

Accreting millisecond pulsars

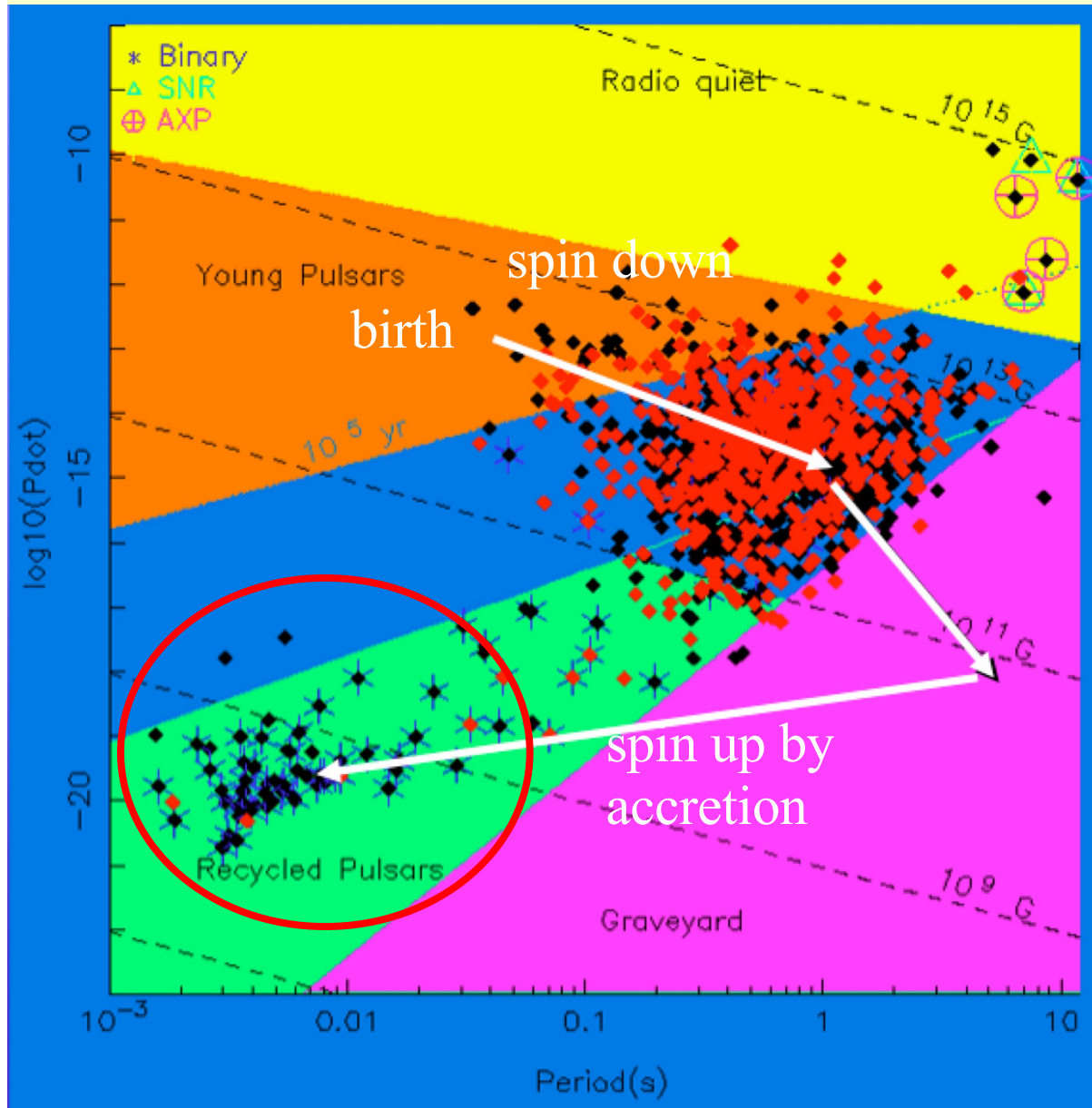
Literature:

1. Poutanen J. 2010,
<http://arxiv.org/abs/0809.2400>
2. Patruno A., Watts A.L., 2012,
<http://arxiv.org/abs/1206.2727>
3. Campana S., Di Salvo T., 2018,
<https://arxiv.org/abs/1804.03422>
4. Poutanen, Beloborodov 2006
<http://arxiv.org/abs/astro-ph/0608663>

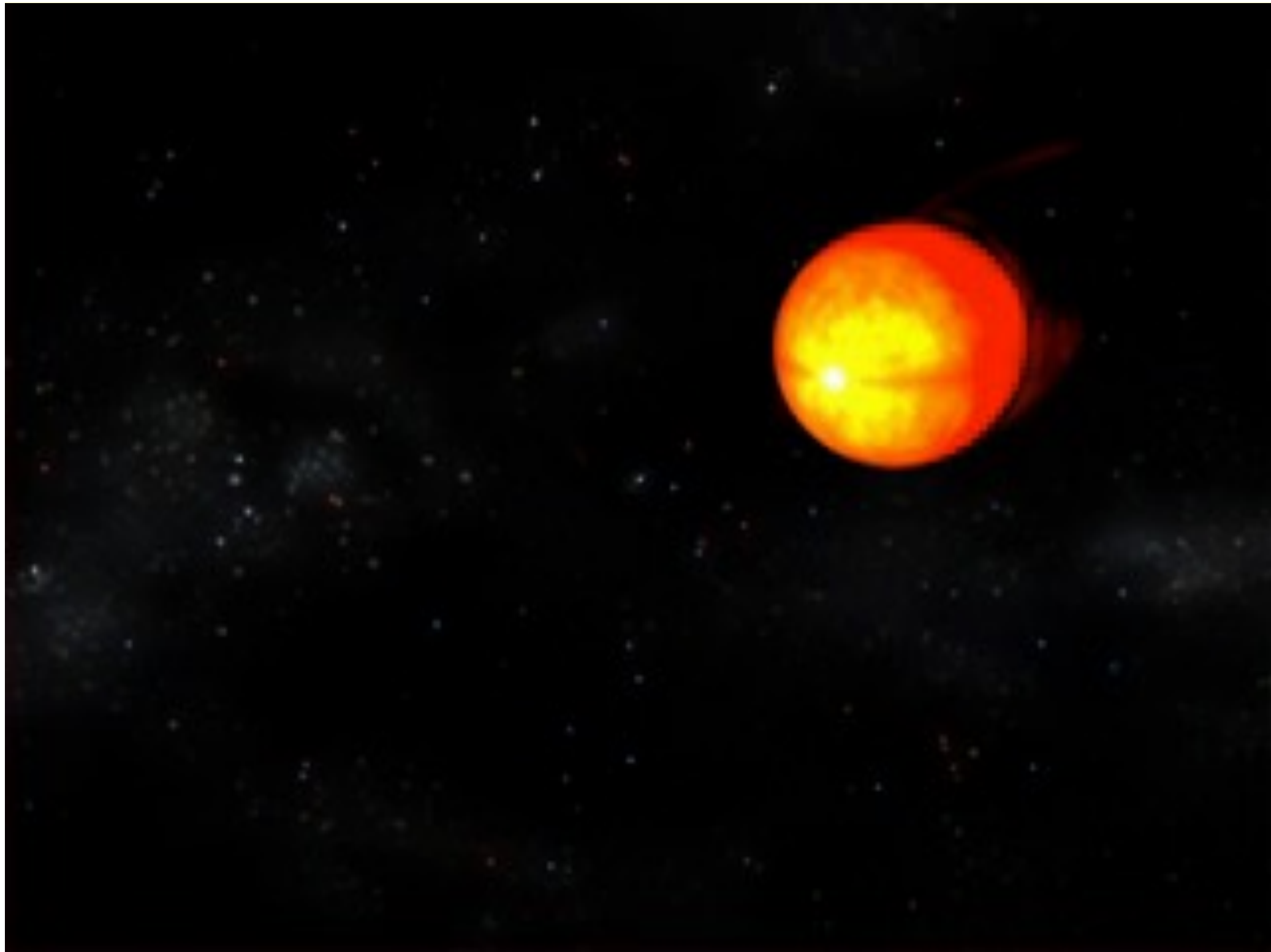


Rossi X-ray Timing Explorer
30 Dec 1995 – 5 Jan 2012

Formation of radio ms pulsars: recycling



- Pulsars are born with short spin periods ***P*** and high ***B***.
- *Spin down* by emission of relativistic wind, waves, photons, magnetic field interaction with the surrounding.
- *Spin-up* by accretion of gas from the companion in low-mass X-ray binaries (LMXRBs).
- LMXRBs are progenitors of ms radio pulsars?
- How to prove that?
- Find X-ray millisecond pulsars!



X-ray millisecond pulsars

Searched for for 15 years without success.

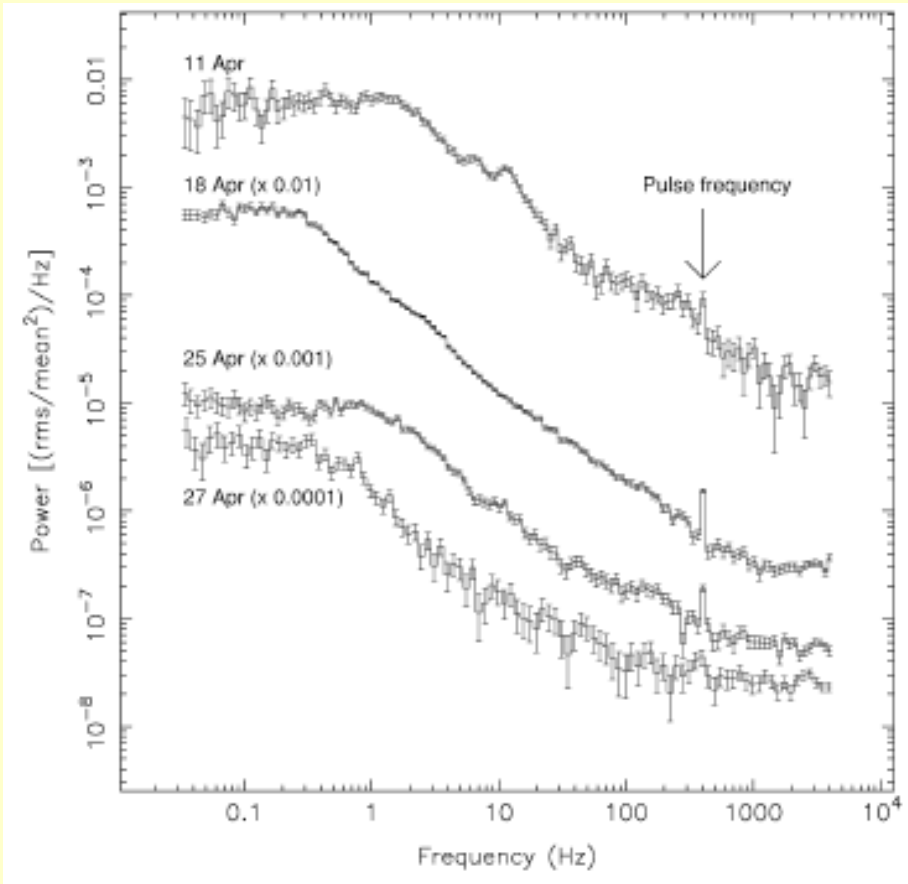
Rossi X-ray Timing Explorer launched in Dec 1995.



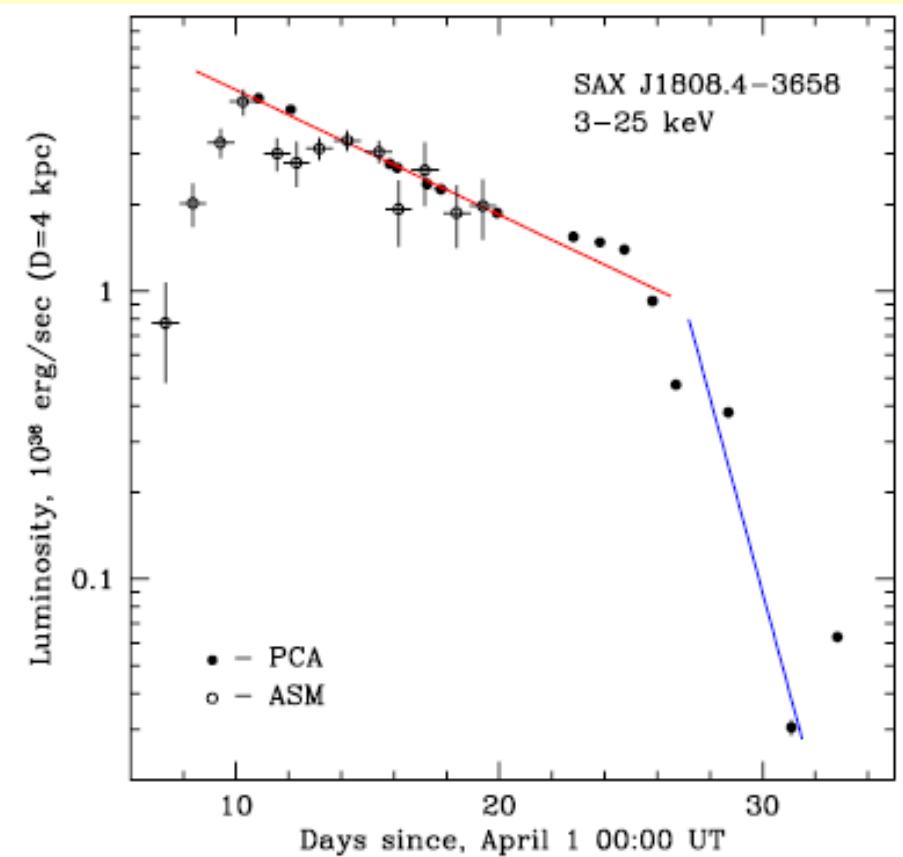
Three scientific instruments,:

1. Proportional Counter Array (PCA), 2-60 keV. Small deadtime, high time resolution.
2. High Energy X-ray Timing Experiment (HEXTE), 20-200 keV
3. All-Sky Monitor (ASM), 1.5-12 keV

SAX J1808.4-3658: the First Accreting Millisecond Pulsar



Power density spectrum (Fourier spectrum of a time-series).
From Wijnands & van der Klis (1998).



April 1998 outburst.
From Gilfanov et al. (1998).

Binary orbit of SAX J1808.4-3658

- Barycentric pulse frequency
 $\nu = 400.9752106$ Hz
- Projected semi-major axis
 $a_x \sin i = 62.809$ ms
- Orbital period $P_{\text{orb}} = 7249.119$ s
- Eccentricity $e < 5 \cdot 10^{-4}$
- Pulsar mass function
 $f_x = 3.7789 \cdot 10^{-5}$

$$f_x = \frac{M_c^3 \sin^3 i}{(M_c + M_x)^2} = \frac{4\pi^2 (a_x \sin i)^3}{GP_{\text{orb}}^2}$$

- For a given neutron star M_x , we can constrain the mass of companion M_c (as a function of inclination i)

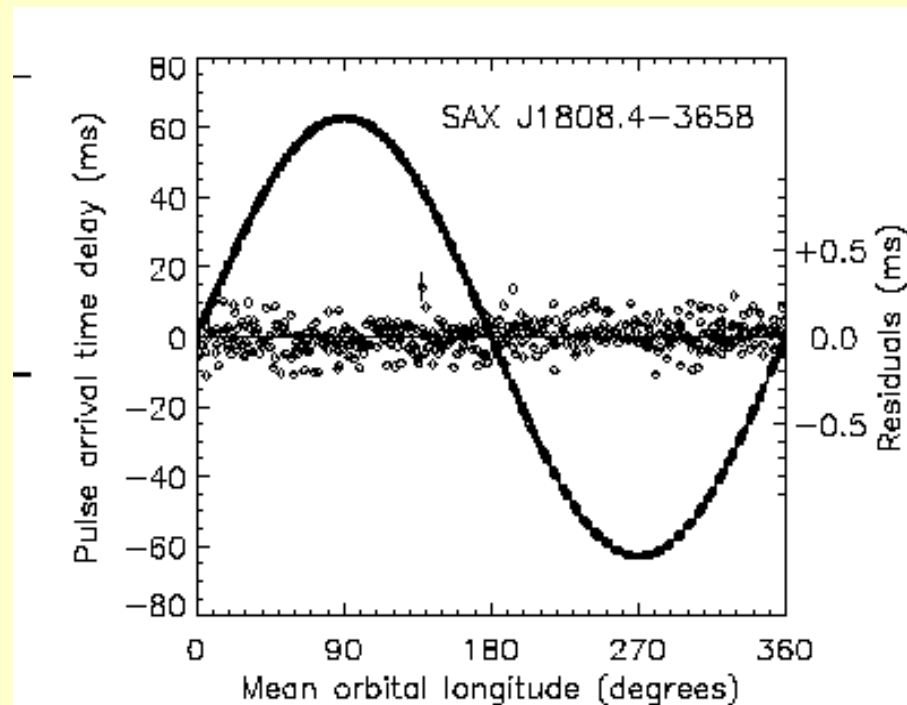
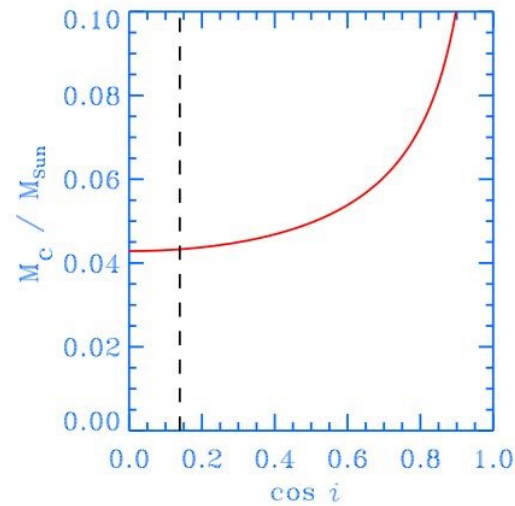
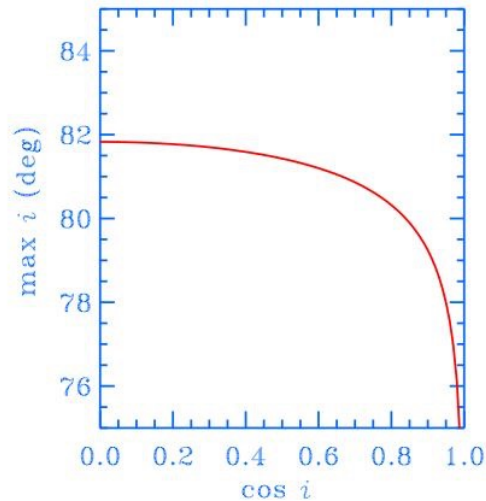
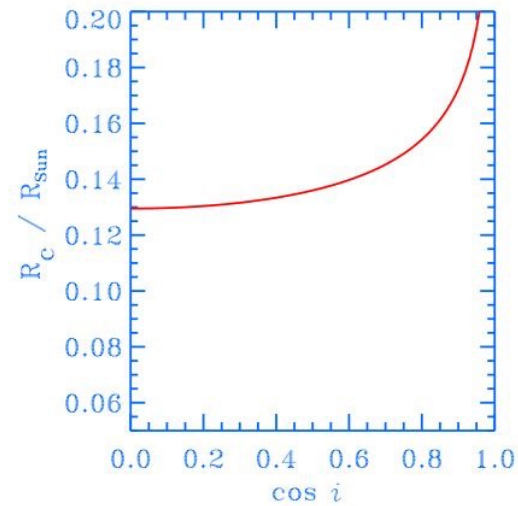


Figure 1. Pulse arrival time delays due to the 2 hr binary orbit, with respect to a constant pulse frequency model. Mean longitude is the orbital phase angle measured from the ascending node, so that the pulsar is behind the companion at 90° . The squares are the measured time delays, the solid curve is the best-fit orbital model, and the diamonds are the model residuals (on a $50\times$ expanded scale). The residuals have an r.m.s. uncertainty of $75 \mu\text{s}$, shown for one of the points.

Companion masses and radii



SAX J1808.4-3658



- For a given inclination $\Rightarrow$ companion mass $\Rightarrow$ size of the companion (i.e. its Roche lobe, Paczynski 1971, Faulkner et al. 1972; assuming NS mass)

$$R_c = 0.462 \left(\frac{GM_c P_{\text{orb}}^2}{4\pi^2} \right)^{1/3} = 0.462 a \left(\frac{M_c}{M_c + M_x} \right)^{1/3}$$

- $\Rightarrow$ maximum inclination angle from the absence of eclipses

Nature of the companions

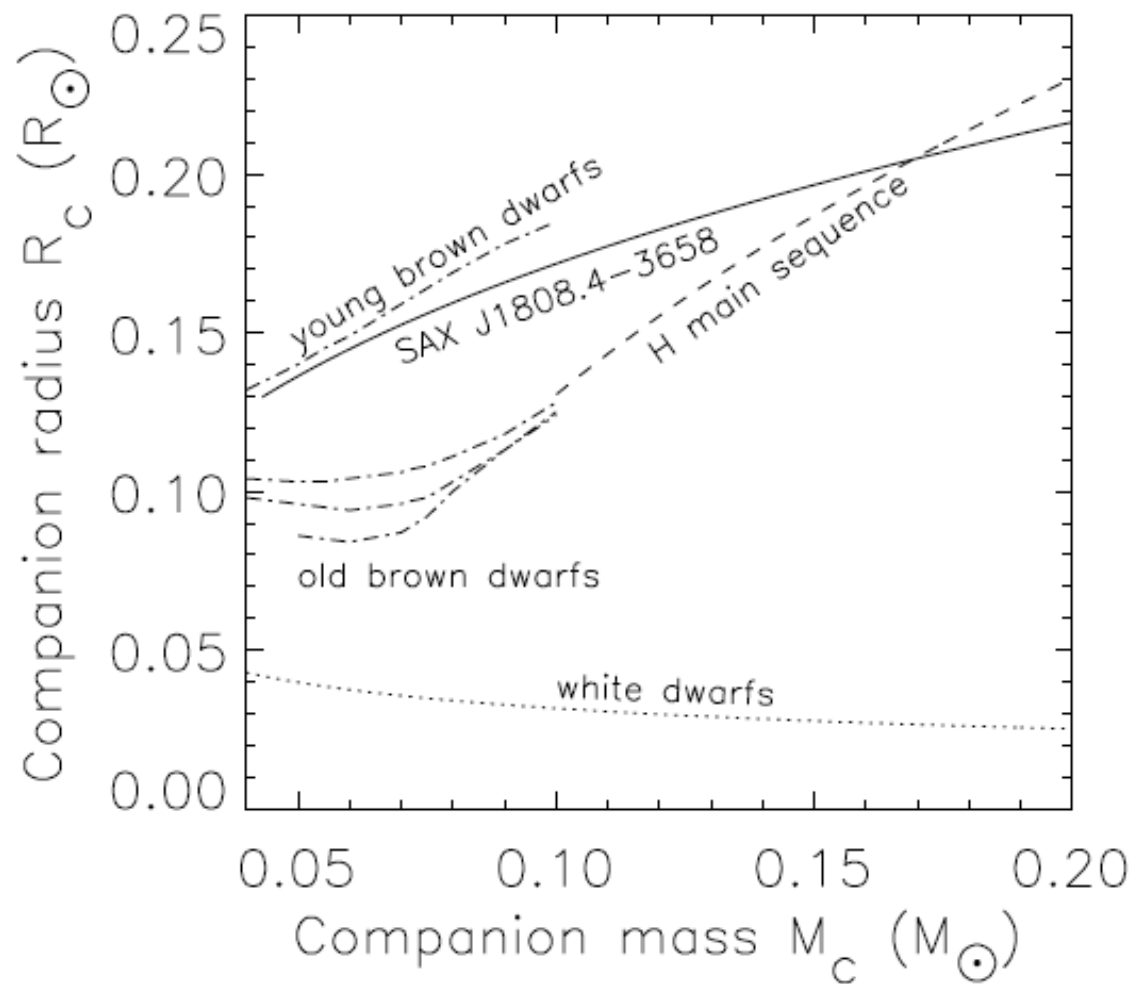


FIG. 2.—Possible companion types for SAX J1808.4–3658. The dark solid curve shows the mass-radius relation for SAX J1808.4–3658 allowed by the observations. The other curves show the theoretical mass-radius relations for cold He white dwarfs (*dotted curve*), low-mass hydrogen mass sequence stars (*dashed curve*), and brown dwarfs of ages (*dot-dashed curves*) of 0.1, 0.5, 1.0, and 5.0 Gyr, from top to bottom.

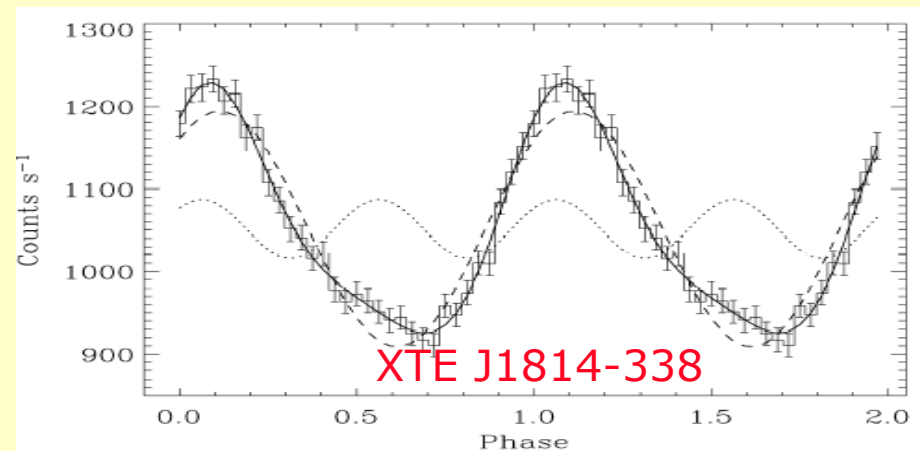
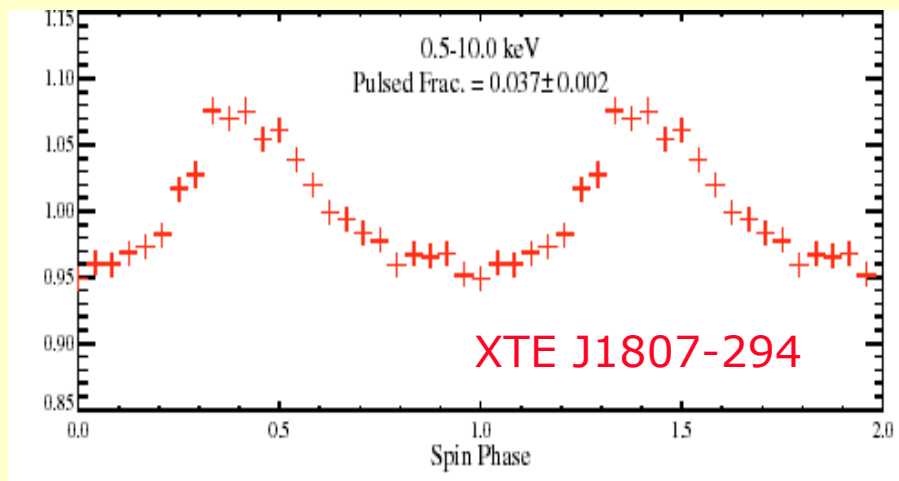
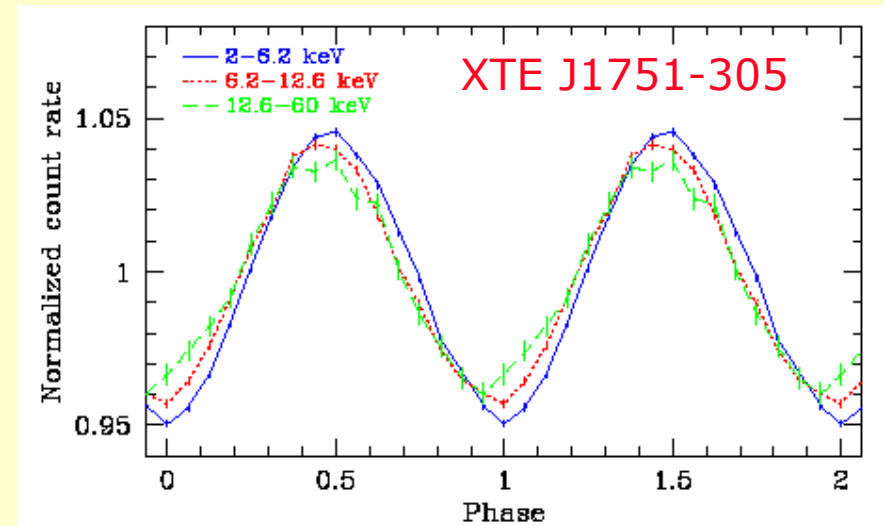
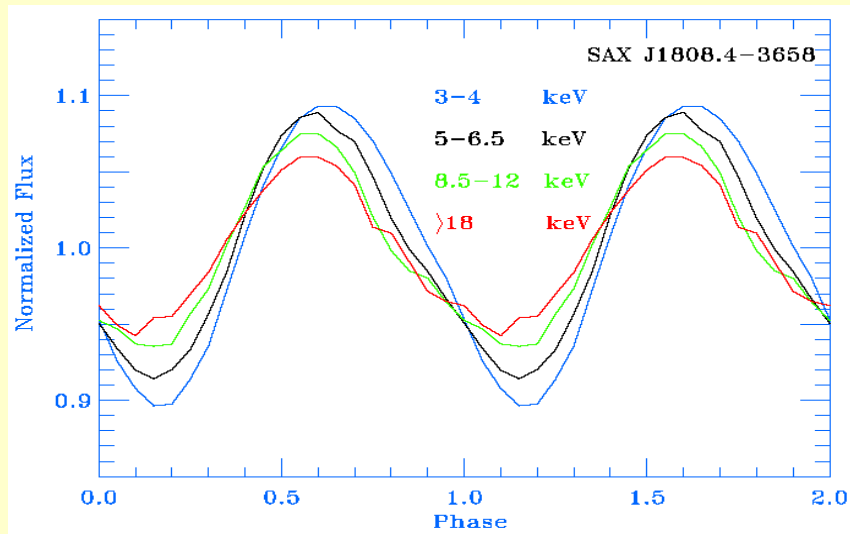
- In SAX J1808.4–3658 the companion is too large to be a white dwarf and even too large for the main sequence stars.
- Since the companion is being heated by the pulsar, it is larger than it would be otherwise.

Accreting millisecond pulsars

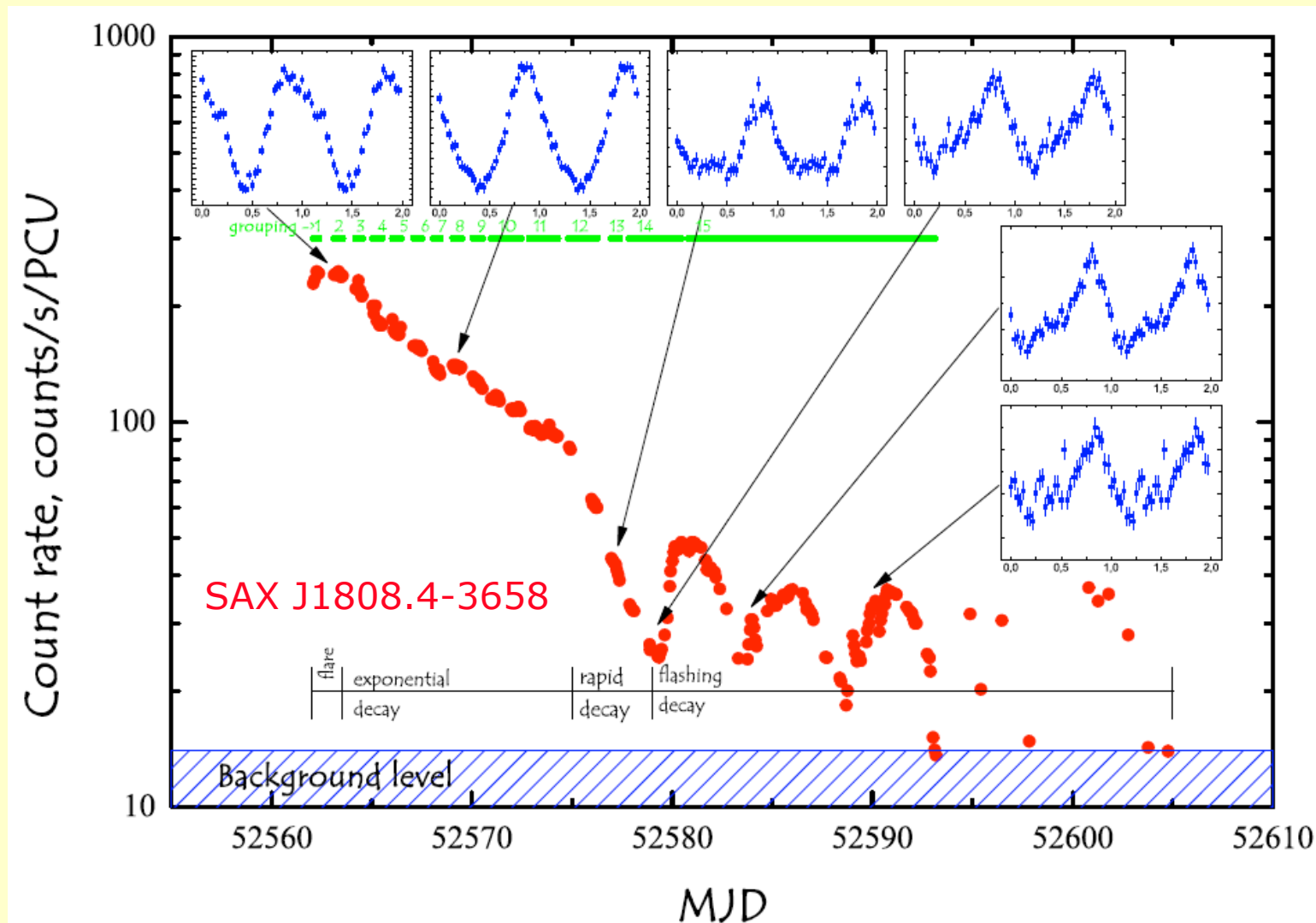
Source	P_{orb} (min)	Spin (Hz)	$M_{\text{c,min}}/M_{\odot}$	When?
1. SAX J1808.4–3658	120	401	0.043	April 1998
2. XTE J1751–305	42	435	0.014	April 2002
3. XTE J0929–314	44	185	0.083	April 2002
4. XTE J1807–294	40	190	0.0066	Feb 2003
5. XTE J1814–338	258	314	0.17	June 2003
6. IGR J00291+5934	148	599	0.039	Dec 2004
7. HETE J1900.1–2455	84	377	0.016	June 2007
8. SWIFT J1756.9–2508	54.7	182	0.0067	June 2007
9. Aql X-1	1140	551	?	August 2007
10. SAX J1748.9–2021	526	442	0.1	August 2007
11. in glob.clust. NGC6440	57.3	206	0.0067	August 2009
12. IGR J17511–305	208	245	0.13	Sept 2009
13. Swift J1749.4–2807	529	518	0.6	April 2010
14. IGR J17498–2921	231	401	0.17	August 2011
15. IGR J18245–2452	662	254	0.17	March 2013
16. MAXI J0911–655	44.3	340	0.024	April 2016
17. IGR J17062–6143	38	164	0.005	Feb 2017
18. IGR J16597–3704	46	105	0.006	Oct 2017
19. IGR J17379–3747	113	468	0.056	April 2018
20. IGR J17591–2342	528	527	0.42	August 2018
21. IGR J17494–3030	75	376	0.02	Oct 2020
22. MAXI J1816–195	290	529	0.1	Apr 2022
23. MAXI J1957+032	61	314	0.017	June 2022
24. SRGA J144459.2–604207	313	448	0.1	Feb 2024

Pulse profiles

Profiles are usually very simple, close to sinusoidal, but...



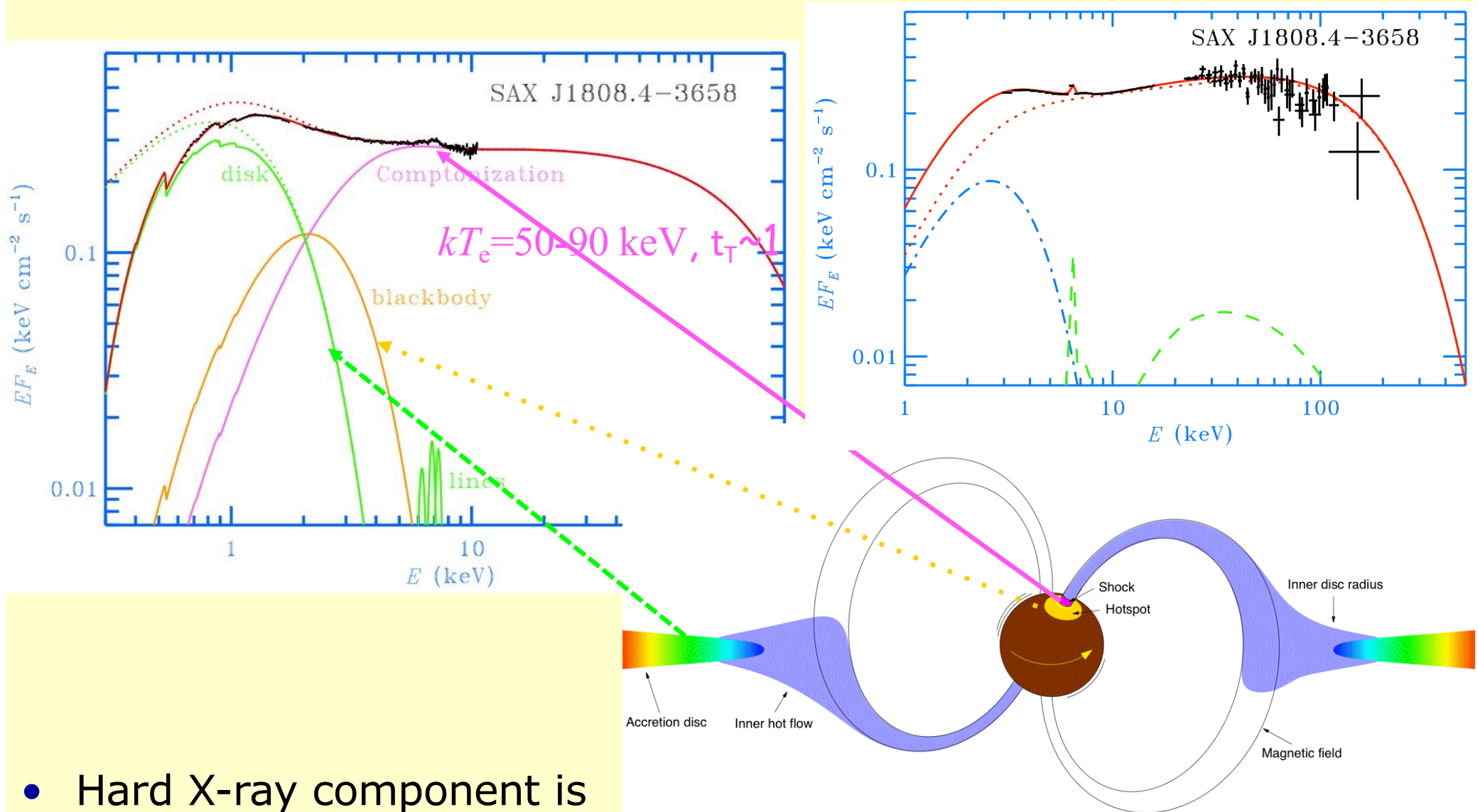
...sometimes vary during the outburst...



- Secondary spot is visible?

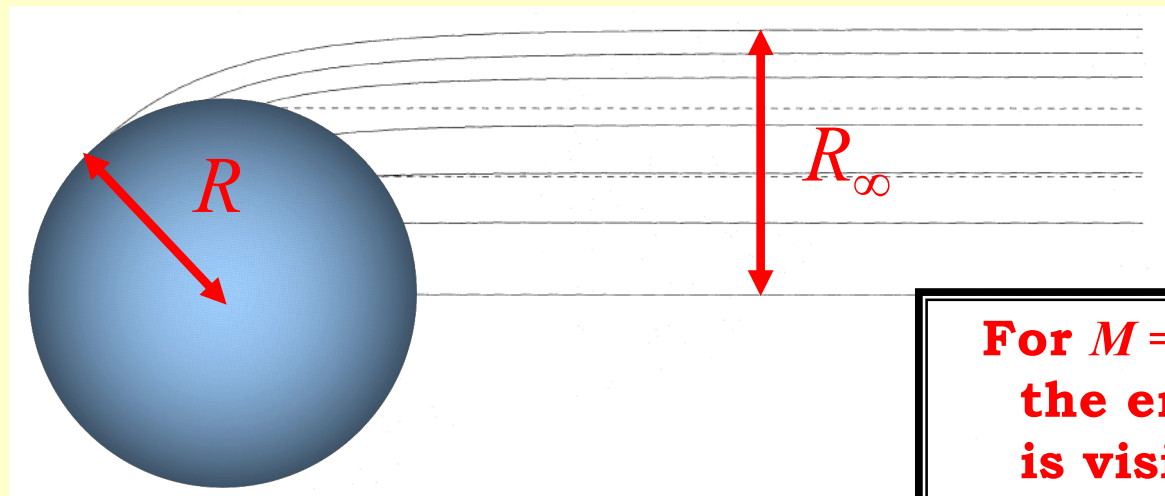
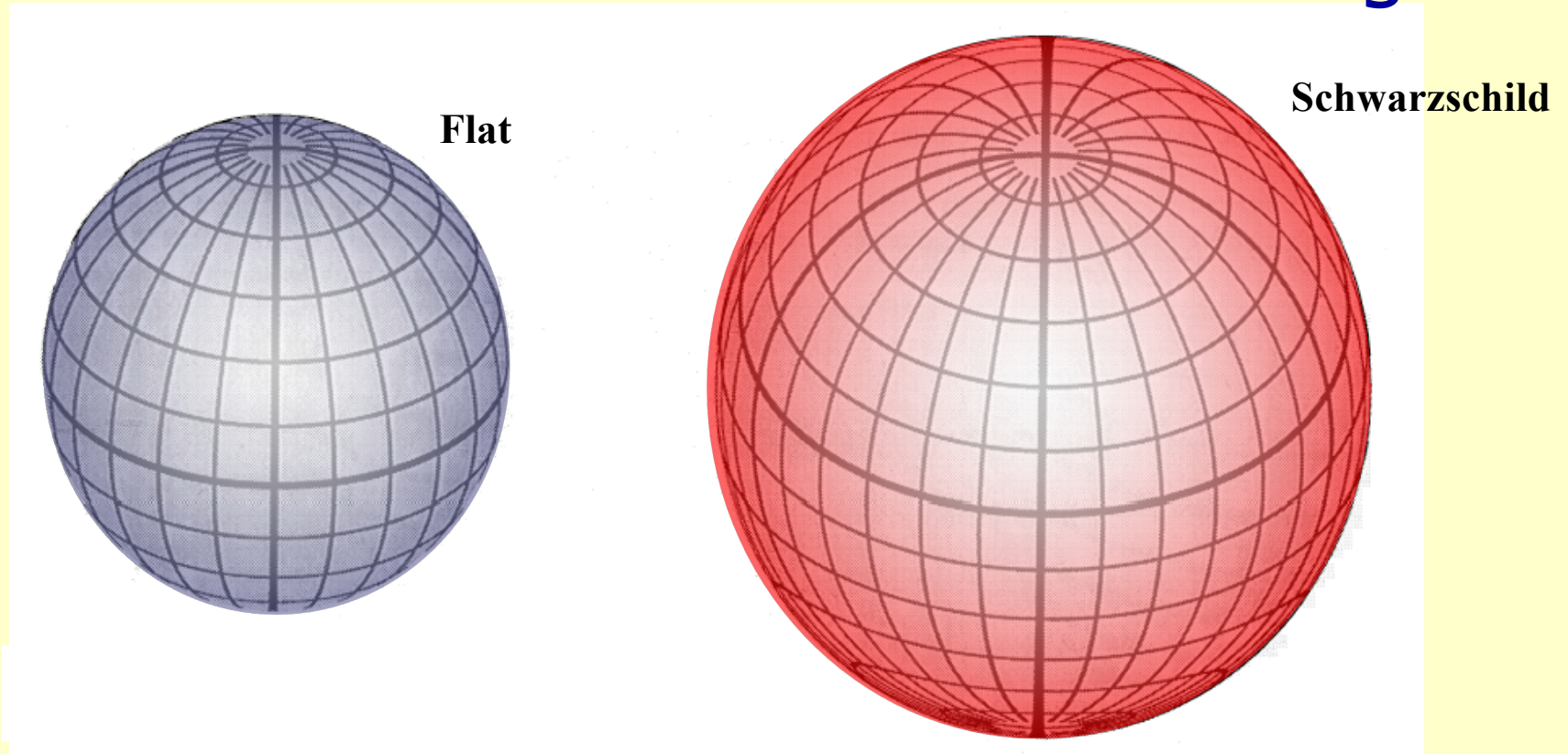
Ibragimov & Poutanen (2009)
Hartman et al. (2008)

Spectral energy distribution



- Hard X-ray component is pulsating and is therefore produced at the shock.
- The blackbody is also pulsating → neutron star surface
- Low-energy 'blackbody' is not pulsating → accretion disk

Gravitational redshift and bending



Nollert et al. (1989)

For $M = 1.4 M_\odot$, $R = 10$ km ~80% of the entire neutron star surface is visible at a given instant.

Gravitational redshift and bending

- The photon energy is redshifted by a factor $1+z$
- The temperature of blackbody radiation then also changes by the same factor
- The time (inverse of frequency) is dilated by factor $1+z$
- The observed luminosity is smaller than luminosity at the stellar surface by $(1+z)^2$ as both the rate of photon arrival and photon energy decrease by $1+z$.

$$1+z = (1 - R_S / R_*)^{-1/2}$$

$$\nu = \nu_* (1+z)^{-1}$$

$$T_\infty = T_* (1+z)^{-1}$$

$$dt = dt_* (1+z)$$

$$L_\infty = L_* (1+z)^{-2}$$

$$L_* = 4\pi R_*^2 \sigma_{SB} T_*^4$$

$$L_\infty = 4\pi D^2 F = 4\pi D^2 \sigma_{SB} T_\infty^4 \frac{R_\infty^2}{D^2}$$

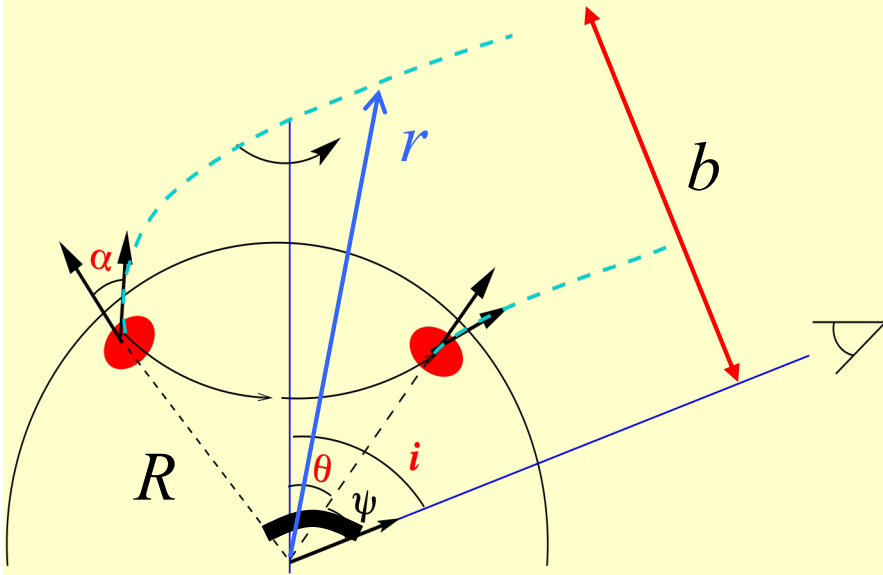
$$= 4\pi R_\infty^2 \sigma_{SB} T_\infty^4$$

- The apparent radius at “infinity” thus can be related to the real stellar radii as

$$R_\infty = R_* (1+z) = R_* (1 - R_S / R_*)^{-1/2}$$

$$R_S = 2GM/c^2$$

Light trajectory



- Equation of photon motion derived in GR:

$$\left(\frac{1}{r^2} \frac{dr}{d\psi} \right)^2 + \frac{1}{r^2} \left(1 - \frac{R_s}{r} \right) = \frac{1}{b^2}$$

where b is impact parameter.

- Light bending

$$\psi(R, \alpha) = \int_R^\infty \frac{dr}{r^2} \left[\frac{1}{b^2} - \frac{1}{r^2} \left(1 - \frac{R_s}{r} \right) \right]^{-1/2}$$

- Impact parameter is related to the angle between the photon momentum and local radial direction as

$$b = R \sin \alpha / \sqrt{1 - \frac{R_s}{R}}$$

Light curves from a "spotted" neutron star

- The observed flux from a small spot of area dS : $dF = I d\Omega$

where $d\Omega$ is the solid angle covered by the spot on the sky and I is the intensity of radiation reaching the observer.

- Observed intensity of radiation can be related to the emitted one

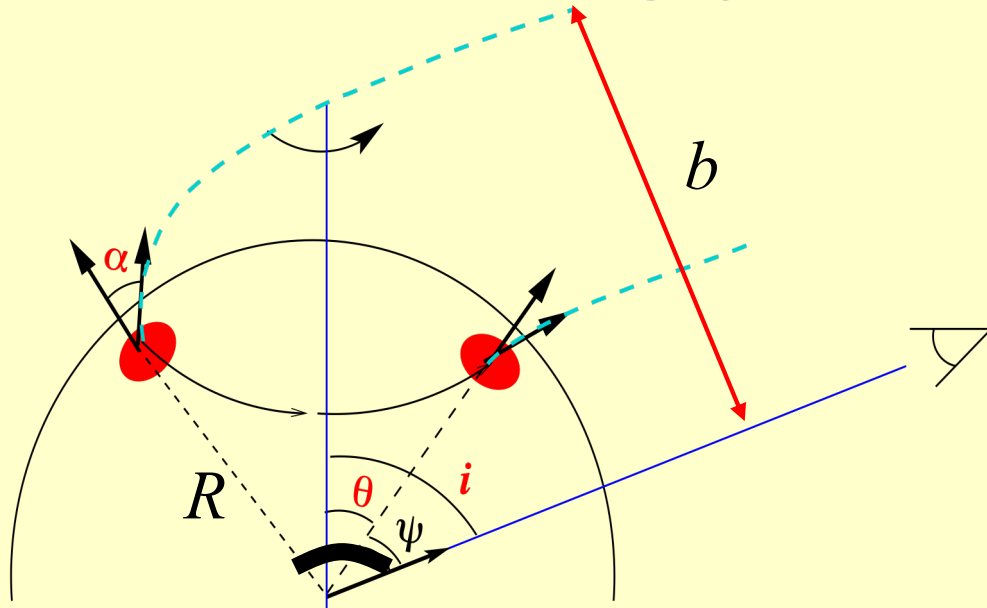
Specific intensity

Bolometric intensity

$$I_\nu / \nu^3 = I_{0,\nu} / \nu_0^3 \Rightarrow I = I_0 (\nu / \nu_0)^4$$

$$\nu = \nu_0 \sqrt{1 - R_s / R} \Rightarrow I = I_0 (1 - R_s / R)^2$$

Observed flux



- Solid angle occupied by spot on the sky

$$d\Omega = b db d\phi / D^2$$

- Surface element

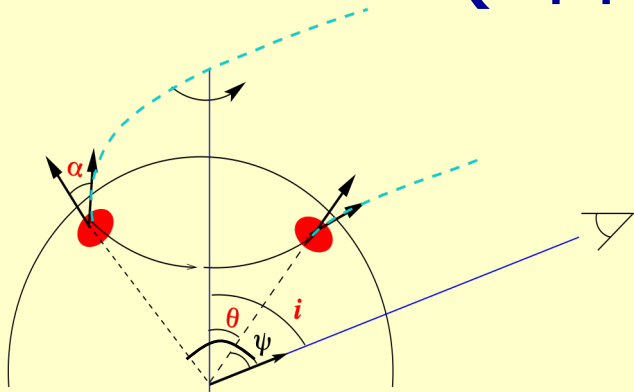
$$dS = R^2 d\mu d\phi, \mu = \cos \psi$$

- We get the observed flux

$$dF = \frac{I b}{R^2} \left| \frac{db}{d\mu} \right| \frac{dS}{D^2} = \left(1 - \frac{R_s}{R} \right) I_0(\alpha) \cos \alpha \frac{d \cos \alpha}{d\mu} \frac{dS}{D^2}$$

α is the angle between the light ray reaching the observer and the normal to the surface

Gravitational Light Bending (approximate treatment)



Approximate light bending formula exists (Beloborodov 2002)

$$\cos \alpha \approx \frac{R_s}{R} + \left(1 - \frac{R_s}{R}\right) \cos \psi$$

where $\cos \psi = \cos i \cos \theta + \sin i \sin \theta \cos \omega t$

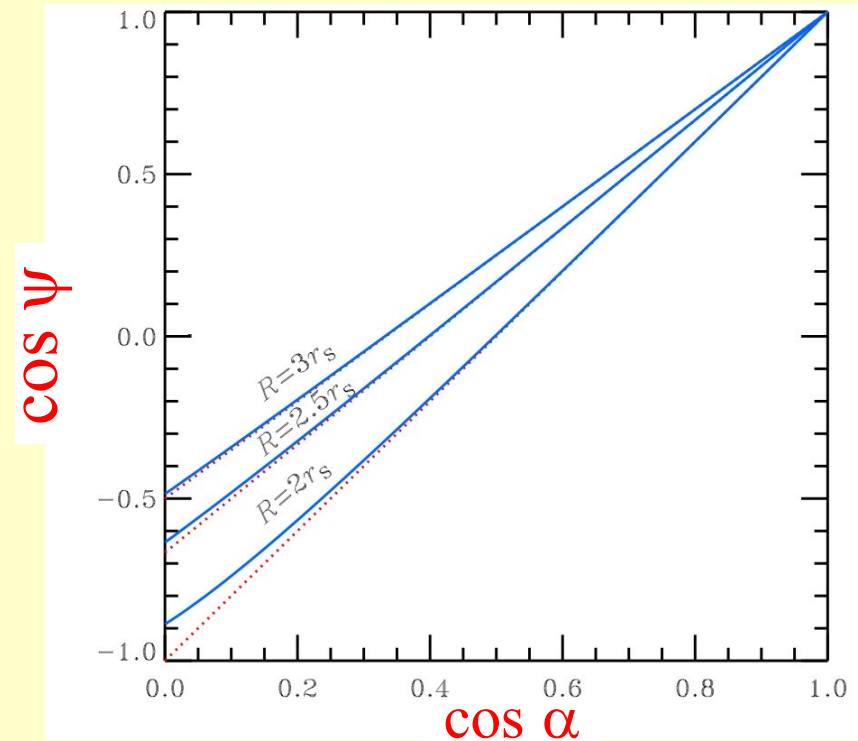
varies with pulsar phase $\phi = \omega t$

The observed flux

$$dF \approx \left(1 - \frac{R_s}{R}\right)^2 I_0(\alpha) \cos \alpha \frac{dS}{D^2}$$

varies almost as a sine-wave of the pulsar phase ϕ .

Visibility condition $\cos \alpha > 0$.



- Assume isotropic surface emission (e.g. black body) with bolometric intensity $I(\alpha)=I_0$.
- Assume two identical antipodal spots.
- For the antipodal secondary spot

$$\cos \psi_s = -\cos \psi_p.$$

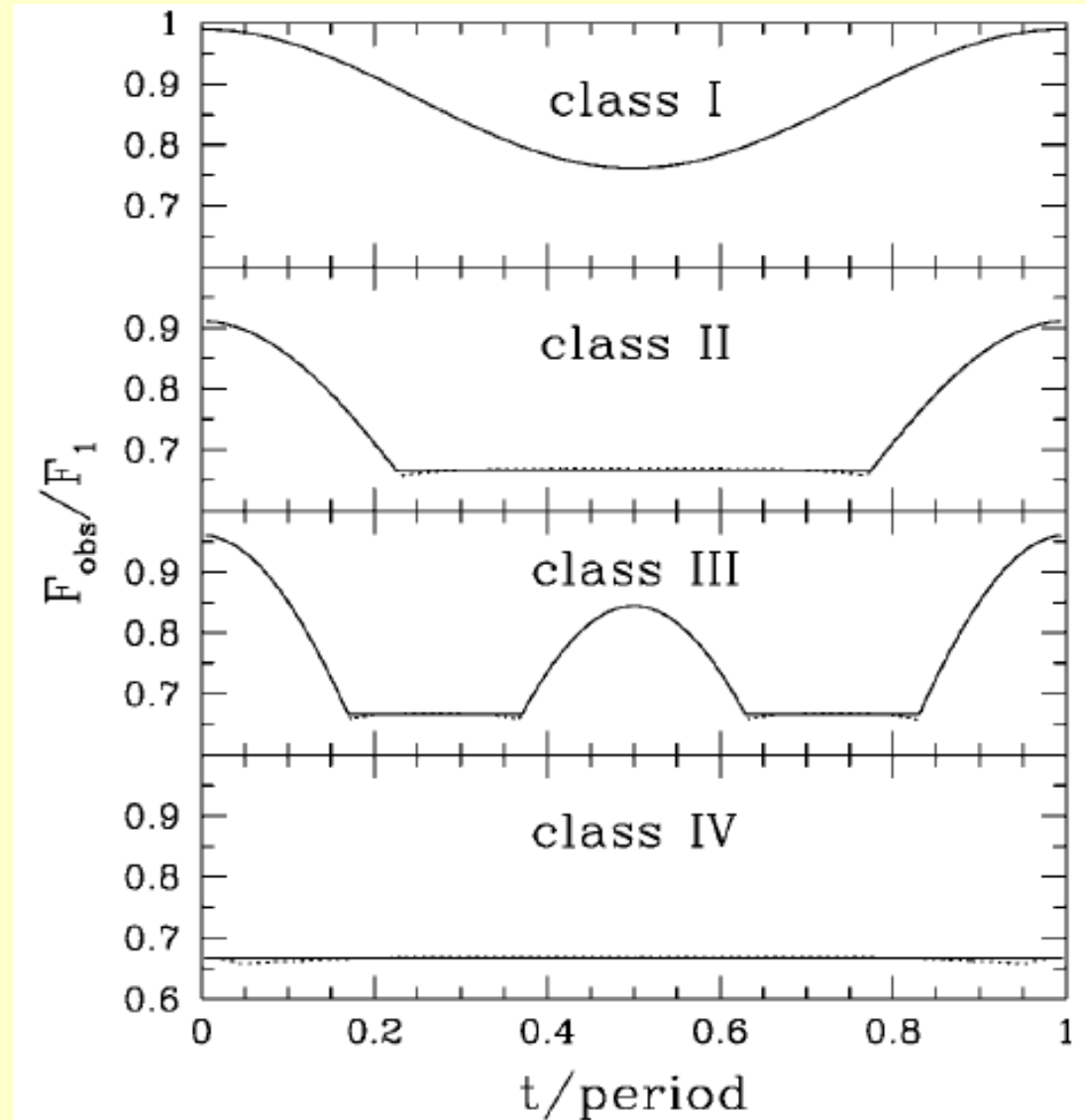
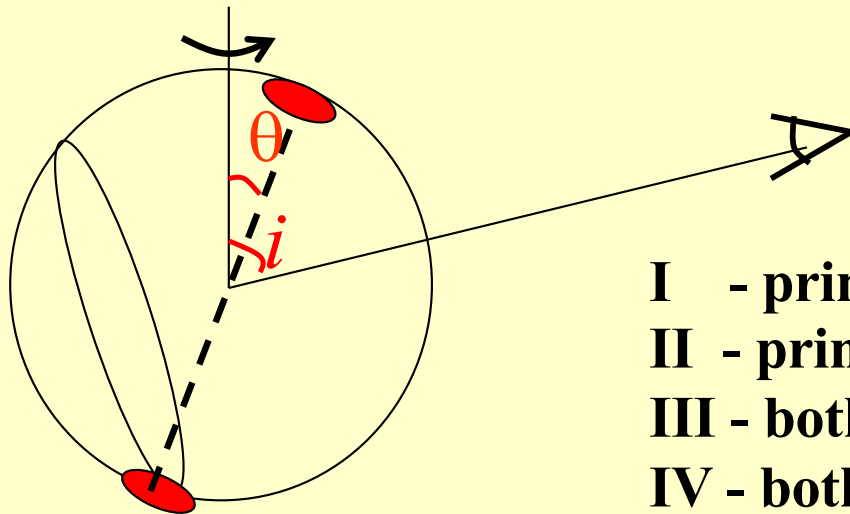
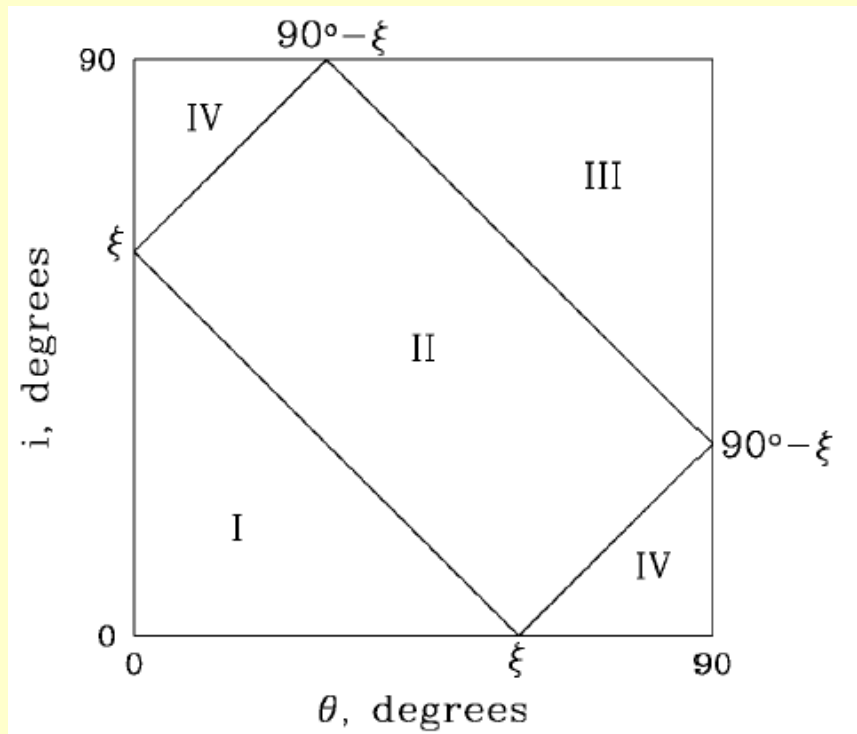
- Primary spot, visibility condition $\cos \alpha > 0$, i.e.
 $\cos \psi_p > -\kappa = -R_s/(R - R_s)$.

- Flux from primary $F_p \propto I_0 \cos \alpha_p = I_0 \left(\frac{R_s}{R} + \left[1 - \frac{R_s}{R} \right] \cos \psi_p \right)$

- Secondary spot, visibility condition
 $\cos \alpha > 0$, i.e. $\cos \psi_p < \kappa$.

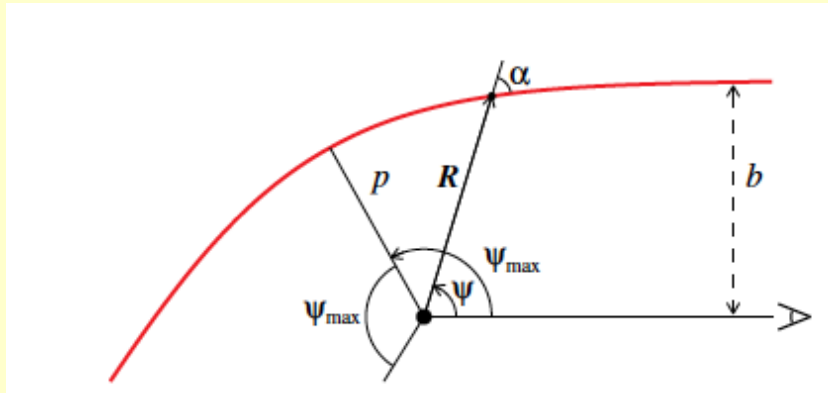
- Flux from secondary $F_s \propto I_0 \cos \alpha_s = I_0 \left(\frac{R_s}{R} - \left[1 - \frac{R_s}{R} \right] \cos \psi_p \right)$

Classes of pulsars



- I - primary visible, secondary 2-never visible;**
- II - primary visible, secondary – sometimes visible;**
- III - both sometimes invisible;**
- IV - both are always visible.**

Approximate light bending formula



$$\psi_P(R, \alpha) = \int_R^\infty \frac{dr}{r^2} \left[\frac{1}{b^2} - \frac{1}{r^2} \left(1 - \frac{R_S}{r} \right) \right]^{-1/2}$$

$$b = \frac{R}{\sqrt{1-u}} \sin \alpha$$

Impact parameter

Beloborodov 2002

$$u = R_S/R$$

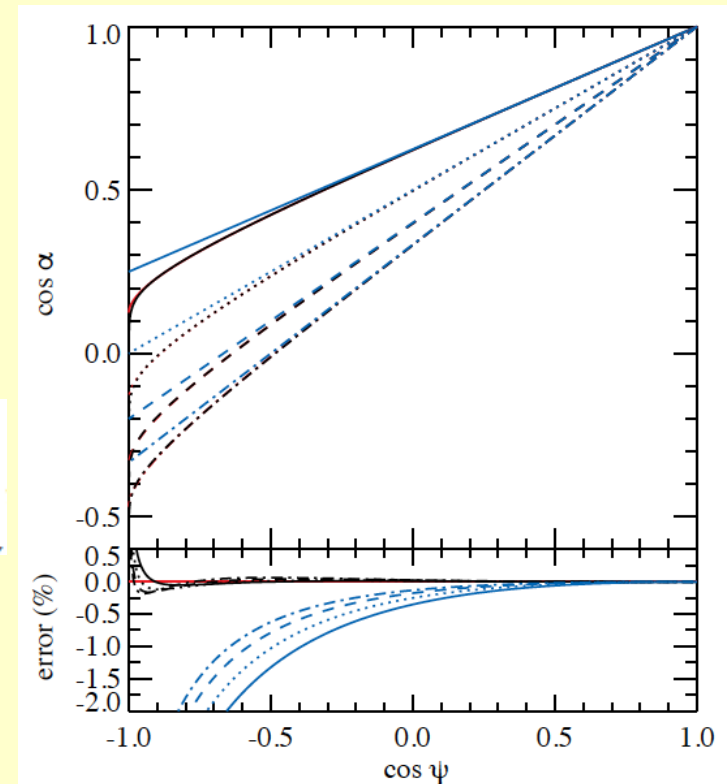
$$1 - \cos \alpha = (1 - u)(1 - \cos \psi) = (1 - u)y.$$

Poutanen 2020

$$1 - \cos \alpha = (1 - u)y \left\{ 1 + \frac{u^2 y^2}{112} - \frac{e}{100} u y \left[\ln \left(1 - \frac{y}{2} \right) + \frac{y}{2} \right] \right\}$$

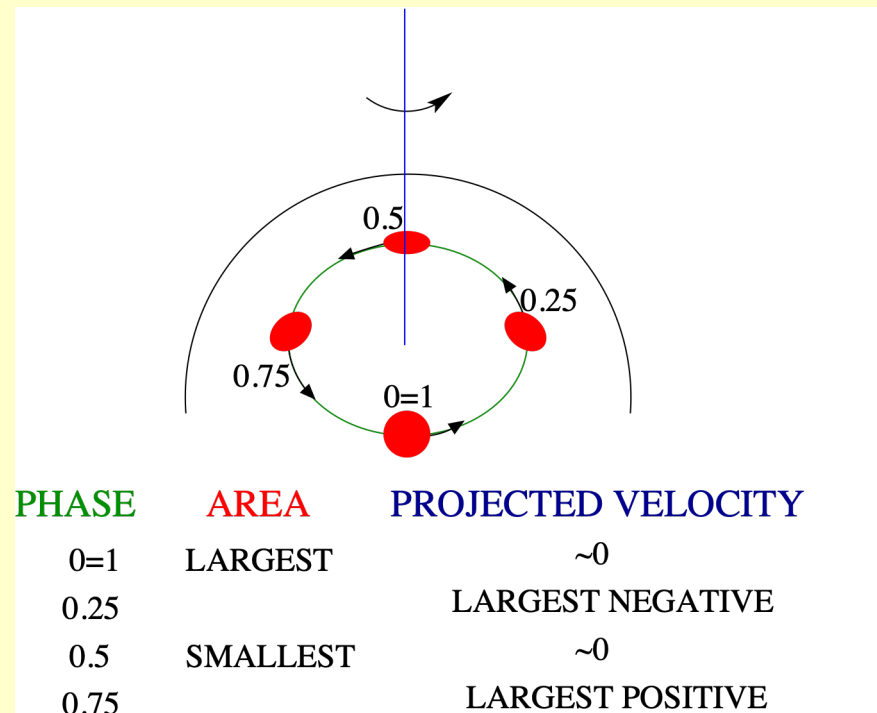
error < 0.06% for $\cos \psi > -0.5$

error < 0.2% for $\cos \psi > -0.95$

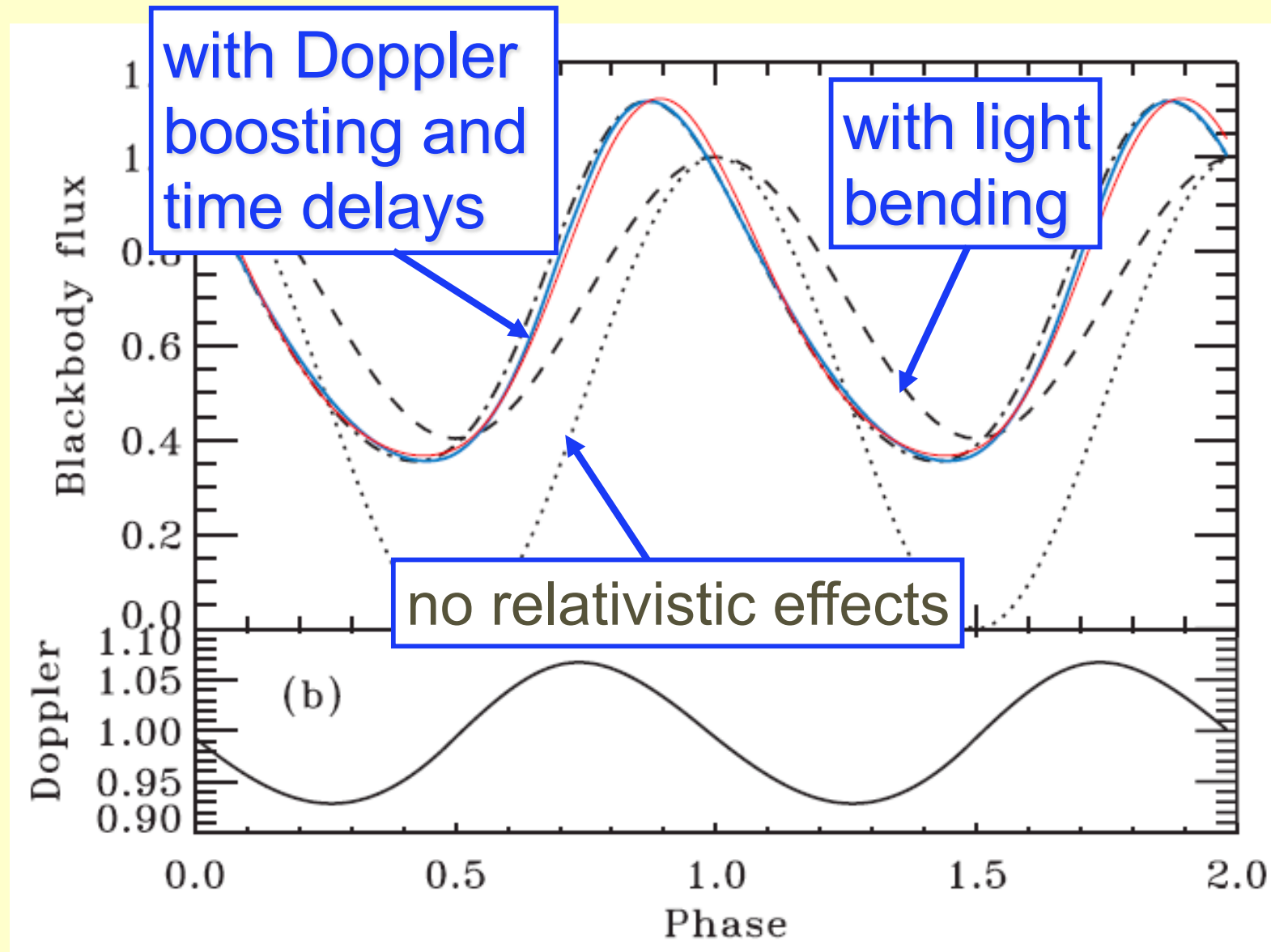


Effects of rapid rotation: Doppler effect and aberration

- In ms pulsars, rotational velocities are high ($v/c \sim 0.1$), the Doppler effect plays an important role.
- Doppler boosting increases the observed flux when the source moves towards the observer and decreases it when the source moves away.
- Relativistic aberration of light changes the angle we see the spot.



Effects of rapid rotation

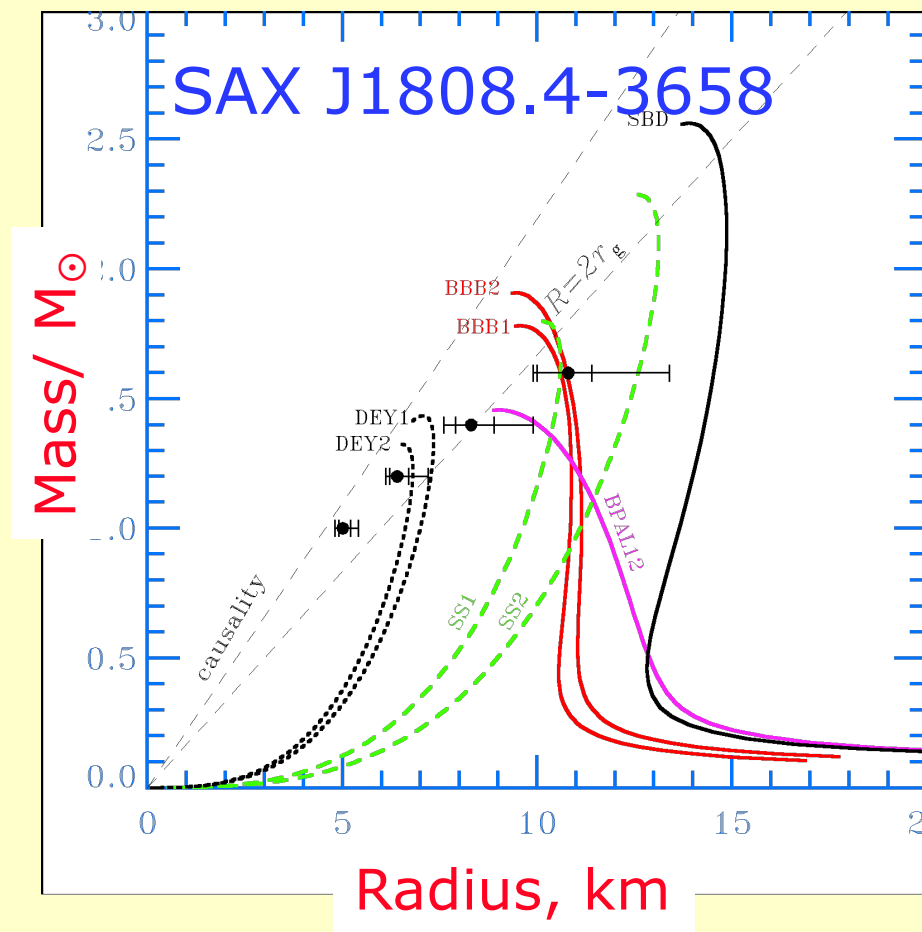


Light curves reflect M and R

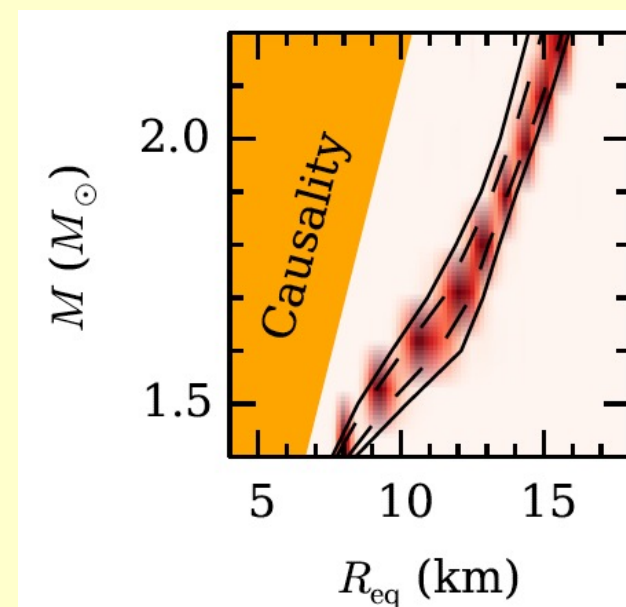
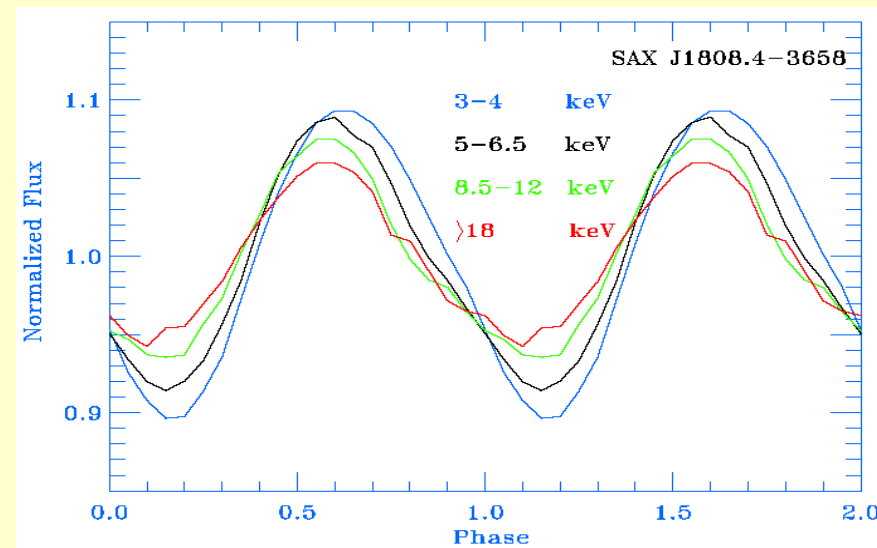
What determines pulse profiles?

- Rotation frequency (ν)
- Inclination (i), magnetic co-latitude (θ)
- Emissivity pattern (radiation processes, anisotropy: black body vs. Comptonized)
- Light bending (M/R)
- Doppler boosting and relativistic aberration (ν, θ, i, R)
- Time delay (ν, θ, i, R)
- Spots' size and shape (θ, i, ρ)
- Stellar oblateness (ν, M, R)
- Eclipse by accretion disk and stream ($\theta, i, B, \dot{M}$)

Determining M , R from pulse profiles



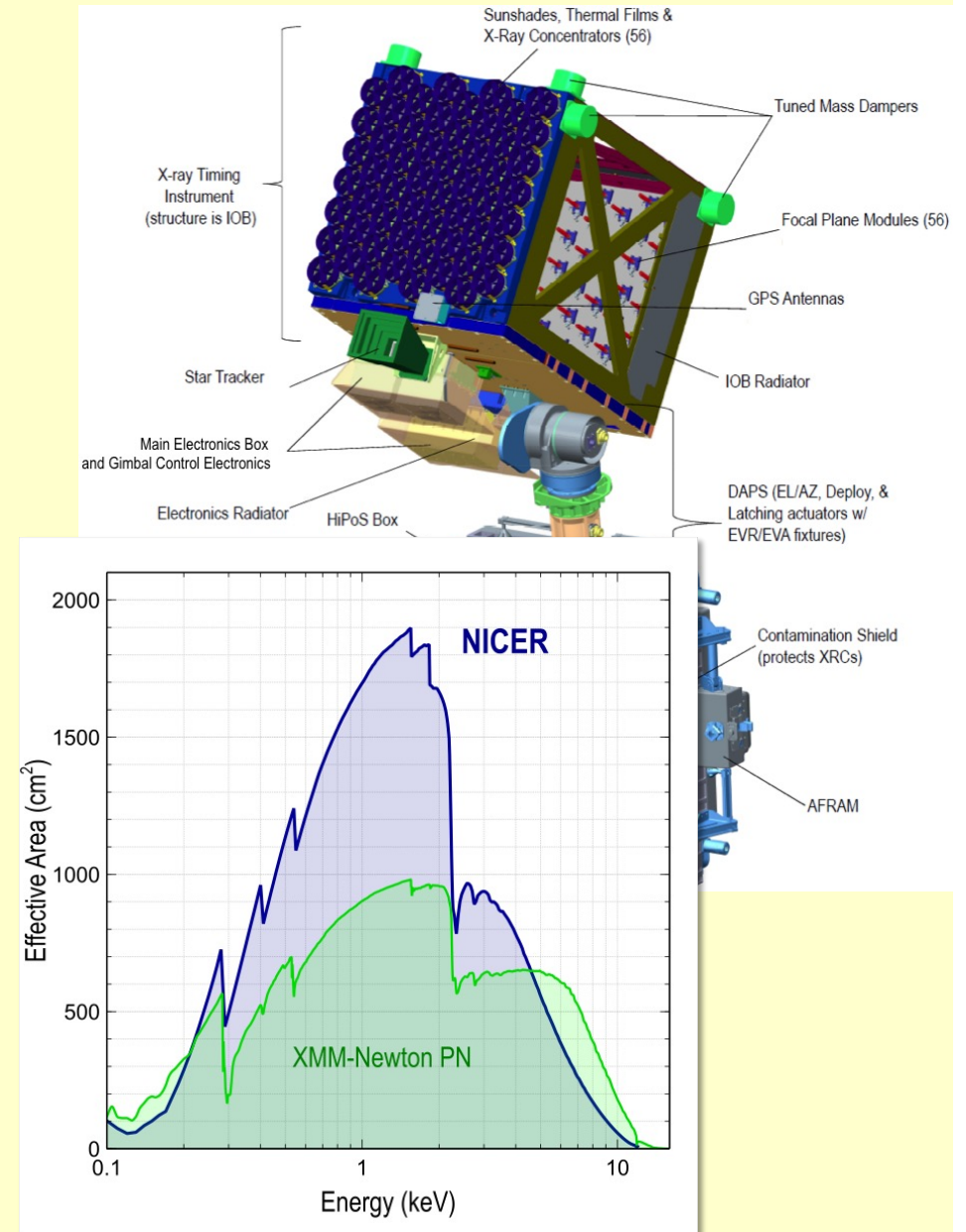
Poutanen & Gierlinski 2003



Salmi et al. 2018

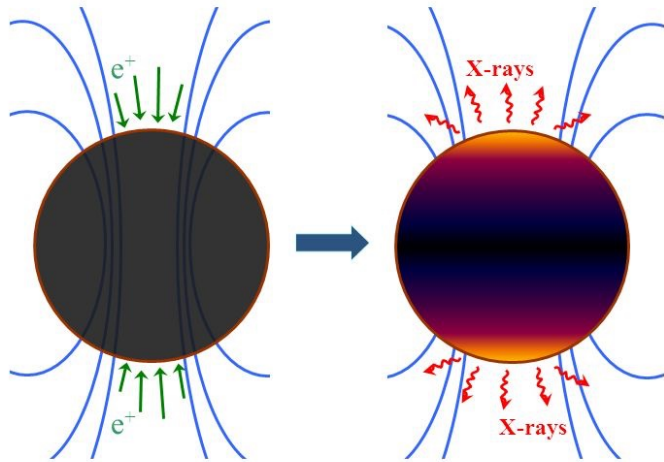
Neutron Star Interior Composition Explorer (NICER)

- Launched on 3 June 2017
- Stopped operations in 2025
- Low Earth orbit (ISS), inclination 52 deg
- Energy range 0.2-12 keV
- Energy resolution: 85 eV@ 1 keV, 137 eV @ 6 keV
- Spatial resolution 5'
- Enhancement: SEXTANT, demonstrator for X-ray navigation

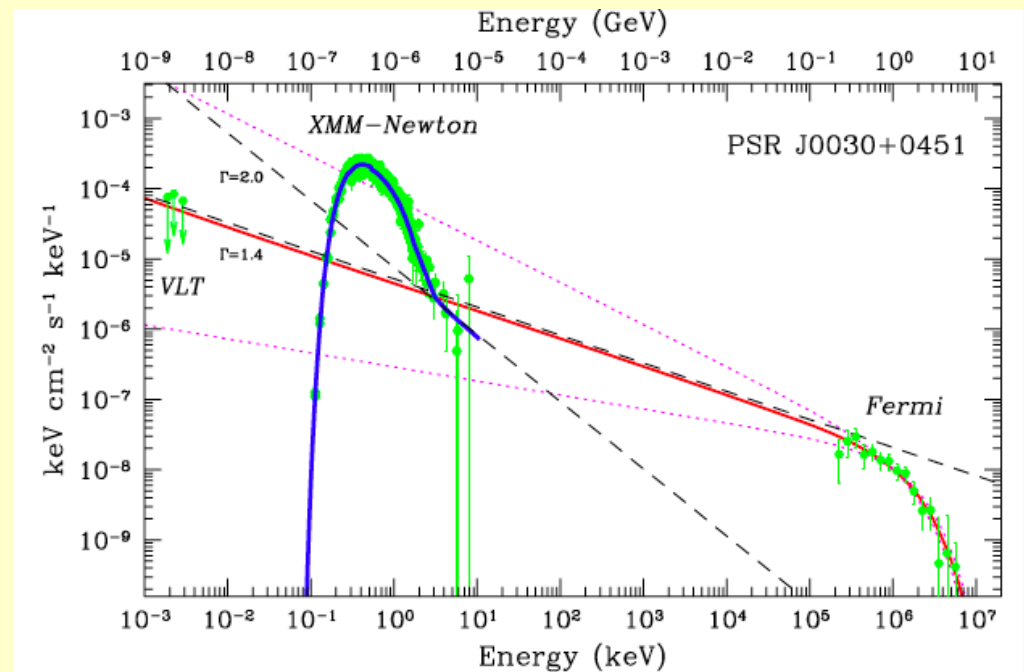


Determining M , R of neutron stars using thermal X-ray emission from rotation-powered millisecond pulsars

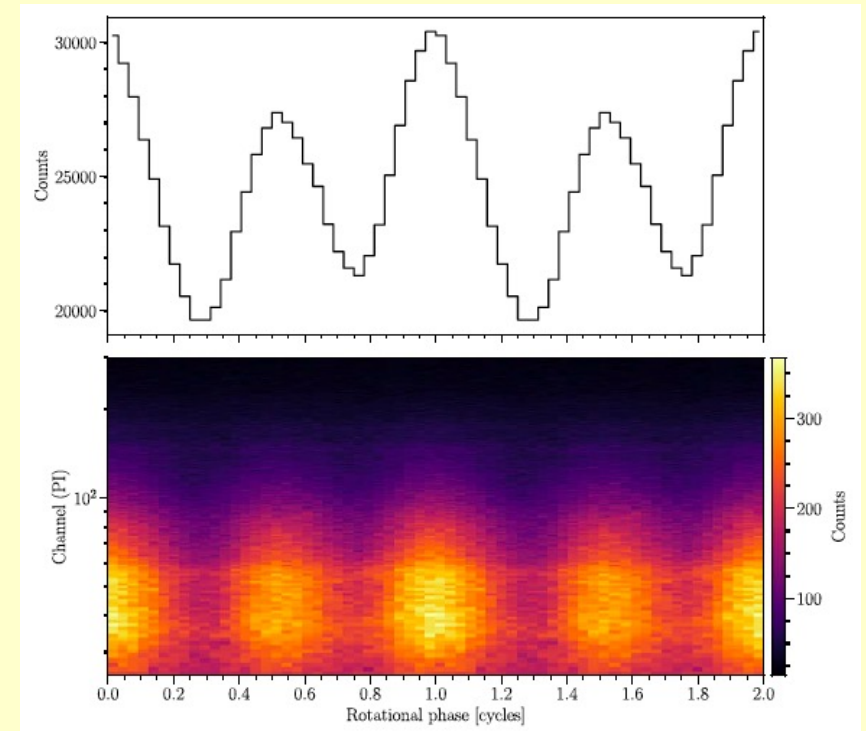
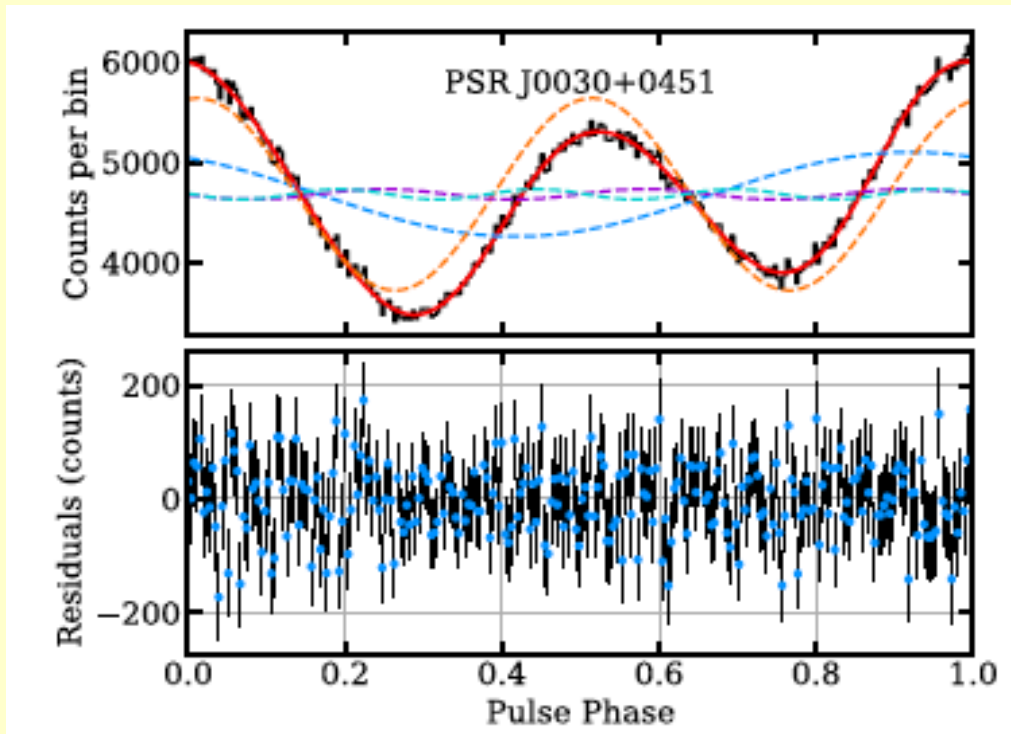
- Thermal X-ray emission due to polar cap heating by a return current of relativistic particles from pulsar magnetosphere



- The surface radiation can serve as a valuable probe of neutron star properties (compactness, magnetic field geometry, surface composition,...)



Pulse profile data for PSR J0030+0451



Bogdanov et al. 2019

$P=4.87$ ms

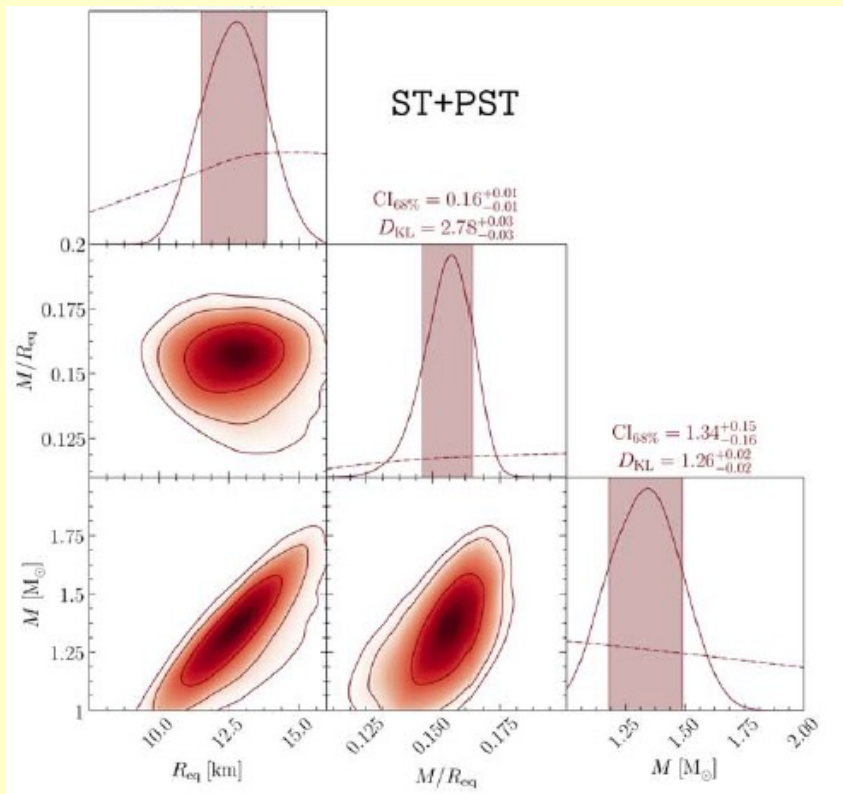
counts $\approx 1,3$ Million

Pulse fitted with 4 sines waves

Results

Amsterdam group
analysis

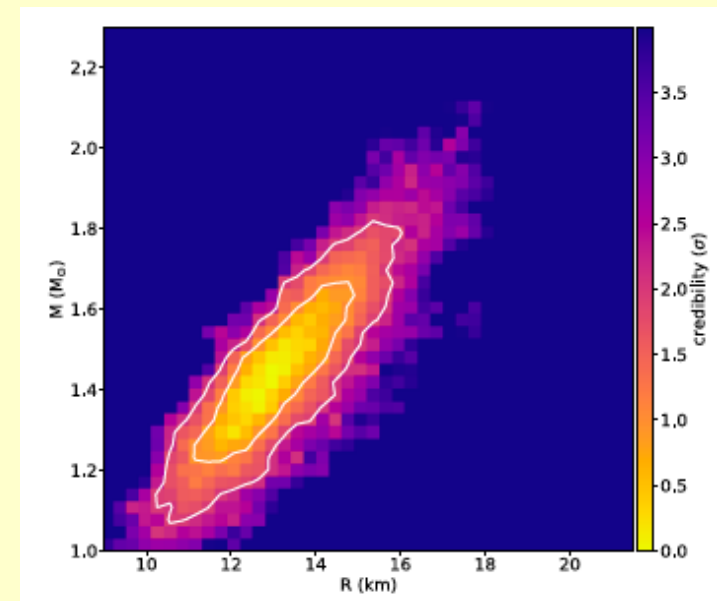
$$R = 12.7 \pm 1.2 \text{ km}$$
$$M = 1.34 \pm 0.15 M_{\odot}$$
$$i = 54 \pm 6 \text{ deg}$$



Riley et al. 2019

Maryland group
analysis

$$R = 13.0 \pm 1.2 \text{ km}$$
$$M = 1.44 \pm 0.15 M_{\odot}$$
$$i = 50 \pm 6 \text{ deg}$$



Miller et al. 2019