{\rm app}^2}$, we get from Eq. (11):

$$\beta_{\rm min} = \frac{\beta_{\rm app}}{\frac{1}{\sqrt{1+\beta_{\rm app}^2}} + \frac{\beta_{\rm app}^2}{\sqrt{1+\beta_{\rm app}^2}}} = \frac{\beta_{\rm app}}{\sqrt{1+\beta_{\rm app}^2}}. \tag{13}$$

The same result can be obtained directly from Eq. (10), expressing β via $\beta_{\rm app,max}$.

8.4: The observed proper motion of approaching and receding blobs ejected from GRS 1915+105 are $\mu_a = 17.6$ mas day^{-1} and $\mu_r = 9.0$ mas day^{-1}, respectively. Assuming the distance to the source of $D = 12$ kpc, compute the velocity $\beta = v/c$ and the angle θ between line of sight and jet direction.

Solution:

The proper motion for an approaching and receding part of the jet are

$$\mu_a = \frac{\beta \sin\theta}{1 - \beta\cos\theta}\frac{c}{D}, \quad \mu_r = \frac{\beta \sin\theta}{1 + \beta\cos\theta}\frac{c}{D}. \tag{14}$$

Since arcsec$\times$pc$=$ 1 AU$=$ 1.5×10^{13} cm, and mas$\times$day$^{-1} \times D/c = 10^{-3} \times 1.5 \times 10^{13} \times 12000/3 \times 10^{10} \times 86400 = 0.06944$, we get the respective apparent dimensionless velocities

$$\beta^a_{\rm app} = \frac{\beta \sin\theta}{1 - \beta\cos\theta} = 0.06944 \times 17.6 = 1.222, \tag{15}$$

$$\beta^r_{\rm app} = \frac{\beta \sin\theta}{1 + \beta\cos\theta} = 0.06944 \times 9.0 = 0.625. \tag{16}$$

Noting that

$$\beta\cos\theta = \frac{\beta^a_{\rm app} - \beta^r_{\rm app}}{\beta^a_{\rm app} + \beta^r_{\rm app}}, \quad \beta\sin\theta = 2\frac{\beta^a_{\rm app}\beta^r_{\rm app}}{\beta^a_{\rm app} + \beta^r_{\rm app}}, \tag{17}$$

we get

$$\tan\theta = 2\frac{\beta^a_{\rm app}\beta^r_{\rm app}}{\beta^a_{\rm app} - \beta^r_{\rm app}} \approx 2.56, \tag{18}$$

i.e. $\theta \approx 69$ deg and

$$\beta = \sqrt{(\beta\cos\theta)^2 + (\beta\sin\theta)^2} = \frac{\sqrt{(\beta^a_{\rm app} - \beta^r_{\rm app})^2 + (2\beta^a_{\rm app}\beta^r_{\rm app})^2}}{\beta^a_{\rm app} + \beta^r_{\rm app}} \approx 0.89. \tag{19}$$

8.5: The jet in the nearby active galaxy M87 is probably inclined at 40° to the line of sight. Superluminal motion has been seen by radio astronomers within the core of the jet with $v_{\rm app} = 2.5c$. Estimate the velocity of the jet β, the bulk Lorentz factor of the jet Γ and the Doppler factor for each side of the jet. Note that even the approaching side has a Doppler factor smaller than 1; what is the physical reason for that?

Solution:
Substituting $\beta_{\rm app} = 2.5$ and $\theta = 40$ deg to Eq.(11) we get

$$\beta = \frac{\beta_{\rm app}}{\sin\theta + \beta_{\rm app}\cos\theta} = 0.977. \tag{20}$$

Thus $\Gamma = /\sqrt{1-\beta^2} = 4.7$, and the Doppler factors for the approaching and receding jets:

$$\mathcal{D}_a = \frac{1}{\Gamma(1-\beta\cos\theta)} = 0.85, \quad \mathcal{D}_r = \frac{1}{\Gamma(1+\beta\cos\theta)} = 0.12. \tag{21}$$

For small θ and large Γ, using expansions $\cos\theta \approx 1-\theta^2/2$ and $\beta = \sqrt{1-1/\Gamma^2} \approx 1-1/2\Gamma^2$, the Doppler factor is

$$\mathcal{D} \approx \frac{2\Gamma}{1+(\theta\Gamma)^2}. \tag{22}$$

We see that when $\theta\Gamma \ll 1$, the Doppler factor is large $\mathcal{D} \sim \Gamma$. In our case, the inclination $\theta = 40\,$deg is much larger than $1/\Gamma = 0.21$ rad, i.e. 12 deg. Therefore even the approaching jet Doppler factor is smaller than unity.

8.6: (a) Show that an observer moving with respect to a blackbody field of temperature T will see blackbody radiation with a temperature that depends on the angle according to

$$T_{\rm obs} = \frac{T}{\Gamma(1-\beta\cos\theta')}, \tag{23}$$

where θ' is the angle between direction of motion and observation in the observer's frame.
(b) Cosmic microwave background radiation ($T = 2.7$ K) shows the anisotropy due to solar motion relative to that radiation. Estimate solar velocity if the anisotropy of radiation intensity at $\lambda = 3$ cm is

$$\frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} \approx 10^{-3}. \tag{24}$$

Solution:

Due to the Doppler effect the photon frequency ν as measured in the external frame can be related to the frequency ν' as measured in the comoving frame as

$$\nu = \frac{\nu'}{\Gamma(1-\beta\cos\alpha)} = \nu'\ \Gamma(1+\beta\cos\alpha') = \mathcal{D}\nu', \tag{25}$$

where α and α' are the angles between the velocity vector of the observer $\boldsymbol{\beta}$ and the photon momentum in the external and comoving frames, respectively, $\beta = v/c$ is the observer velocity relative to the blackbody field, $\Gamma = 1/\sqrt{1-\beta^2}$ is the corresponding Lorentz factor, and $\mathcal{D} = \Gamma(1+\beta\cos\alpha')$ is the Doppler factor. Noting that $\alpha' = \pi - \theta'$, the photon frequency as measured in the comoving frame is

$$\nu' = \frac{\nu}{\Gamma(1+\beta\cos\alpha')} = \frac{\nu}{\Gamma(1-\beta\cos\theta')} = \frac{\nu}{\mathcal{D}}. \tag{26}$$

The observed specific intensity (in comoving frame of the observer) is related to the intensity in the external frame (where CMB radiation is described by the Planck function $B_\nu(T)$) as

$$I_{\nu'}^{\rm obs} = \mathcal{D}^{-3} B_\nu(T) = \frac{\nu'^3}{\nu^3}\frac{2h\nu^3/c^2}{\exp(h\nu/kT)-1} = \frac{2h\nu'^3/c^2}{\exp[h\nu'\mathcal{D}/kT]-1} = B_{\nu'}(T_{\rm obs}), \tag{27}$$

where now the temperature of the blackbody

$$T_{\rm obs} = \frac{T}{\mathcal{D}} = \frac{T}{\Gamma(1-\beta\cos\theta')}. \tag{28}$$

For the blackbody of 2.7 K, the wavelength 3 cm lies in the Rayleigh-Jean part of the spectrum, i.e. $\lambda = 3\,\text{cm} \gg \lambda_{\max} = 0.29/T = 0.11$ cm, where the Planck function scales linearly with the temperature $B_\nu \approx 2(\nu/c)^2 kT$. Therefore, variation of the intensity at a given wavelength follows variations of the temperature. The observed temperature has maximum and minimum of $\approx T/\Gamma(1-\beta)$ and $T/\Gamma(1+\beta)$ at $\cos\theta' = 1$ and -1, respectively. The intensity anisotropy is then

$$\frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} = 10^{-3} \approx \frac{\frac{1}{1-\beta} - \frac{1}{1+\beta}}{\frac{1}{1-\beta} + \frac{1}{1+\beta}} = \beta. \tag{29}$$

Thus the solar system velocity is $v = \beta c \approx 300$ km/s.