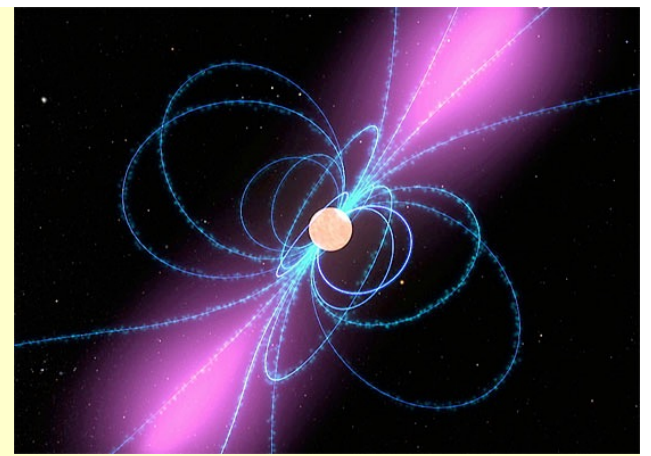


Radio



Pulsars



References:

1. A. G. Lyne & F. Graham-Smith, Pulsar Astronomy , Cambridge University Press, 1998
2. Shapiro & Teukolsky, WD, NS & BHs, Chapters 9 & 10
3. D. Lorimer and M. Kramer, Handbook of pulsar astronomy, Cambridge University Press, 2005



Jocelyn Bell Anthony Hewish

**1967: Jocelyn Bell
discovers radio
pulsars**

**1974: Nobel prize to
her supervisor
Antony Hewish
and Martin Ryle (for
aperture synthesis)**

Discovery of pulsars

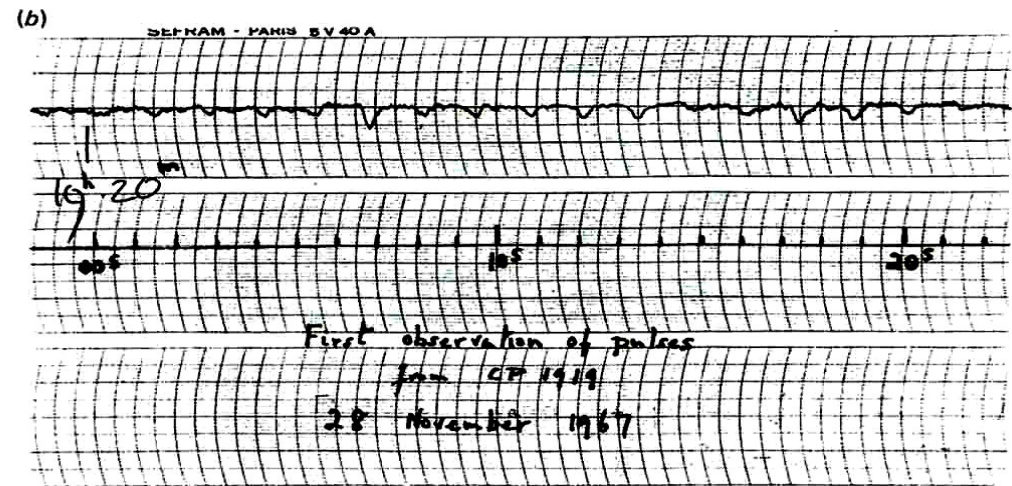
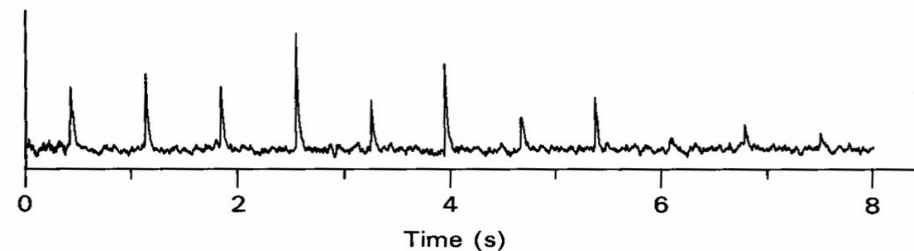
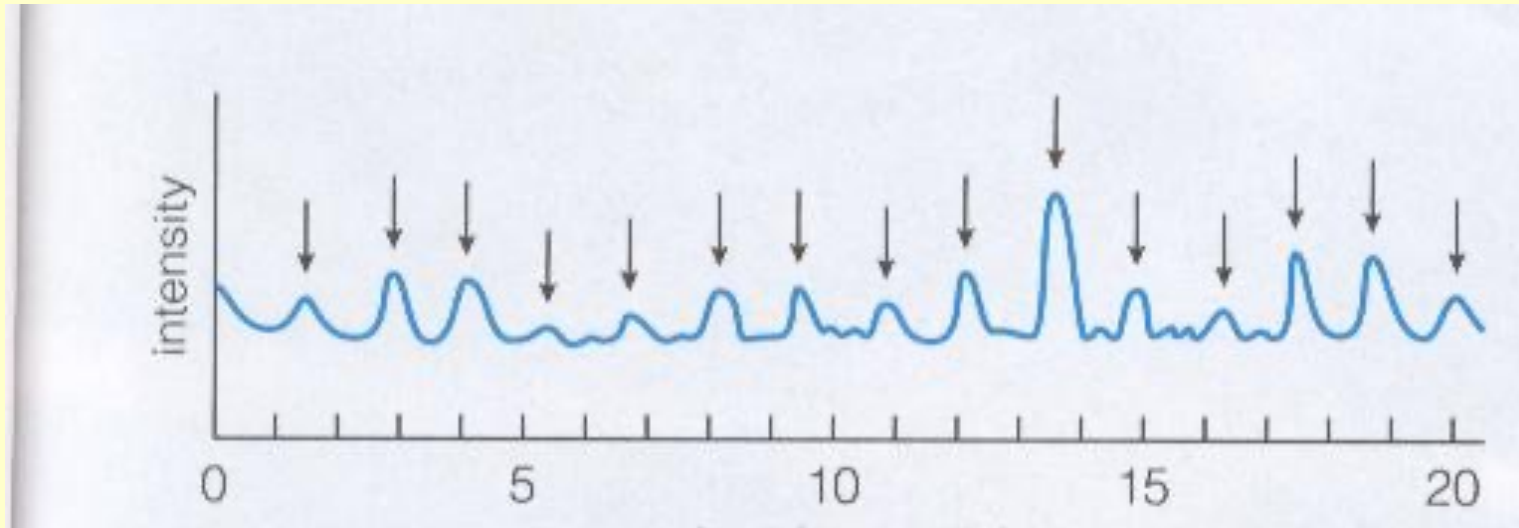


Fig. 1.1. Discovery observations of the first pulsar. (a) The first recording of PSR B1919+21; the signal resembled the radio interference also seen on this chart: (b) Fast chart recording showing individual pulses as downward deflections of the trace (Hewish *et al.* 1968).



1-3 Chart record of individual pulses from one of the first pulsars discovered, PSR 0329 + 54. They were recorded at a frequency of 410 MHz and with an instrumental time constant of 20 ms. The pulses occur at regular intervals of about 0.714 s.

Little Green Men!



- October-November 1967, Jodrell Bank radiotelescope, graduate student Jocelyn Bell and Dr. Anthony Hewish discover a strong source of radio waves in Cygnus pulsating with a period of 1.337301 s. As precise as atomic clocks.
- The mysterious source of radio emission was dubbed LGM for a while.

Pulsar Zoo

- **Radiopulsars** – isolated neutron stars (>4000 known in 2026). Radiate rotational energy and slow down (periods P usually $\leq 1\text{s}$, $dP/dt > 0$). Are observed also in other wavebands.
- **Millisecond radio pulsars** – most have companions; spun-up by accreting gas from the companion (>600 known in 2026). Recently discovered as strong gamma-ray emitters.
- **X-ray pulsars** – neutron stars in binary systems. Accrete matter from normal star companion ($P \sim 1-1000\text{ s}$). >300 known in 2026.
- **X-ray millisecond pulsars** – “future” radio ms pulsars; accreting matter from companions ($P \sim \text{a few ms}$); 24 “accretion-powered” & 18 “nuclear-powered” (some are both).
- **Transitional millisecond pulsars** – discovered in 2009. Transit from the radio-emitting and X-ray dim stage to the X-ray bright, accreting (?) stage ($\sim 3-9$ known in 2026).
- **Magnetars** (AXP - anomalous X-ray pulsars, SGR - soft gamma-ray repeaters) – radiate energy of their strong magnetic field. ~ 30 known in 2026, see <http://www.physics.mcgill.ca/~pulsar/magnetar/main.html>

Pulsar formation

Collapse of a star with conserving angular momentum and magnetic flux leading to formation of the NS gives rise to a few ms P and $B \sim 10^{12}$ G

$$MR^2\Omega = MR_*^2\Omega_* \Rightarrow P = P_* \left(R / R_* \right)^2$$

$$P \sim 1 \text{ ms} \quad (P_* \sim 1 \text{ month} \sim 3 \cdot 10^6 \text{ s}; R_* / R \sim 10^5)$$

$$R^2 B = R_*^2 B_* \Rightarrow B = B_* \left(R_* / R \right)^2$$

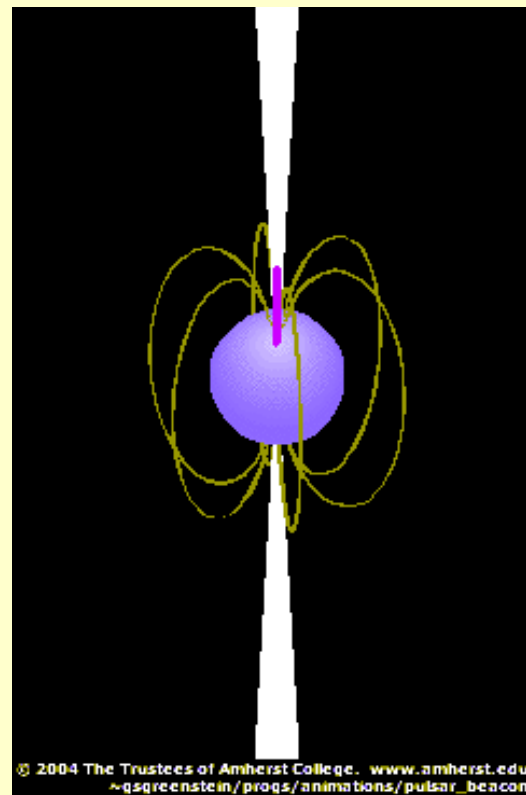
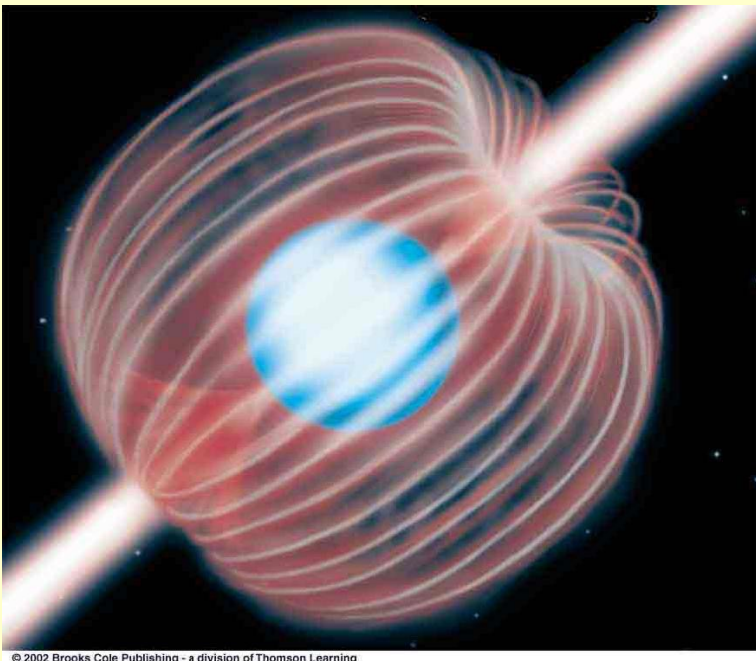
$$B \sim 10^{12} \text{ Gauss} \quad (\text{if } B_* \sim 100 \text{ G})$$

As a result of dynamo action (Duncan & Thompson 1992), the magnetic field can be amplified and the magnetar can be formed with $B \sim 10^{15}$ G

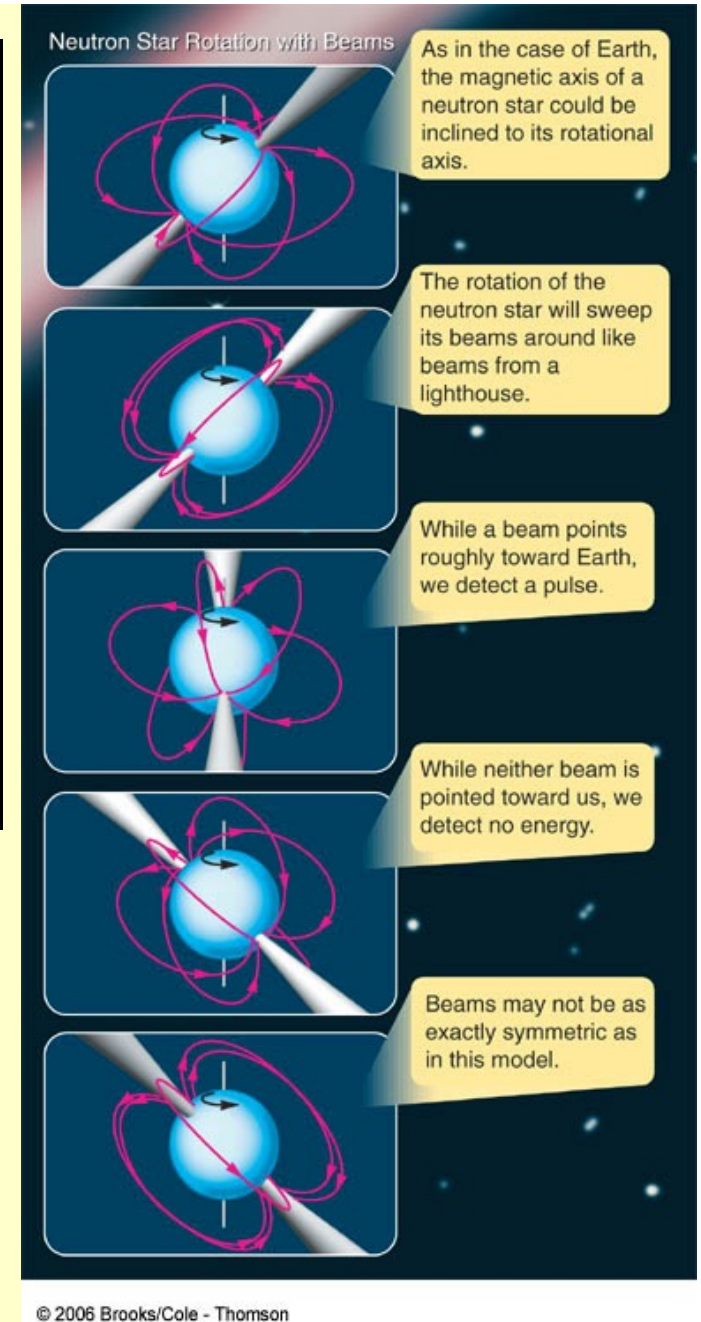
Model of a pulsar:

Pulsars do not pulse, but rather emit beams of radiation that sweep around the sky as the neutron star rotates

Strong magnetic and electric fields are likely the cause of the intense beams of radiation



Note that we only can see the pulsars whose beams sweep over Earth.



- 1968: Thomas Gold proposes rotating NS model for Pulsars

- Why neutron stars? Pulsation timescale for WD is:

$$P \sim 2\pi\sqrt{l/g} = 2\pi\sqrt{R/(GM/R^2)} = 17(M/M_{Sun})^{-1/2} R_9^{3/2} \text{ s}$$

[$Q=10^x Q_x$ in cgs] (The period of the closest orbit is similar; moreover, these timescales decrease with time – not increase as for pulsars).

- The break-up rotation period for WDs is also $\sim 1\text{s}$.
- Not possible to get highly stable periodic signal from BHs.
- The break-up rotation period, pulsation or dynamical time for a NS is $\sim$ millisecond; rotation can explain the observed period range and stability.

Minimum period: gravity > centrifugal force

$$GM / R^2 > \Omega^2 R \Rightarrow$$

$$P = 2\pi / \Omega > 2\pi / \sqrt{GM / R^3}$$

$$P_{NS} > 0.5 (M / M_{\text{sun}})^{-1/2} R_6^{3/2} \text{ ms}$$

Minimum density:

$$\rho = \frac{M}{\frac{4}{3}\pi R^3} > \frac{3\pi}{P^2 G}$$

for Crab $P=0.033$ s, $\rho > 1.3 \cdot 10^{11}$ g/cc

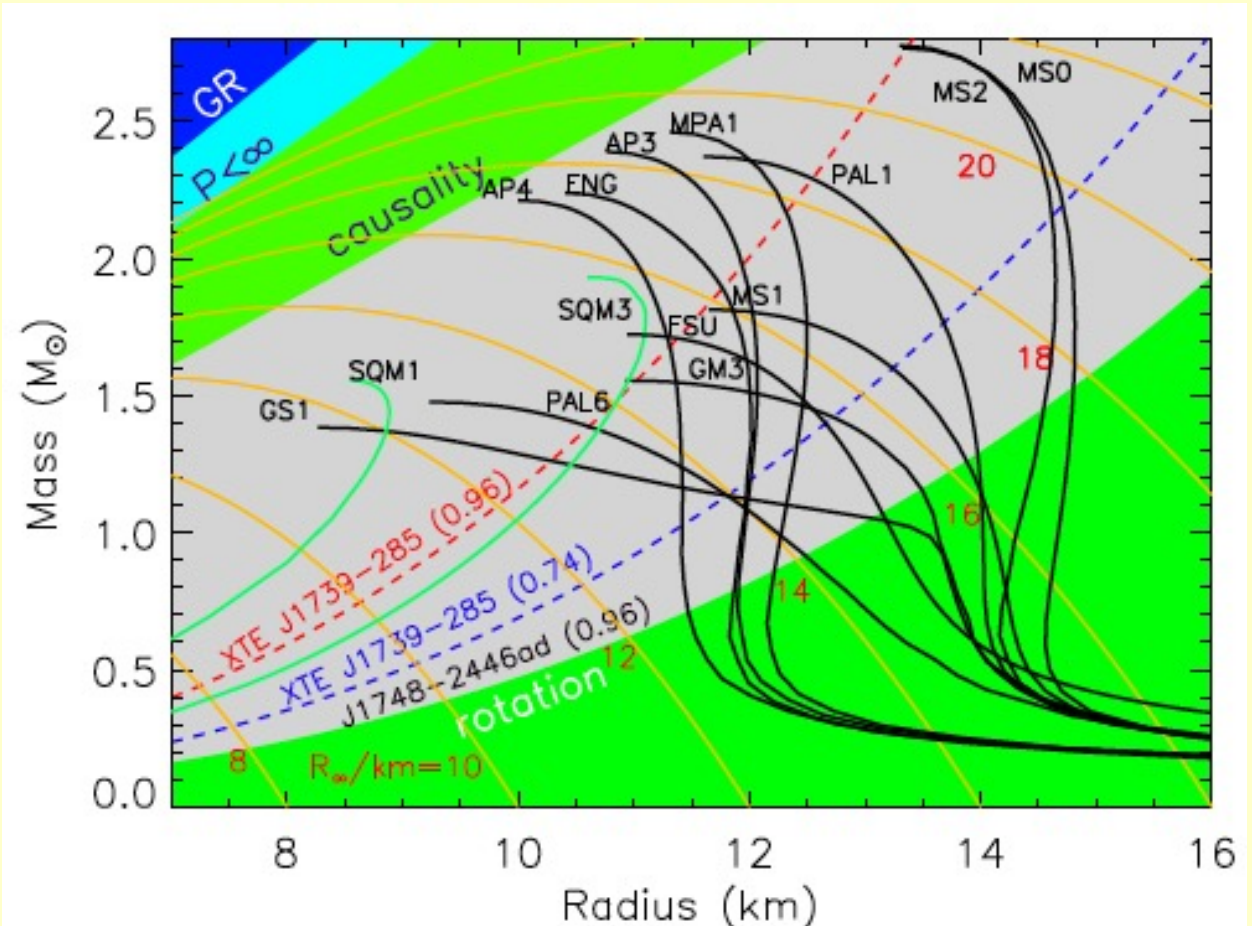
This is too high for a white dwarf (which has a density of $\sim 10^6$ g/cc), so it must be a **neutron star**.

Constraints on neutron star M and R from rapid rotation

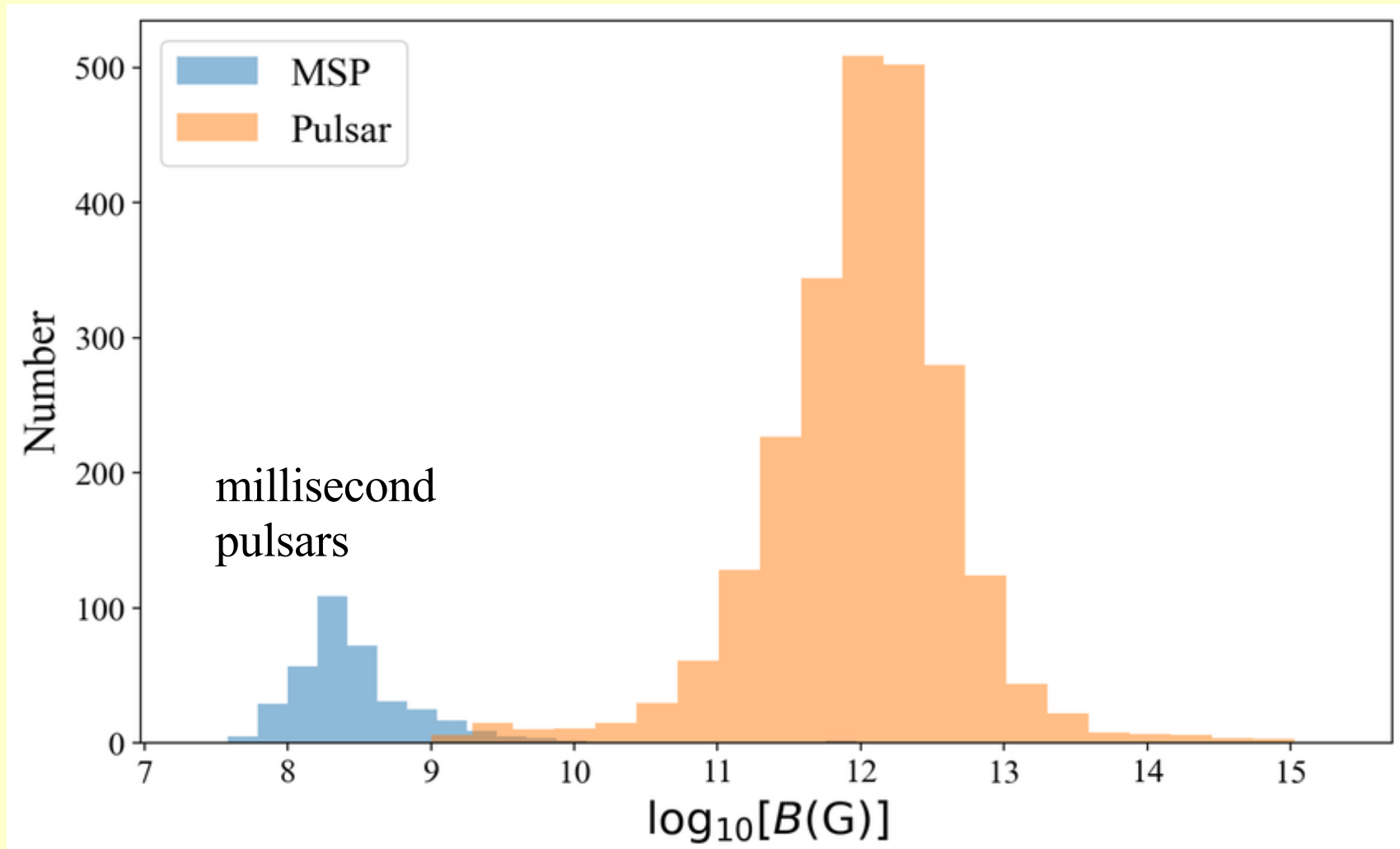
Existence of a pulsar
J1748-2446ad, which
rotates at 716 Hz,
does not allow too
large radii for a given
mass.

The mass-shedding
limit is (Haensel et al.
2009)

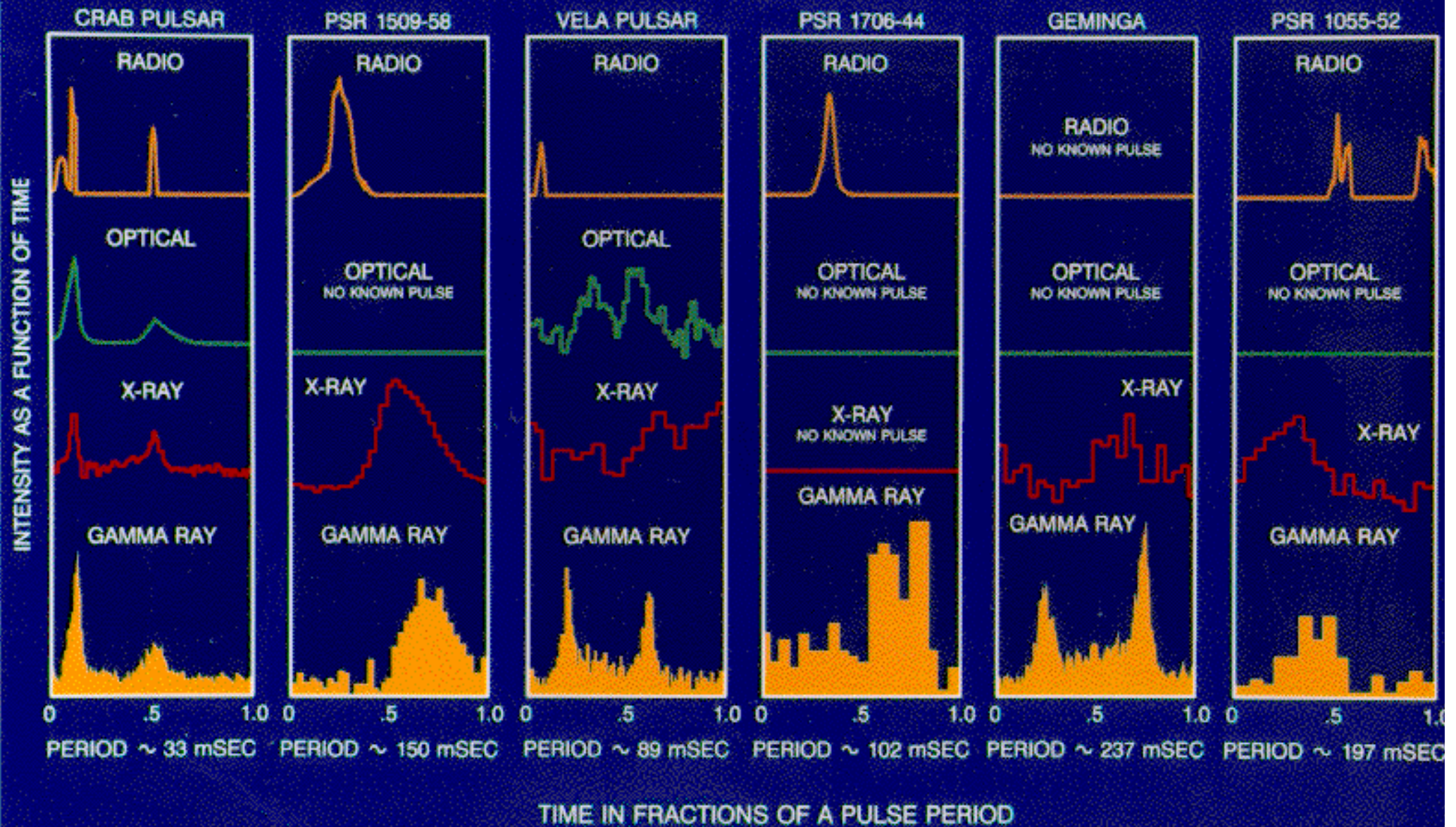
$$f \leq 1.04 \left(M_{\text{sph}} / M_{\text{sun}} \right)^{1/2} \left(R_{\text{sph}} / 10 \text{ km} \right)^{-3/2} \text{ kHz}$$



Pulsar period distribution

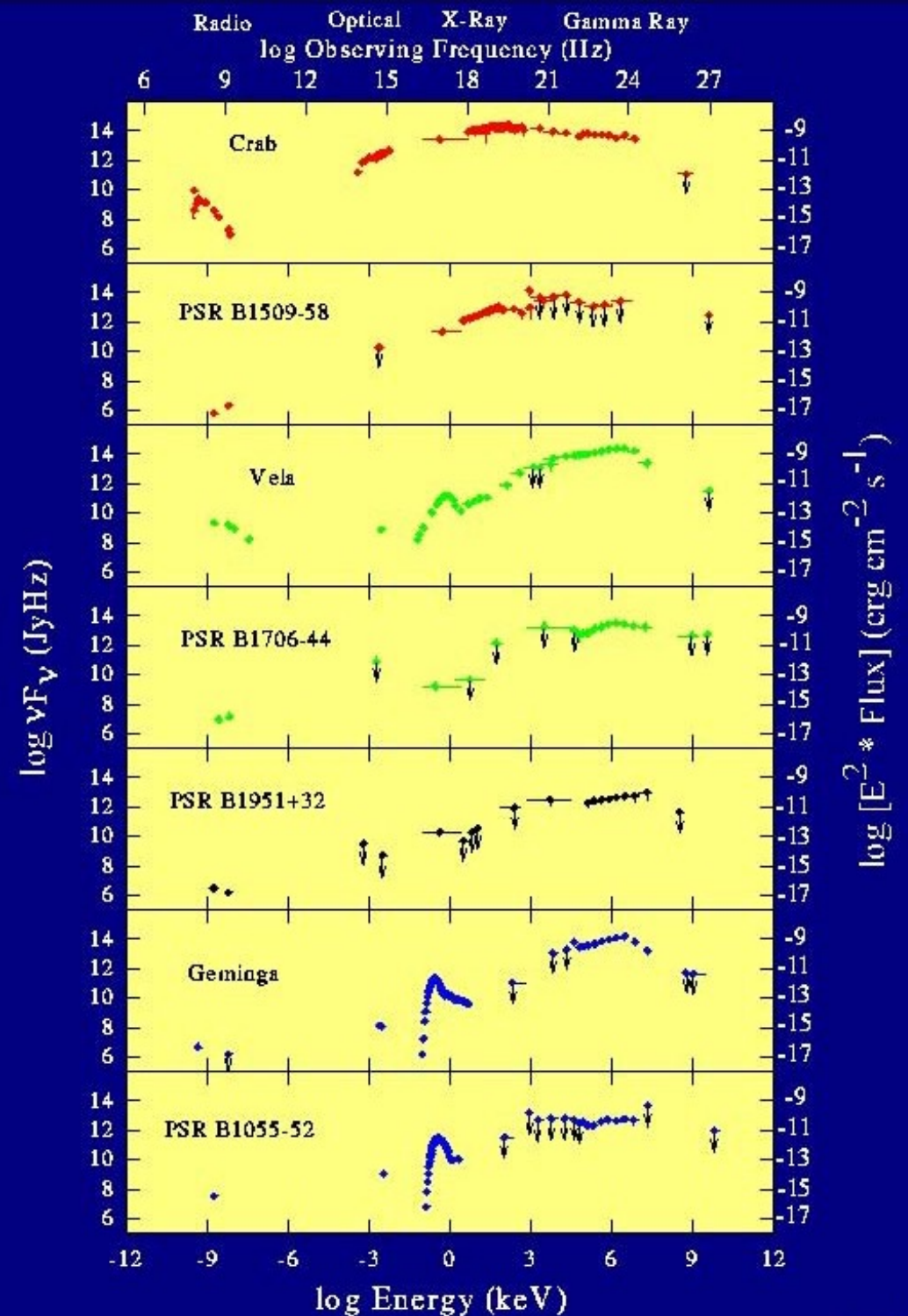


GAMMA-RAY PULSARS

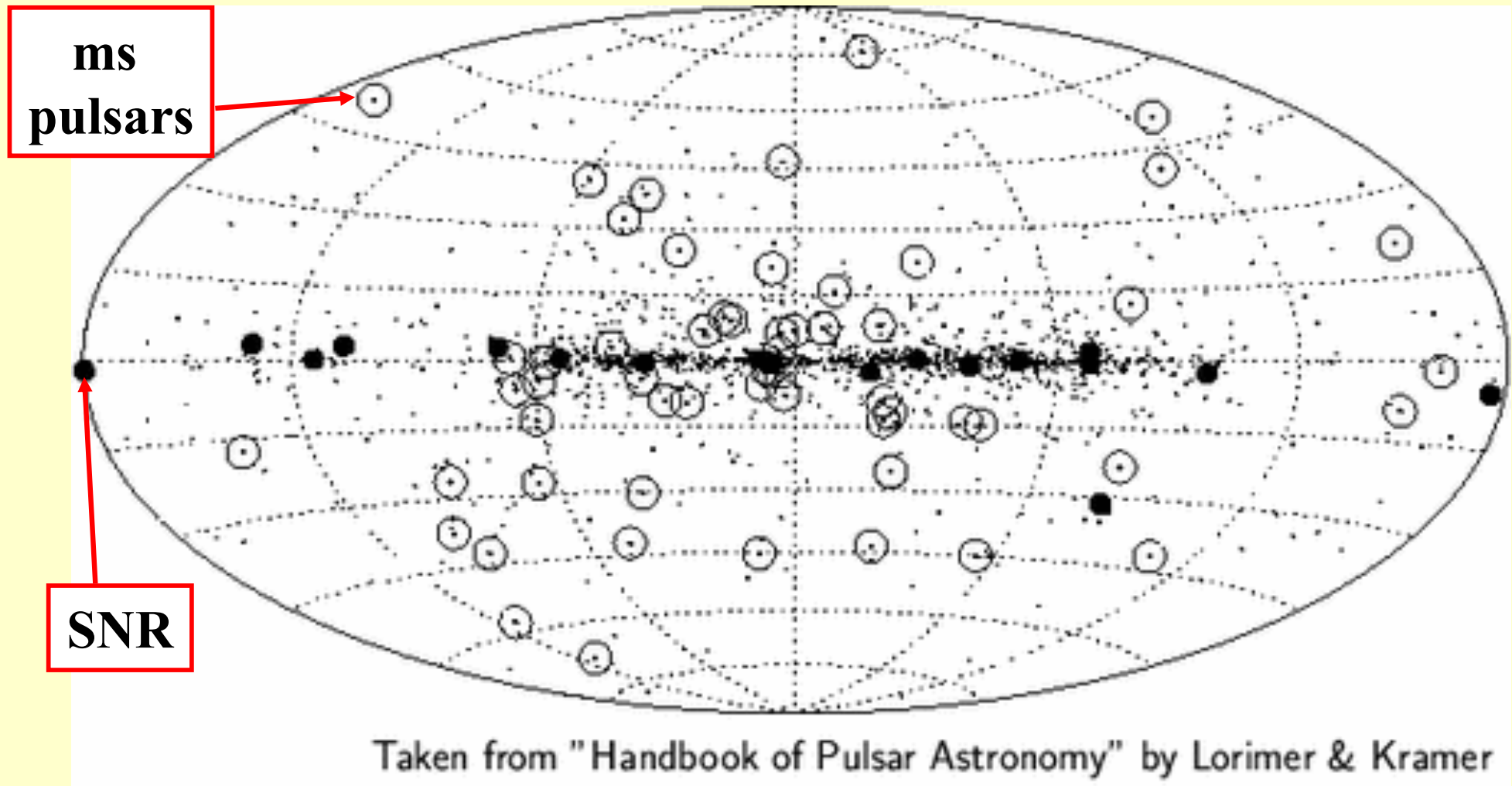


Spectral energy distribution

Most of the energy is emitted in X-rays and gamma-rays



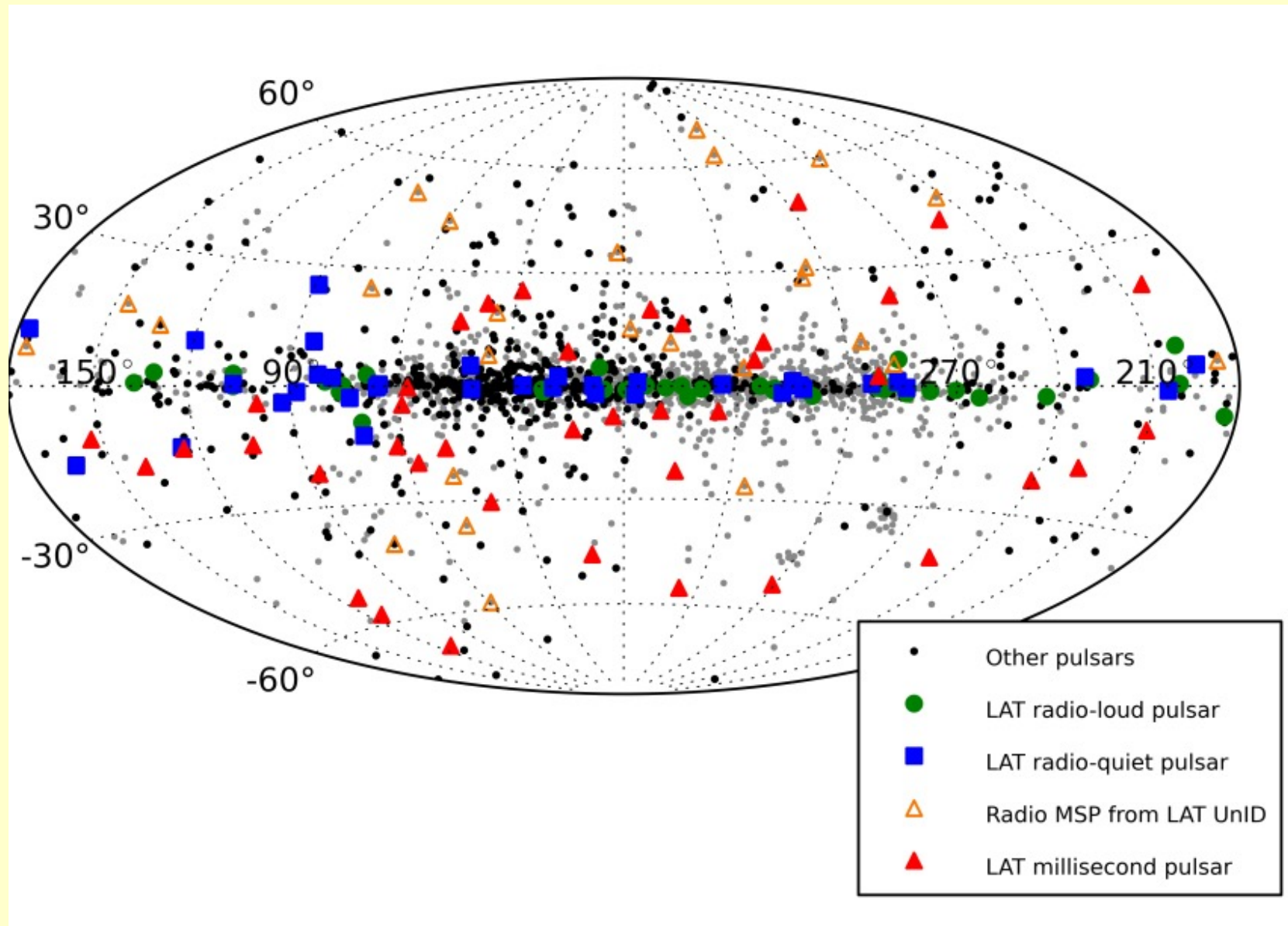
Pulsar galactic distribution



Scale-height $h=300$ pc, velocities a few 100 km/s

Significant fraction will escape the Galactic gravitational potential

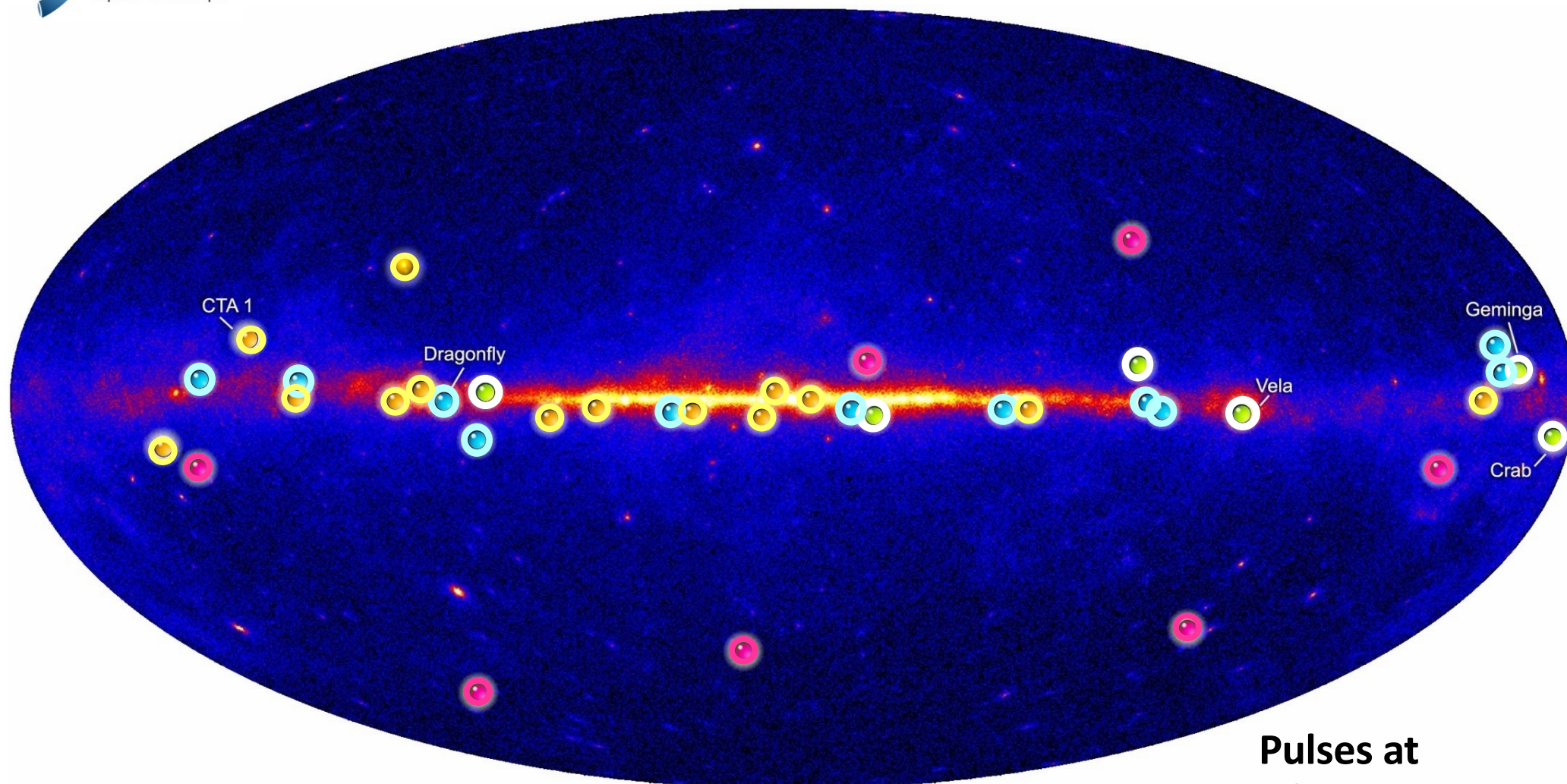
Pulsar galactic distribution



Fermi gamma-ray space telescope observations of pulsars

Fermi LAT γ -ray Sky

more than 200 gamma-ray pulsars were known in 2017

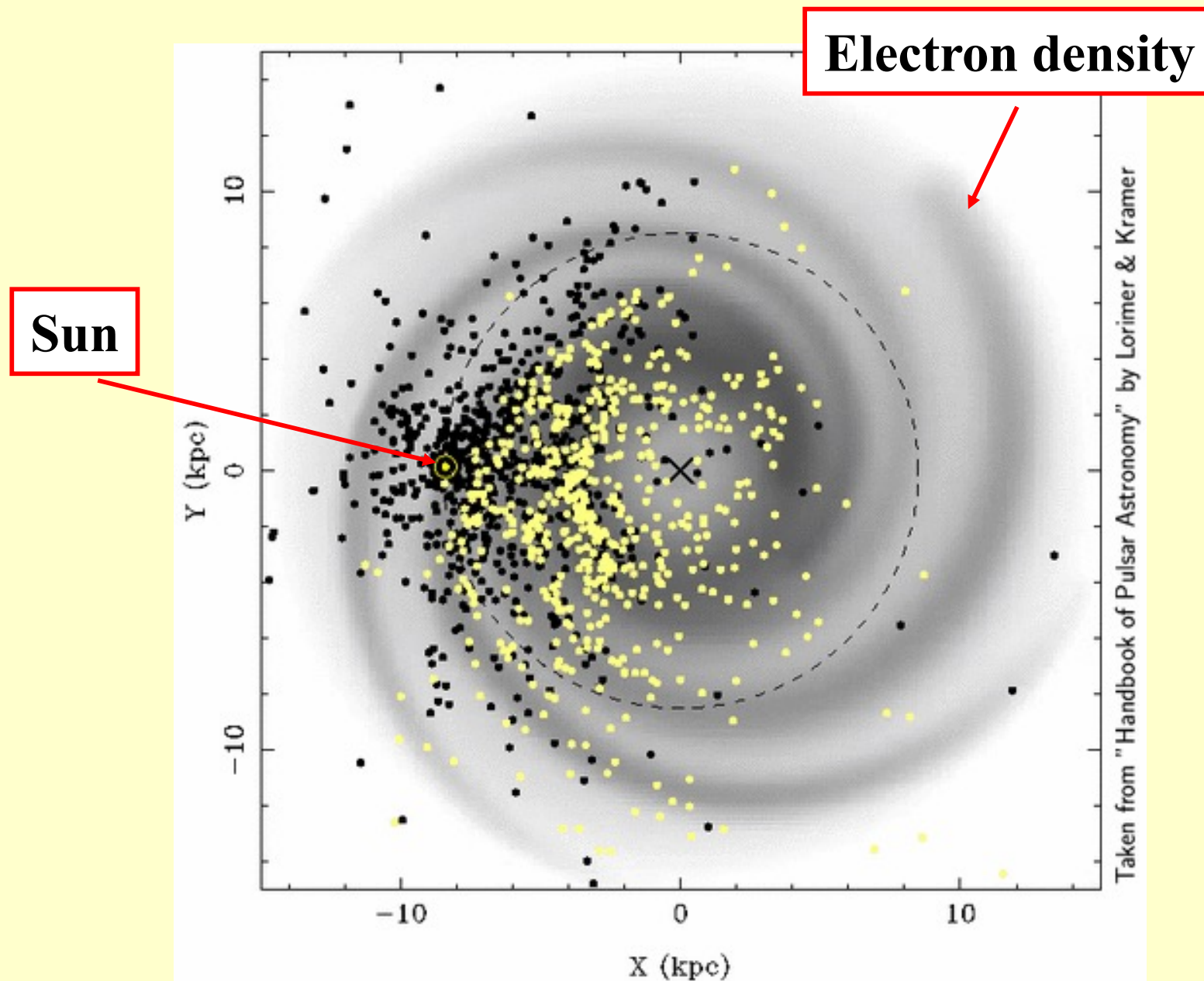


Fermi Pulsar Detections

- New pulsars discovered in a blind search
- Millisecond radio pulsars
- Young radio pulsars
- Pulsars seen by Compton Observatory EGRET instrument

Pulses at
 $1/10^{\text{th}}$ true rate

Pulsar galactic distribution



Pulsar Population

- Pulsars lose their energy and “die”. To sustain the observed population, 1 pulsar must form every 50 years.
- cf. supernovae (SN) rate of 1 every 50-100 years
- only a dozen of pulsars are associated with visible SN remnants (pulsar lifetime 1-10 Myr, SN remnants 10-100 kyr ... so about 1% of pulsars should be within SNRs, which is consistent with the data)

Pulsar Distance Determination

- Parallax
- Neutral H absorption at 21 cm
The Doppler shift of the 21cm absorption line together with the dynamical model of the Galaxy can be used to identify the location of the hydrogen clouds and determine the distance to the pulsar
- Dispersion measure (*pulses at different frequencies arrive at different times*)

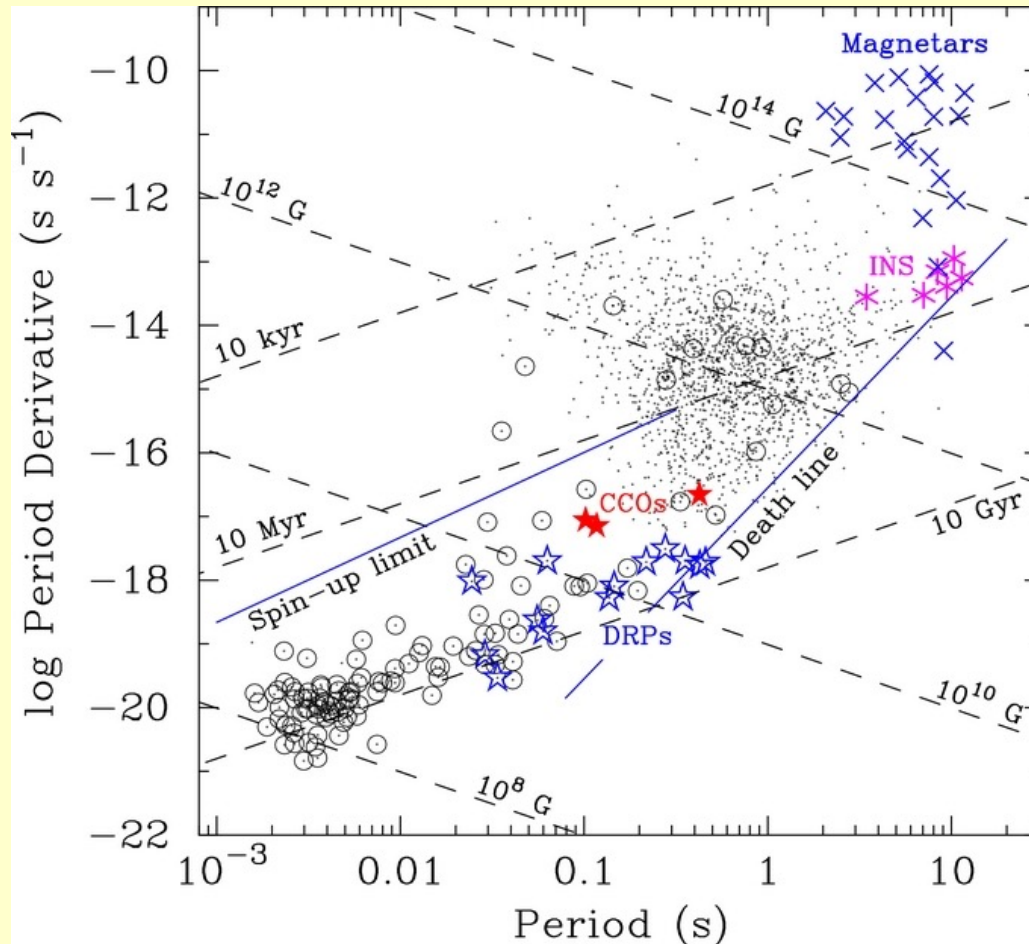
Table 4.1. *Pulsar parallax distances from position and timing measurements*

PSR	Parallax (milliarcsec)	Distance (kpc)	DM (cm^{-3} pc)	Derived $\langle n_e \rangle$ (cm^{-3})	Technique	Ref.
J0633+1746	6.4 ± 1.7	0.16	—	—	O	1
B0823+26	2.8 ± 0.6	0.38	19.5	0.054	R	2
B0950+08	7.9 ± 0.8	0.12	3.0	0.024	R	2
B1451-68	2.2 ± 0.3	0.45	8.6	0.019	R	3
J1713+0747	0.9 ± 0.3	1.1	16.0	0.015	T	4
B1855+09	1.1 ± 0.3	0.9	13.3	0.015	T	5
B1929+10	5.0 ± 1.5	0.20	3.2	0.016	R	6
B1937+21	0.12 ± 0.08	8.3	71.0	0.008	T	5
B2021+51	0.95 ± 0.37	1.1	22.6	0.021	R	7

Technique: O—optical astrometry; R—radio astrometry; T—timing

- | | | |
|---|-----------------------------------|---------------------------------|
| 1. Caraveo <i>et al.</i> (1996) | 2. Gwinn <i>et al.</i> (1986) | 3. Bailes <i>et al.</i> (1990a) |
| 4. Camilo, Foster &
Wolszczan (1994) | 5. Kaspi, Taylor &
Ryba (1994) | 6. Campbell (1995) |
| 7. Campbell <i>et al.</i> (1996) | | |

Period- Period derivative ($P-\dot{P}$) diagram



Gotthelf et al.
(2013)

Pulsar populations on the $P-\dot{P}$ diagram, including magnetars (blue crosses), radio quiet isolated NS (INS, magenta asterisks), central compact objects/CCOs (filled red stars), and disrupted recycled pulsars/DRPs (open blue stars). Black dots are isolated pulsars and circled dots are pulsars in binaries.

Dashed lines of constant characteristic age $\tau = P/(2\dot{P})$ and magnetic field are indicated.

**Crab nebula (Plerion;
from Greek “pleres” which means “full”).**

**Blue: X-ray
Red: optical
Green: radio**

**Luminosity $\sim 10^{38}$ erg/s
(mostly in X-rays &
gamma-rays)**

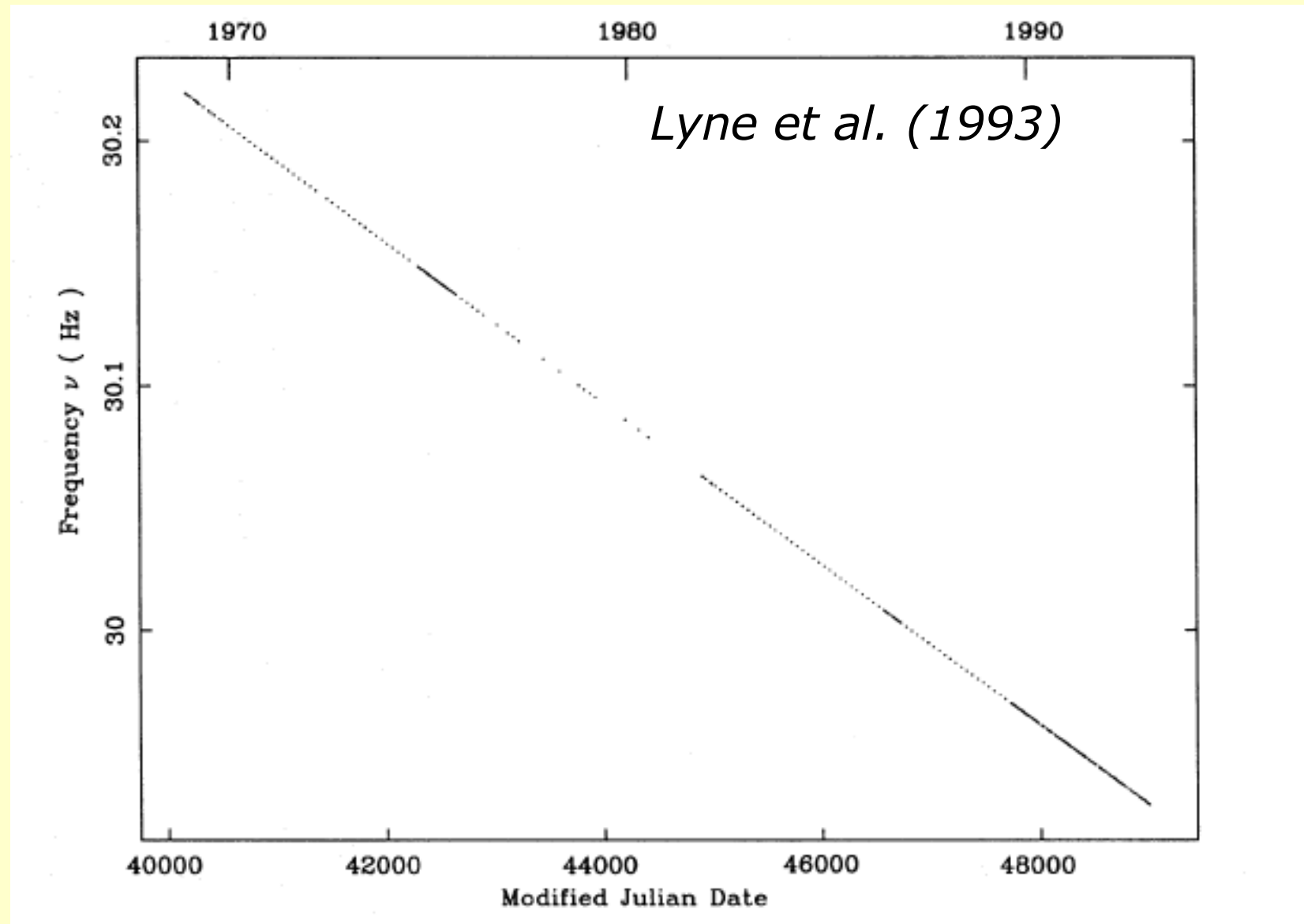
Synchrotron radiation
(optical linear polarization of 9%
averaged over nebula).

Electrons with energy $> 10^{14}$ eV
are needed for emission at 10 keV;
lifetime for these e^- 's < 1 year.
So electrons must be injected
continuously &
not come from SNe.



Crab nebula is the remnant of SNe explosion (perhaps type II) observed by the Chinese astronomers in 1054 (July 4th). The pulsar at the center has a period of 33 milliseconds.

Changes in Crab pulsar spin rate



Pulsar energetics

- Pulsars slow down $\Rightarrow$ lose rotational energy - can this account for observed emission of plerions?
- Rotational energy:

$$E = \frac{1}{2} I \Omega^2 = \frac{I}{2} \left(\frac{4\pi^2}{P^2} \right) = \frac{2\pi^2 I}{P^2}$$

so

$$\frac{dE}{dt} = \frac{d}{dt} \left(\frac{2\pi^2 I}{P^2} \right) = - \frac{4\pi^2 I}{P^3} \frac{dP}{dt}$$

Energetics - Crab pulsar

Crab pulsar: $M \sim 1.4 M_{\odot}$; $P = 0.033$ s; $R \sim 10^6$ cm

Moment of inertia (for a homogeneous sphere):

$$I \approx \frac{2}{5} MR^2 = \frac{2}{5} \times 1.4 \times 2 \times 10^{33} \times 10^{12} \sim 10^{45} \text{ g cm}^2$$

From observations

$$\frac{1}{P} \frac{dP}{dt} \sim 10^{-11} \text{ s}^{-1}$$

the energy lost by the pulsar

$$\frac{dE}{dt} = \frac{-4 \times 10 \times 10^{45}}{(0.033)^2} 10^{-11} = -4 \times 10^{38} \text{ erg/s}$$

Thus the pulsar can power the nebula which has luminosity $L \sim 10^{38}$ erg/s. (Note! $L \sim 10^5 L_{\odot}$)

Magnetic dipole radiation formula

Magnetic dipole radiation energy loss rate

$$\frac{dE}{dt} = -\frac{2}{3c^3} \left(\frac{d^2 m}{dt^2} \right)^2$$

where $m(t) = m_0 [e_3 \cos i + (e_1 \cos \Omega t + e_2 \sin \Omega t) \sin i]$

is the magnetic moment of the neutron star and $m_0 = B R^3/2$
[cf. Larmor formula for electric dipole radiation:
 $dE/dt = -2e^2 a^2/3c^3 = -2(d^2 d/dt^2)^2/3c^3$, d - dipole moment].

$$\text{Thus } \frac{dE}{dt} = -\frac{2}{3c^3} m_0^2 \Omega^4 \sin^2 i$$

More accurate calculation of force-free pulsar magnetospheres
(Spitkovsky 2006) gives $dE/dt \approx -m_0^2 \Omega^4 (1 + \sin^2 i)/c^3$

$$\text{For } B \sim 10^{12} \text{ G} \ \& \ P=0.1 \text{ s} \quad dE/dt \sim 10^{35} \text{ erg/s}$$

Magnetic field estimation

Since $E = I \Omega^2/2$ and $dE/dt = I \Omega d\Omega/dt$, we get:

$$d\Omega/dt = - B^2 R^6 \Omega^3 \sin^2 i / (6 I c^3) = - K \Omega^3$$

or $P dP/dt = K (2\pi)^2$

Observationally: $d\Omega/dt = - K \Omega^n$

where n – "braking index".

$$n = \frac{v\ddot{v}}{\dot{v}^2}$$

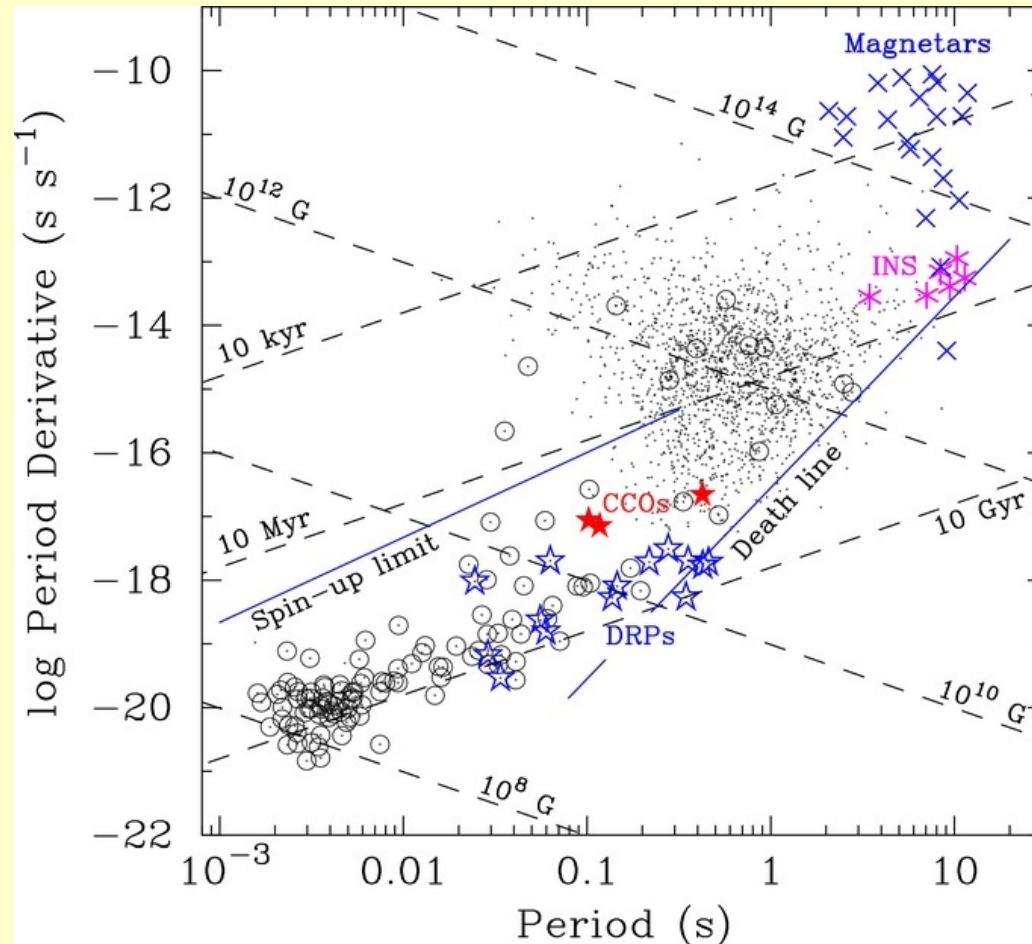
$$\Omega = 2\pi\nu$$

For the magnetic dipole model $n = 3$. [Observations give n between 1.4–2.8. The deviation of the braking index from 3 could probably be due to torque on the pulsar from outflow of particles.] The magnetic field can be estimated as (taking $\sin i=1$):

$$B \approx \sqrt{3c^3 I P \dot{P} / 2\pi^2 R^6} \text{ Gauss} \approx 6 \times 10^{19} \text{ G} \sqrt{P \dot{P}}$$

Here P in seconds, $\dot{P}$ in s/s.

Magnetic field B determined from the magnetic dipole radiation formula



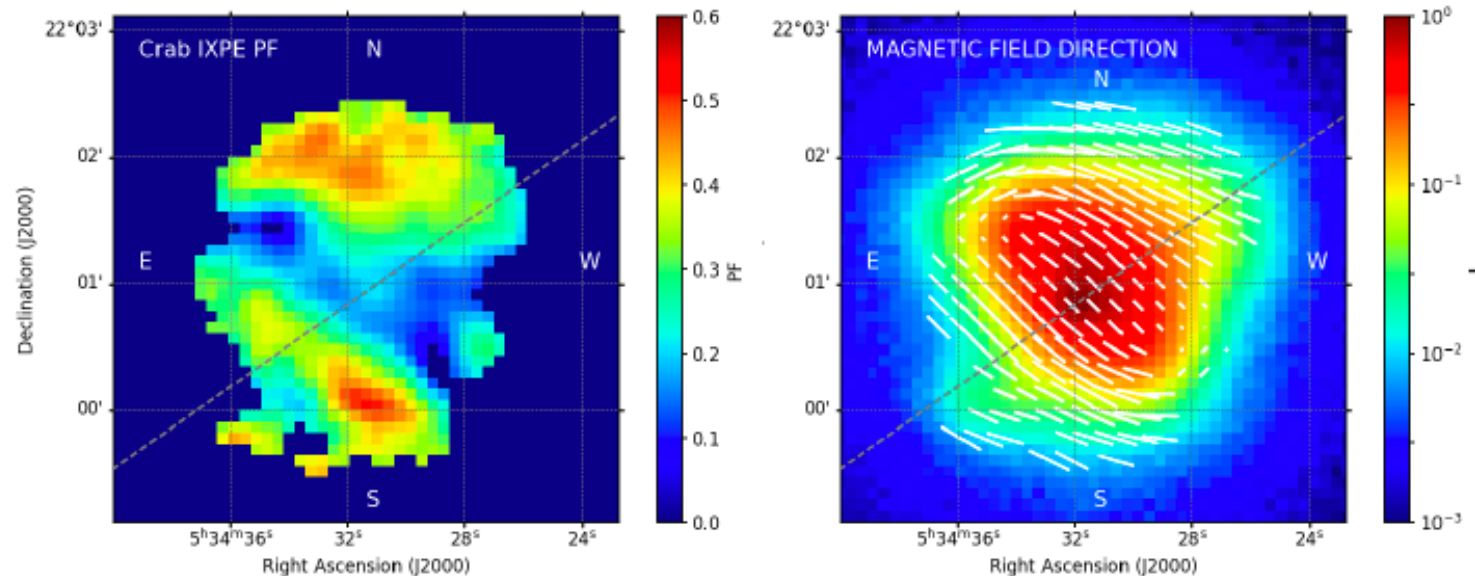
Gotthelf et al.
(2013)

Pulsar populations on the P - $\dot{P}$ diagram, including magnetars (blue crosses), radio quiet isolated NS /INS (magenta asterisks), central compact objects/CCOs in SNRs (filled red stars), and disrupted recycled pulsars/DRPs (open blue stars).

Black dots are isolated pulsars and circled dots are pulsars in binaries.

Dashed lines of constant characteristic age $\tau = P/(2\dot{P})$ and magnetic field are indicated.

X-ray polarimetry of Crab nebula



Polarized Structure of the Crab nebula. Right panel: Total intensity map of the Crab PWN+PSR complex in the [2-8] keV energy band, overlaid with the reconstructed polarisation direction (magnetic field). Left panel: map of the polarized fraction (PD/100) cut above 5σ confidence level. The gray dashed line is the nebular axis inferred from X-ray intensity maps [32]. Overlaid are the Sky directions for ease of reference.

X-ray linear polarization of 20% averaged over nebula was first detected by OSO-8 in 1978 and with much higher accuracy and spatially-resolved by IXPE in 2022.
High polarization implies rather ordered magnetic field.

Age of Pulsars

$$\frac{d\Omega}{dt} = -K\Omega^n \Rightarrow \frac{1}{\Omega^{n-1}(t)} - \frac{1}{\Omega^{n-1}(0)} = K(n-1)t$$

For $\Omega(t) \ll \Omega(0)$, the pulsar spin-down age is

$$t = \frac{1}{K\Omega^{n-1}(t)(n-1)} = \frac{\Omega}{|\dot{\Omega}|(n-1)} = \frac{P}{|\dot{P}|(n-1)}$$

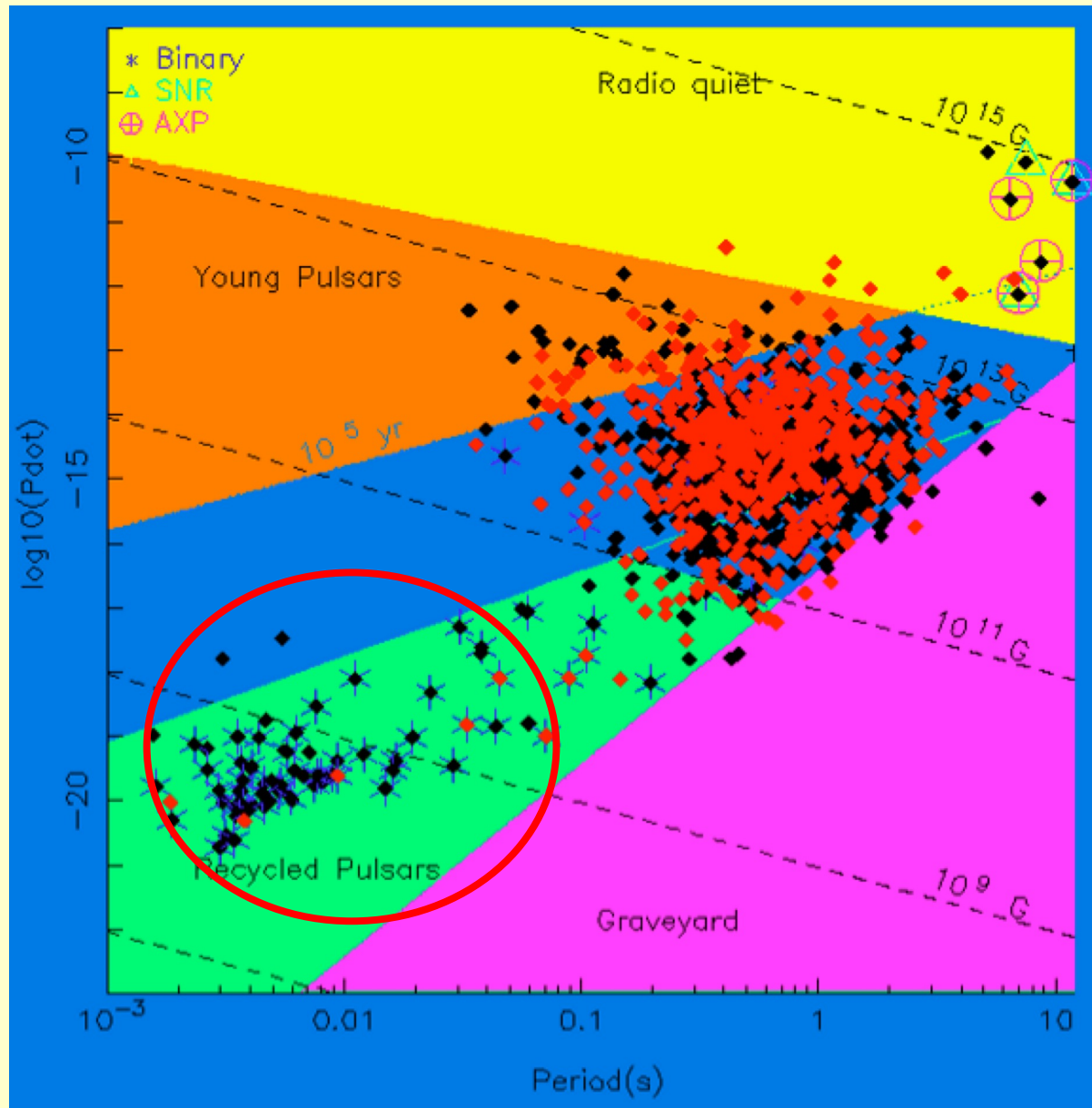
for $n=3$ $t = P / 2\dot{P}$

which is OK for many historical SN pulsars.

In reality, may be longer than the real age.

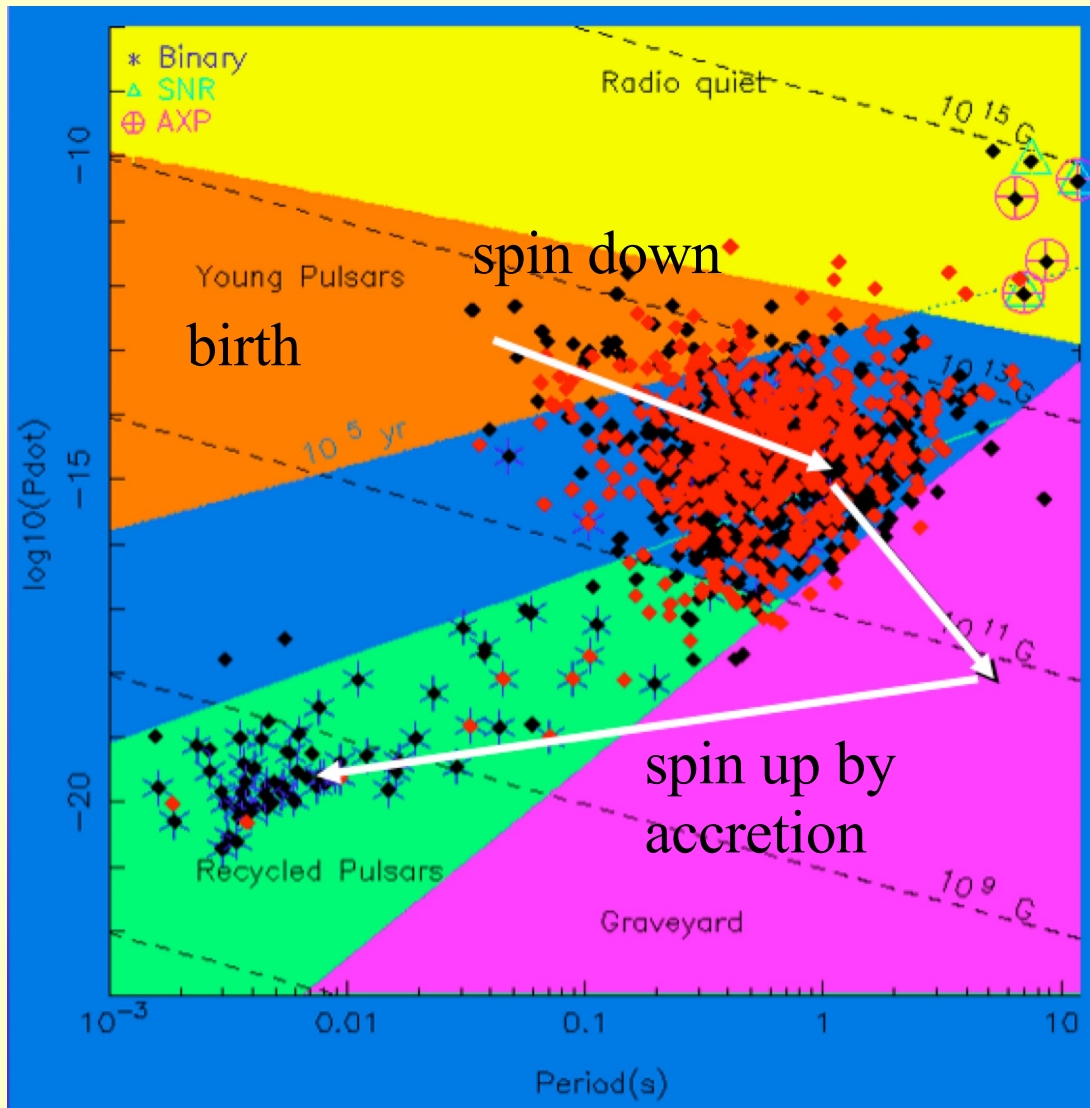
(Note: $\dot{P}$ should be positive to use that formula.)

ms pulsars



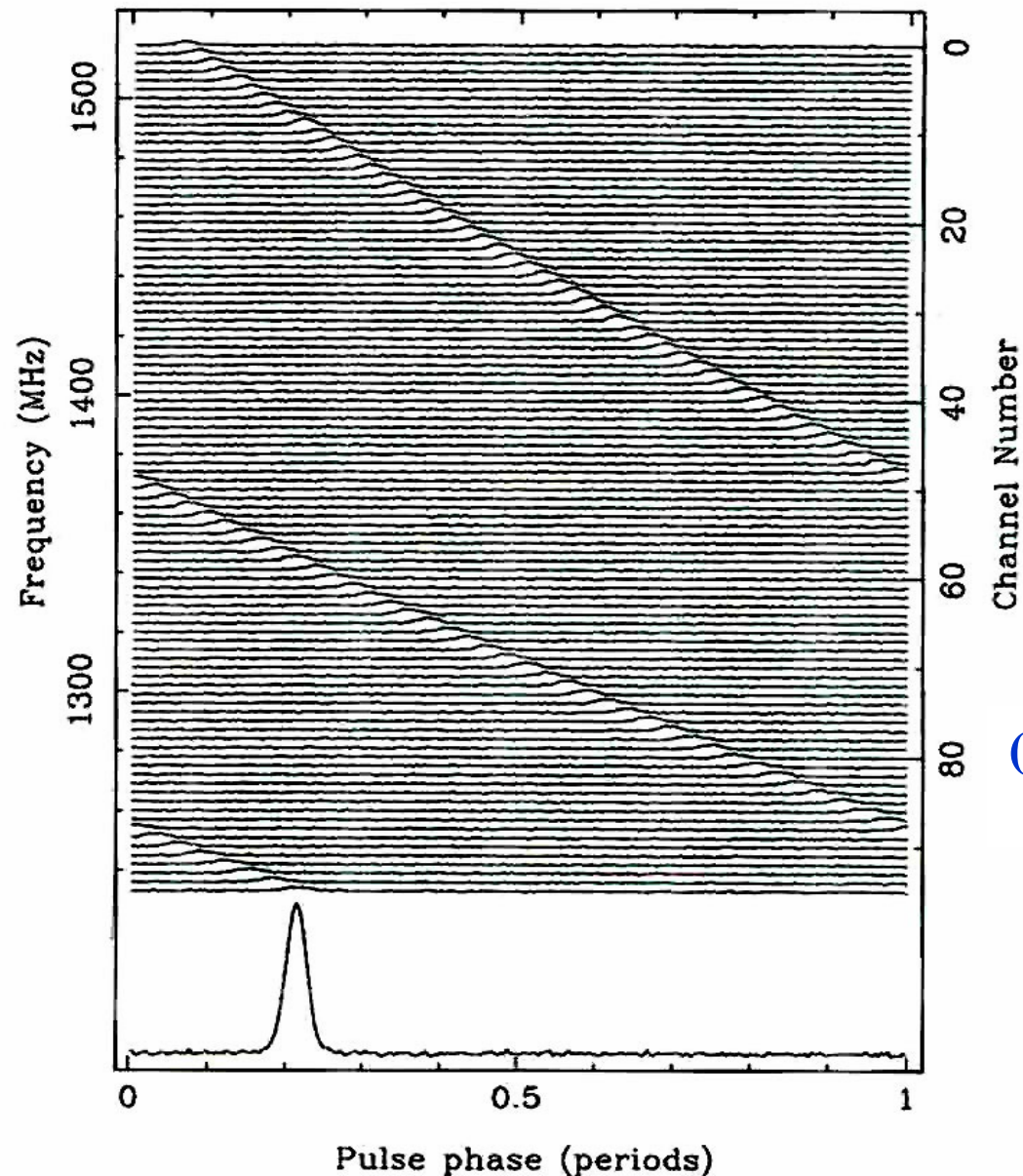
1. The first ms pulsar PSR B1937+21 was discovered in 1982 by Backer et al. $P=1.558$ ms (641 Hz)
2. The fastest pulsar known is PSR J1748-2446ad (716 Hz) discovered in 2004 by Hessels et al.
3. Most of the ms pulsars are in binary system.
4. Magnetic field for ms pulsars is low.
5. ms pulsars lie below the spin-up line which is derived later.
4. ms pulsars are the most stable clock --- $dP/dt \sim 10^{-20}$; i.e. period changes by $\sim 10^{-13}$ s in one year!

Formation of radio ms pulsars



- Pulsars are born with short spin periods P and high B .
- *Spin down* by emission of relativistic wind, waves, photons, magnetic field interaction with the surrounding.
- Magnetic field decays on the time scale of 10^8 years.
- *Spin-up* by accretion of gas from the companion.
- Spin-up line represents the minimum period of recycled pulsars for a given field strength.
- Death-line is an empirical estimate of the maximum period at which pulsars have detectable radio emission for a given field strength.

Pulse Dispersion



(Lyne & Graham-Smith
"Pulsar Astronomy")

Fig. 3.1. Frequency dispersion in pulse arrival time for PSR B1641-45, recorded in 96 adjacent frequency channels, each 3 MHz wide, centred on 1380 MHz.

Propagation of radiation in plasma and plasma frequency

- Maxwell equations

$$\nabla \cdot \vec{D} = 4\pi\rho$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{H} = \frac{4\pi}{c} \vec{j} + \frac{1}{c} \frac{\partial \vec{D}}{\partial t}$$

$$\vec{D} = \epsilon \vec{E}$$

ϵ – dielectric constant

$$\vec{B} = \mu \vec{H}$$

μ – magnetic permeability

In vacuum $\epsilon = 1, \mu = 1$

$$\rho = \lim_{\Delta V \rightarrow 0} \frac{1}{\Delta V} \sum_i q_i \text{ - charge density}$$
$$\vec{j} = \lim_{\Delta V \rightarrow 0} \frac{1}{\Delta V} \sum_i q_i \vec{v}_i \text{ - current density}$$

- Taking $\nabla \cdot$ of the $\nabla \times H$ equation we get conservation of charge

$$\nabla \cdot \vec{j} + \frac{\partial \rho}{\partial t} = 0$$

- Maxwell equations for vacuum can still be used for a plasma if the charge ρ and current j densities due to the plasma are explicitly included.
- Assume variations of all quantities in the form $\exp[i(\vec{k} \cdot \vec{r} - \omega t)]$

$$i\vec{k} \cdot \vec{E} = 4\pi\rho$$

$$i\vec{k} \cdot \vec{B} = 0$$

Note :

$$i\vec{k} \times \vec{E} = i\frac{\omega}{c}\vec{B}$$

$$i\vec{k} \times \vec{B} = \frac{4\pi}{c}\vec{j} - i\frac{\omega}{c}\vec{E}$$

$$\frac{\partial}{\partial x} \rightarrow ik_x, \quad \frac{\partial}{\partial t} \rightarrow -i\omega$$

- We assume that plasma consists of electrons with density n (ions can be neglected here because they are less mobile and contribute negligible to the current). Also, there is no magnetic field, thus plasma is isotropic. Each electron responds to the electric field according to Newton's law:

$$m \frac{d}{dt} \vec{V} = -e\vec{E}$$

- The magnetic force can be neglected as it is of order V/c smaller.

- Thus we get for the velocity: $\vec{V} = \frac{e\vec{E}}{i\omega m}$
- The current density is $\vec{j} = -ne\vec{V}$
and therefore $\vec{j} = \sigma\vec{E}$ where the conductivity $\sigma = \frac{ine^2}{\omega m}$
- From the charge conservation we get $-i\omega\rho + i\vec{k} \cdot \vec{j} = 0$
and the charge density

$$\rho = \omega^{-1}\vec{k} \cdot \vec{j} = \sigma\omega^{-1}\vec{k} \cdot \vec{E}$$

- The Maxwell equations then take the form:

$$i\vec{k} \cdot \epsilon\vec{E} = 0 \qquad i\vec{k} \cdot \vec{B} = 0$$

$$i\vec{k} \times \vec{E} = i\frac{\omega}{c}\vec{B} \qquad i\vec{k} \times \vec{B} = -i\frac{\omega}{c}\epsilon\vec{E}$$

where $\epsilon = 1 - \frac{4\pi\sigma}{i\omega} = 1 - \left(\frac{\omega_p}{\omega}\right)^2$ is dielectric constant

and plasma frequency $\omega_p^2 = \frac{4\pi ne^2}{m} \Rightarrow \omega_p = 5.63 \times 10^4 n^{1/2} \text{ s}^{-1}$

- The equations are identical to source-free Maxwell equations in vacuum. Representing the vectors as

$$\vec{E} = \vec{a}_1 E_0 \quad , \quad \vec{B} = \vec{a}_2 B_0$$

where E_0 and B_0 are complex constants and $\vec{a}_{1,2}$ are unit vectors, we get

$$i\vec{k} \cdot \epsilon \vec{a}_1 E_0 = 0$$

$$i\vec{k} \cdot \vec{a}_2 B_0 = 0$$

$$i\vec{k} \times \vec{a}_1 E_0 = i \frac{\omega}{c} \vec{a}_2 B_0$$

$$i\vec{k} \times \vec{a}_2 B_0 = -i \frac{\omega}{c} \epsilon \vec{a}_1 E_0$$

$\vec{a}_1$ and $\vec{a}_2$ are both $\perp$ to $\vec{k}$, and $\vec{a}_1 \perp \vec{a}_2$,

$\vec{a}_1, \vec{a}_2, \vec{k}$ form right - hand triad of mutually perpendicular vectors

From the second pair of equations we immediately get

$$E_0 = \frac{\omega}{kc} B_0, \quad B_0 = \frac{\omega}{kc} \epsilon E_0 \Rightarrow E_0 = \left(\frac{\omega}{kc} \right)^2 \epsilon E_0 \Rightarrow \epsilon \omega^2 = k^2 c^2$$

- The dispersion relation connecting k and ω is

$$k = c^{-1} \sqrt{\omega^2 - \omega_p^2}, \quad \omega^2 = \omega_p^2 + k^2 c^2$$

- If frequency of the electromagnetic wave is smaller than the plasma frequency, the wave number is imaginary:

$$k = \frac{i}{c} \sqrt{\omega_p^2 - \omega^2}$$

and the amplitude of the wave decreases exponentially on a scale of the order $2\pi c / \omega_p$

- Thus, the plasma frequency defines the plasma cutoff frequency below which there is no electromagnetic propagation. For example, the Earth ionosphere prevents extraterrestrial radiation at frequencies below about 1 MHz ($n \approx 10^4 \text{ cm}^{-3}$).
- The existence of plasma cutoff gives an important method to probe the ionosphere. A pulse in a narrow range of frequencies around ω propagates upwards from the surface. When the layer at which the electron density is large enough to make $\omega_p > \omega$, the pulse will be totally reflected from the layer. The time delay of the pulse provides information on the height of the layer. Making measurements at different frequencies, the electron density can be determined as a function of height.

Pulsar Distance Determination (Dispersion measure)

$$\omega^2 = \omega_p^2 + k^2 c^2 \quad k \text{ -wavenumber}$$

$$\omega_p^2 = 4\pi n_e e^2 / m_e = 3 \cdot 10^9 n_e \text{ (rad/s)}^2, \quad \nu_p = 8.97 n_e^{1/2} \text{ kHz}$$

Arrival time $t_{obs} = \int_0^d \frac{dl}{v_g}$, where v_g - group velocity

In ISM $\omega_p \approx 10^4 \Rightarrow$
 $\omega_p \ll \omega$

$$v_g = \frac{d\omega}{dk} = \frac{kc^2}{\omega} = \frac{c}{\omega} \sqrt{\omega^2 - \omega_p^2} = c \sqrt{1 - \frac{\omega_p^2}{\omega^2}} \approx c \left(1 - \frac{\omega_p^2}{2\omega^2} \right)$$

$$t_{obs} = \frac{d}{c} + \frac{1}{2c\omega^2} \int_0^d dl \omega_p^2 = \frac{d}{c} + \frac{2\pi e^2}{cm_e \omega^2} \int_0^d dl n_e = \frac{d}{c} + \frac{2\pi e^2}{cm_e \omega^2} \text{DM}$$

$$\text{DM} \equiv \int dl n_e, \quad \frac{dt_{obs}}{d\omega} = -\frac{4\pi e^2}{cm_e \omega^3} \langle n_e \rangle d$$

$$d = \frac{cm_e \omega^3}{4\pi e^2} \left| \frac{dt_{obs}}{d\omega} \right| \frac{1}{\langle n_e \rangle}$$

$n_e \sim 0.03 \text{ cm}^{-3}$ in the Galaxy is determined for pulsars with known distances, but it varies along different lines of sight.

