

Physics of X-ray pulsars

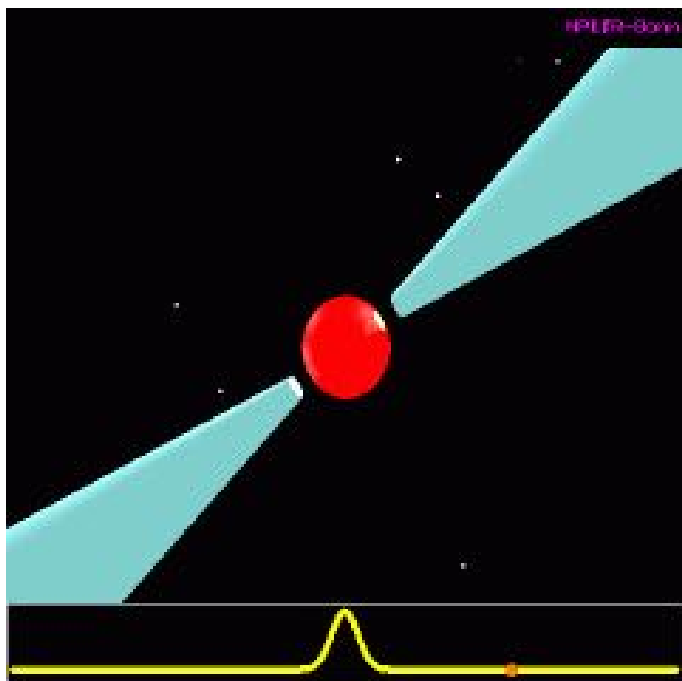
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Magnetic field strengths

Earth	0.5 G
Magnet on fridge	50 G
Stars	10-1000 G
Field for the levitating frogs	10^5 G
Strongest steady field in lab	10^6 G
E_{cycl} comparable to E_{Coulomb}	10^9 G
Young neutron stars	10^{12-15} G



NS magnetic field

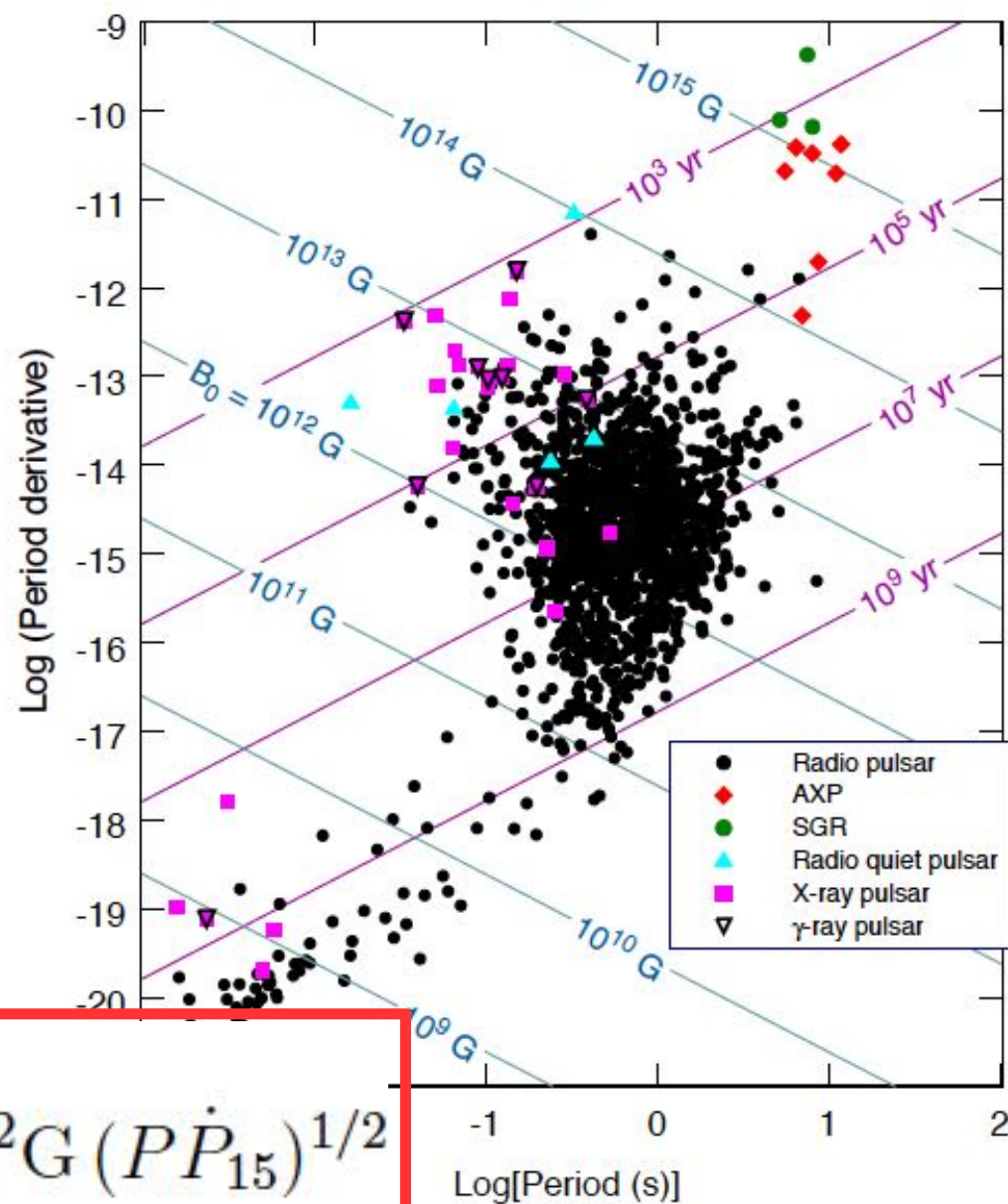


$$R \simeq 10^6 \text{ cm}$$

$$I \simeq 10^{45} \text{ g cm}^2$$

$$\dot{P}_{15} \equiv \dot{P} / (10^{-15} \text{ s s}^{-1})$$

$$B_s = \left(\frac{3Ic^3 P \dot{P}}{2\pi^2 R^6} \right)^{1/2} \simeq 2 \times 10^{12} \text{ G} (P \dot{P}_{15})^{1/2}$$



NS magnetic field

Strong magnetic fields:

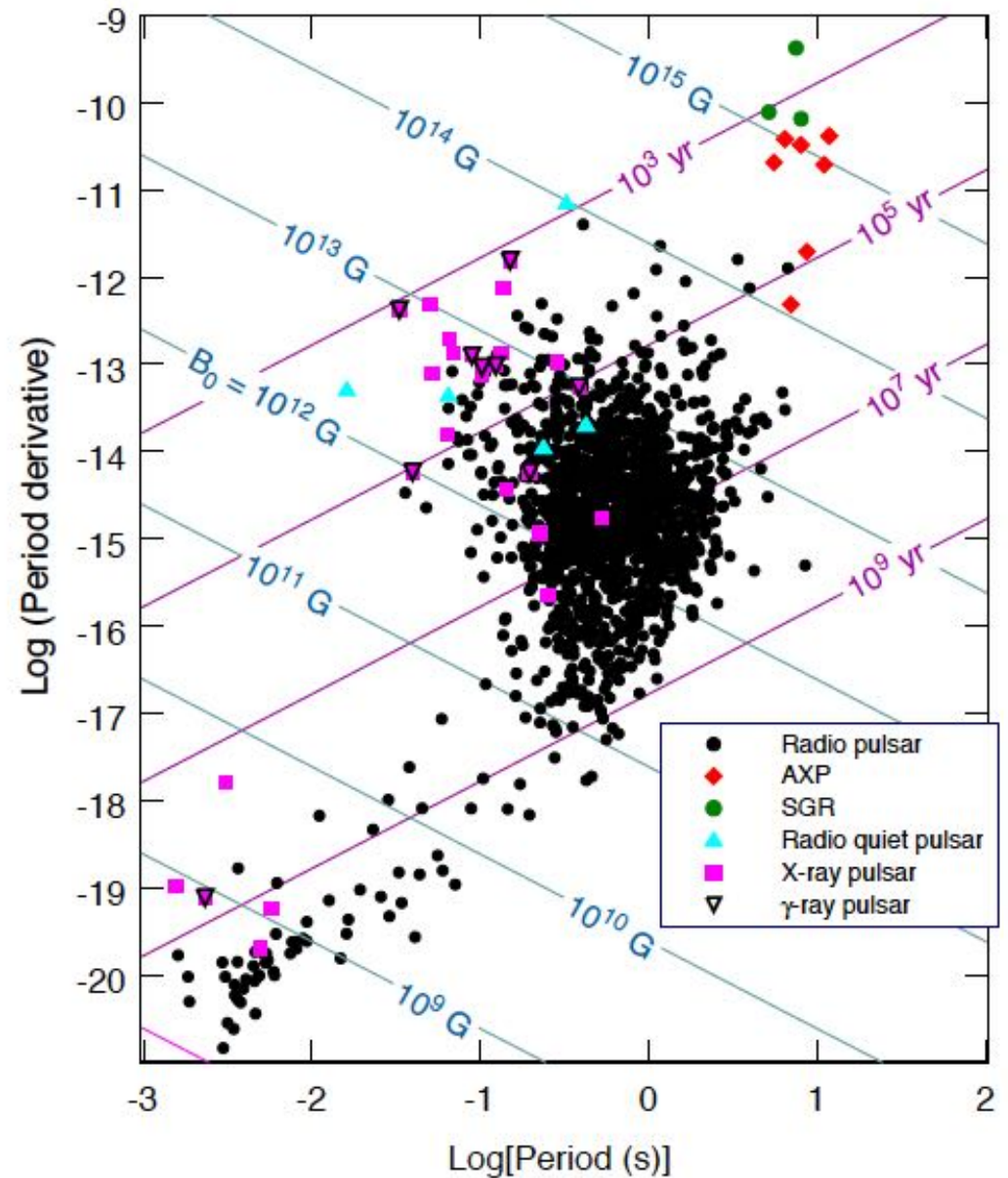
Cyclotron energy is comparable with the typical Coulomb energy at

$$B_0 = 2.3505 \times 10^9 \text{ G}$$

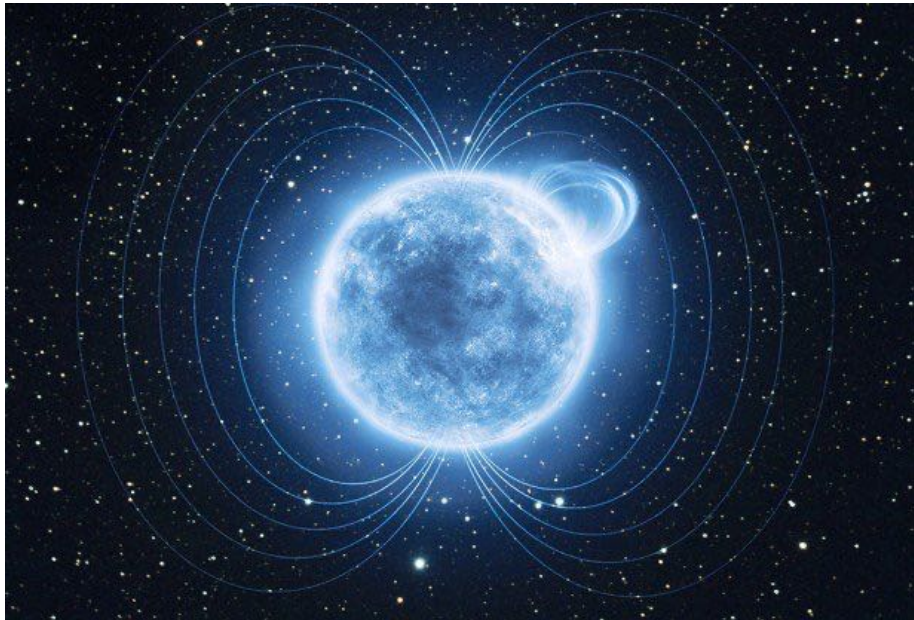
Theoretical upper limit for NSs:

$$\frac{E_{\text{mag}}}{E_{\text{grav}}} \sim \frac{B_{\text{in}}^2 R^3 / 6}{GM^2 / R} \lesssim 1$$

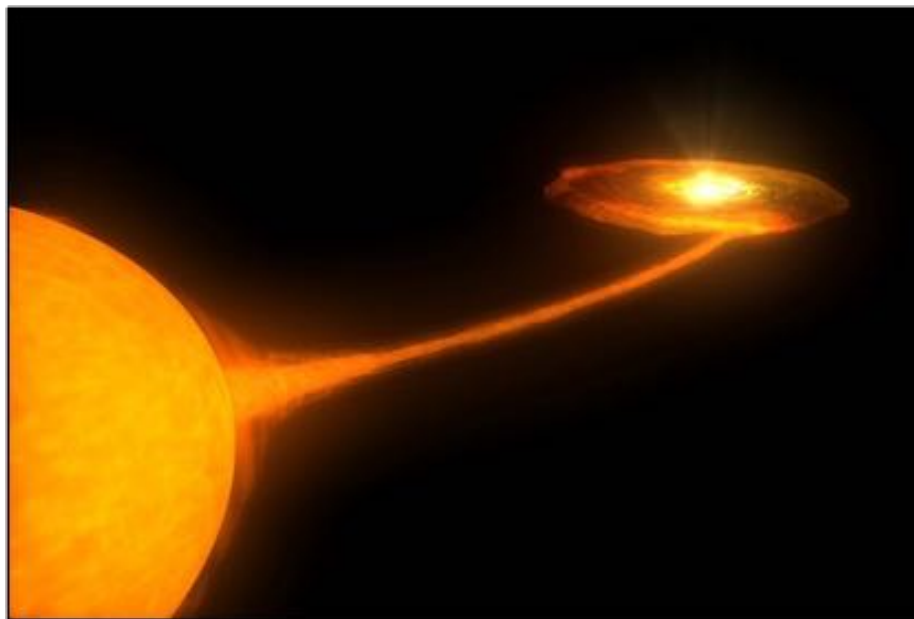
$$B_{\text{in}} \lesssim 10^{18} \text{ G}$$



What if NS has a companion?



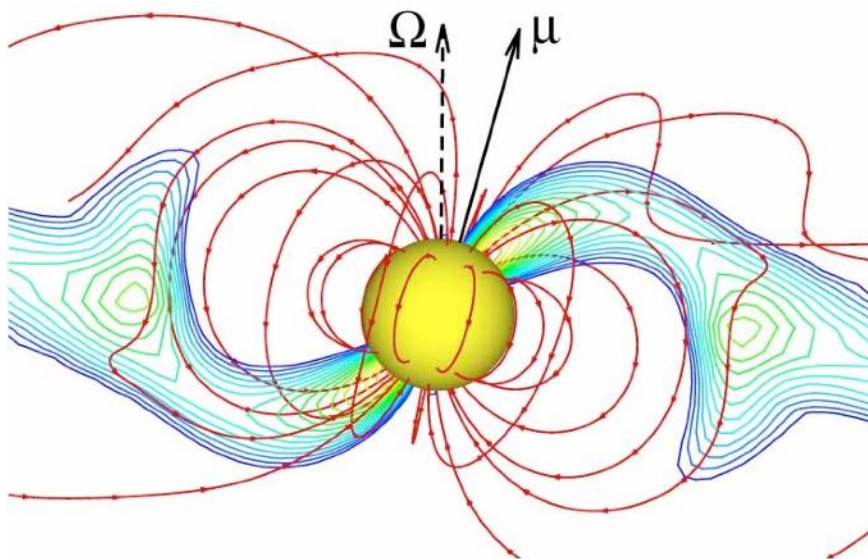
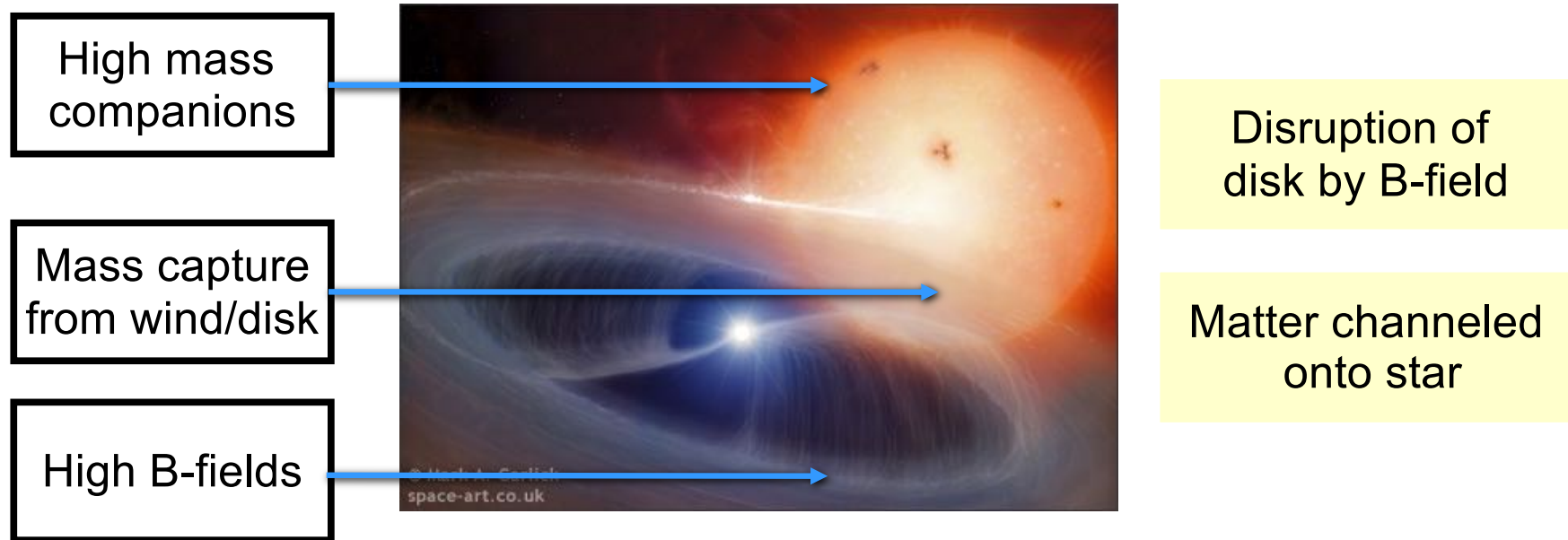
90% of all neutron stars are isolated. The main source of energy is rotation.



If neutron star has a close companion, it will be powered by accretion.

X-ray pulsar

Rotating neutron star in binary systems

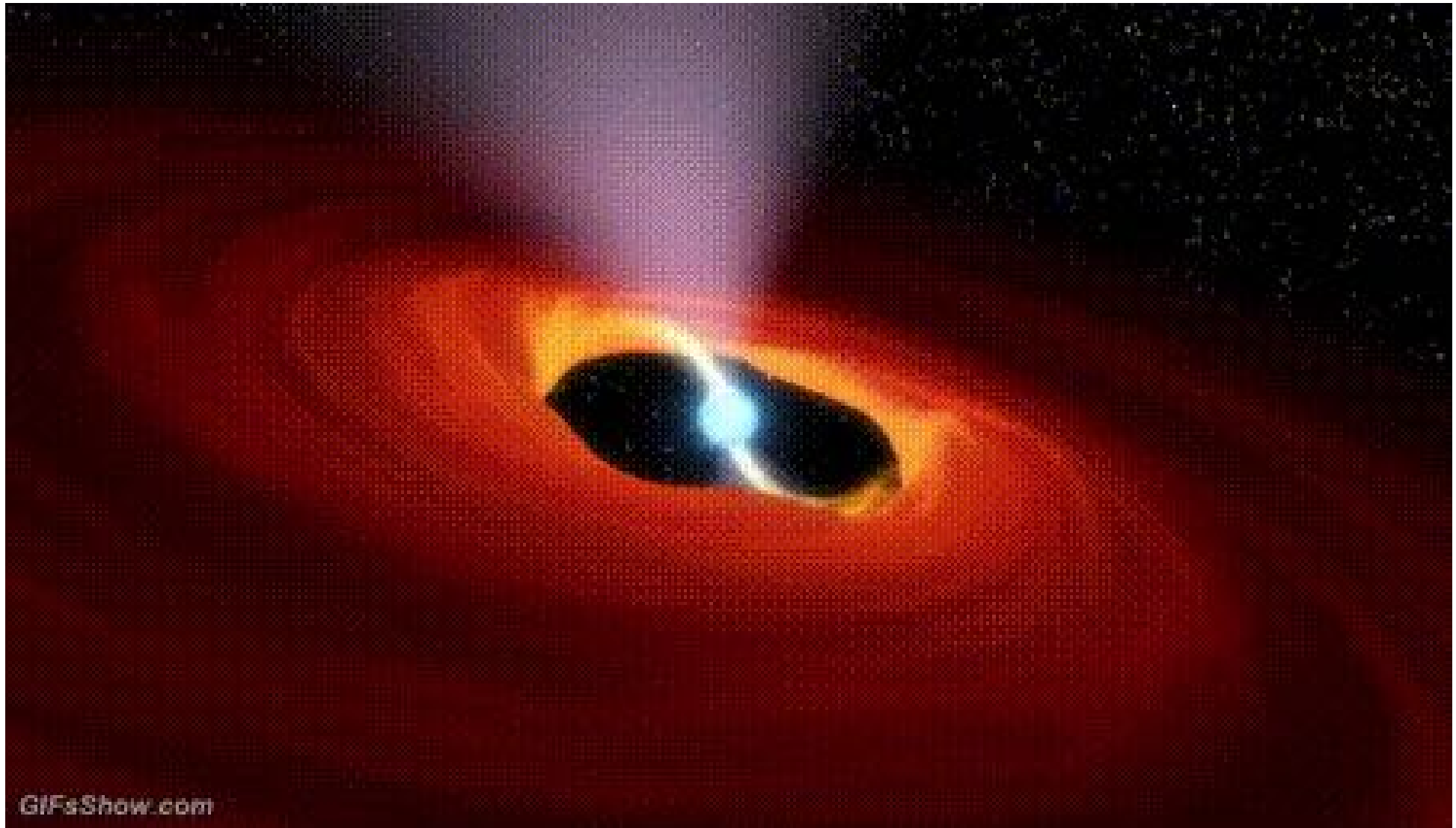


Neutron star parameters:

$$\begin{aligned} M_{\text{NS}} &\sim 1.4 M_{\text{sun}} \\ R_{\text{NS}} &\sim 10 \text{ km} (10^6 \text{ sm}) \\ P_{\text{spin}} &\sim 1 - 10^3 \text{ s} \\ \mathbf{B}_{\text{NS}} &\sim \mathbf{10^{11-13} G} \end{aligned}$$

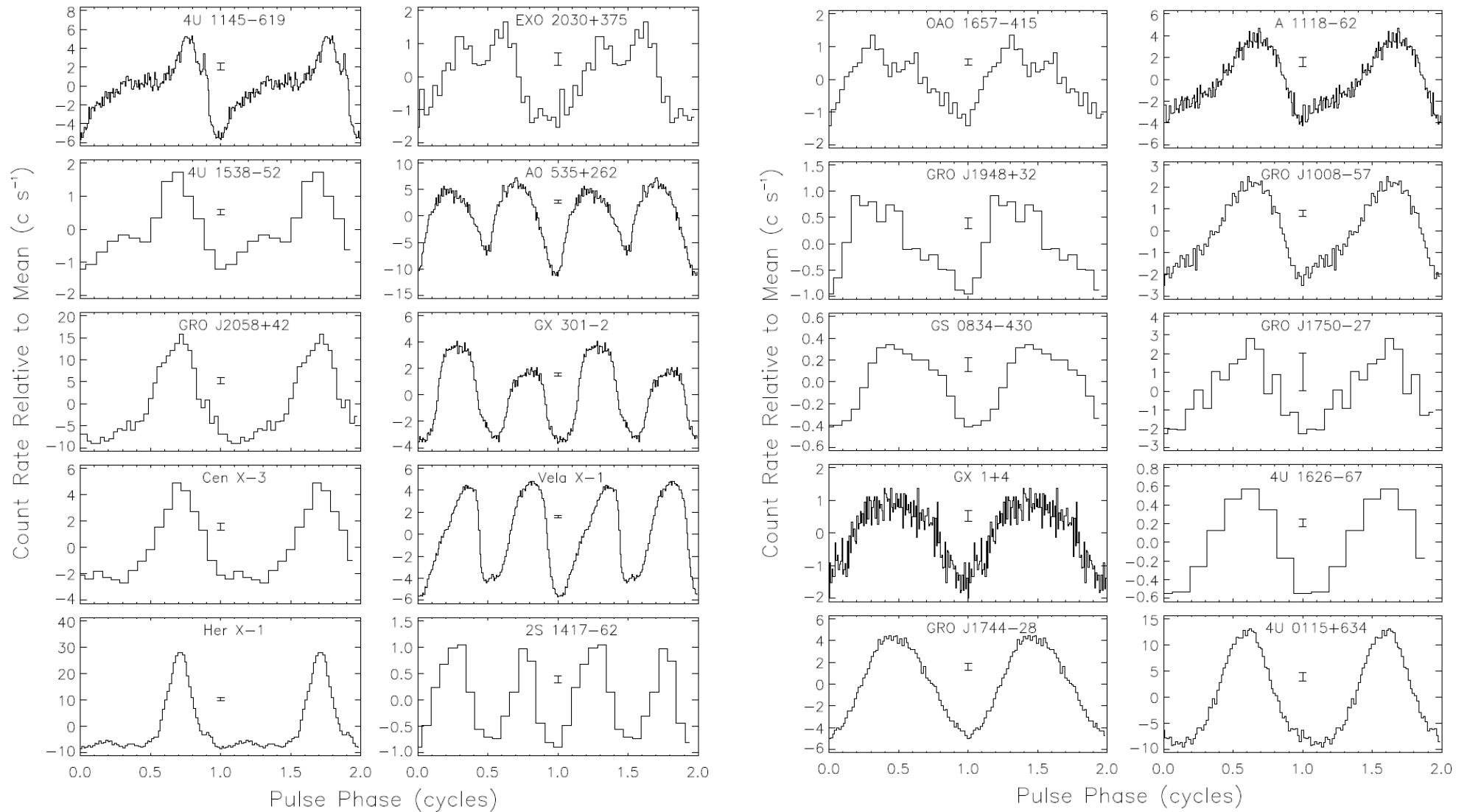
X-ray pulsar

Rotating neutron star in binary systems

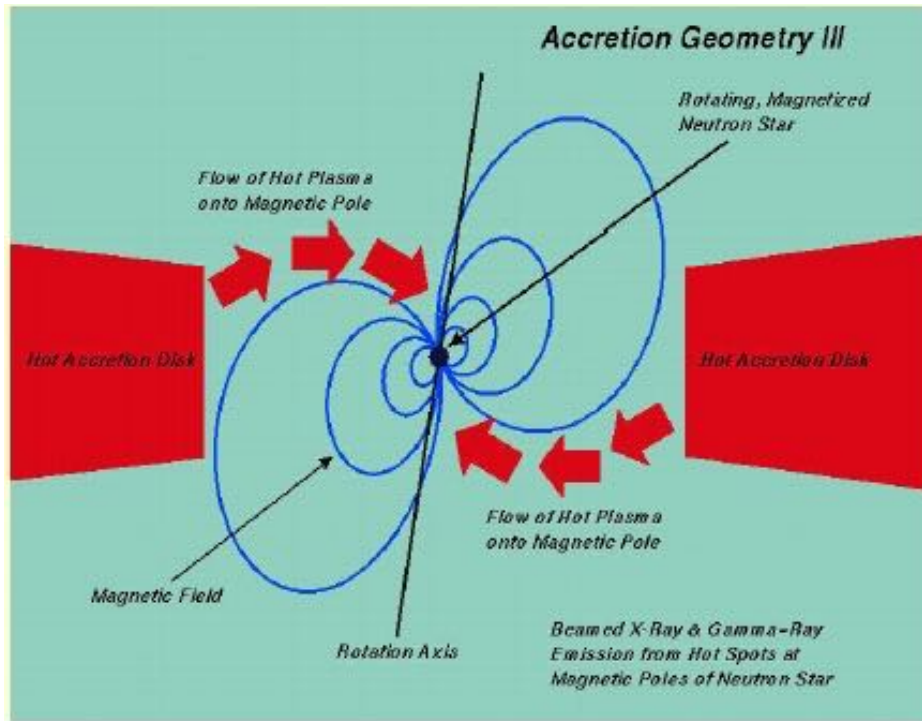


X-ray pulsars

- Pulse periods from ~ 1 s to ~ 1000 s.



X-ray (?) pulsars

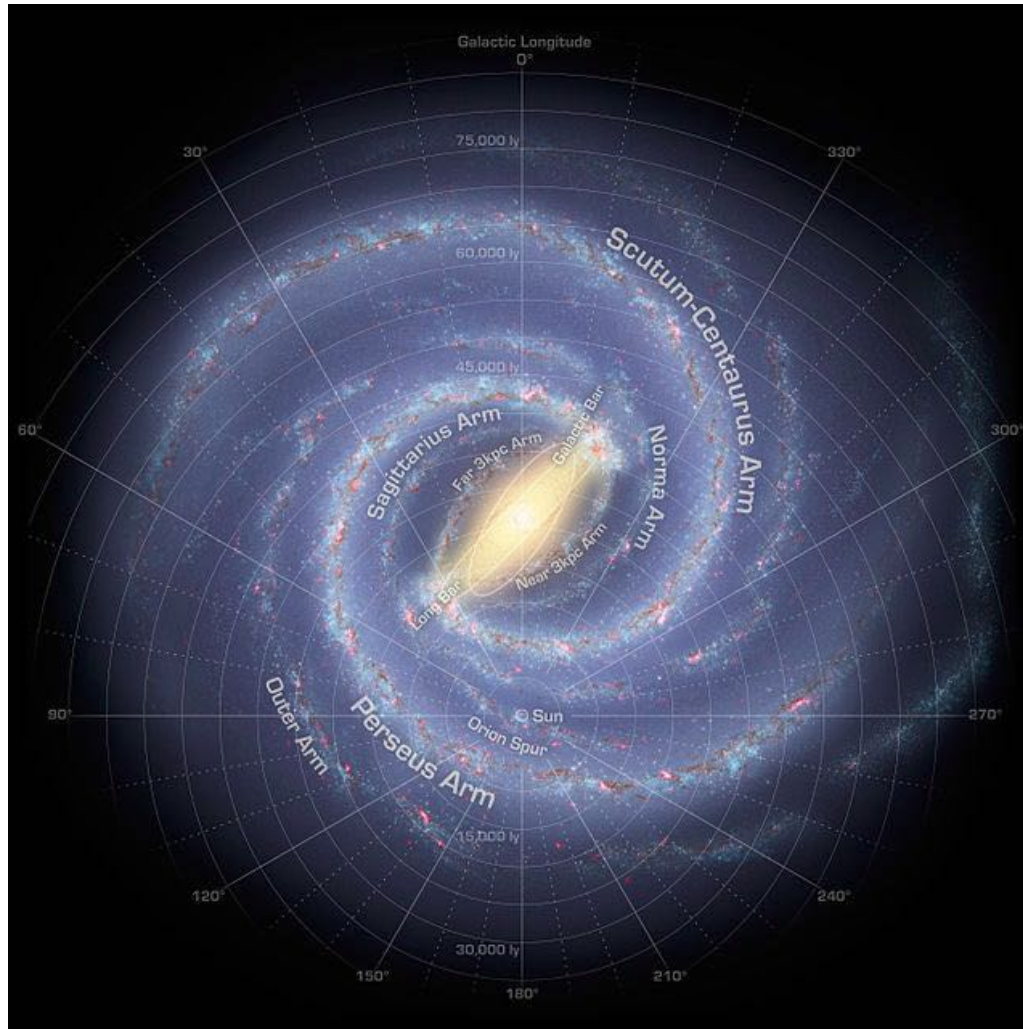


**Emission is not a
black body!**

Stefan-Boltzmann law:

- $L = \sigma T^4 S$
- $\sigma = 5.67 \times 10^{-5} \text{ erg s}^{-1} \text{ cm}^{-2} \text{ K}^{-4}$
(Stefan-Boltzmann constant)
- Radius of the polar cap $\sim 100 \text{ m}$
 $S \sim 10^8 \text{ cm}^2$
- Observed $L \sim 10^{37} \text{ erg s}^{-1}$
- $T = (L / \sigma S)^{1/4} \sim 10^8 \text{ K}$
- $1 \text{ eV} = 11605 \text{ K} \sim 10^4 \text{ K}$
- $T \sim 10 \text{ keV}$

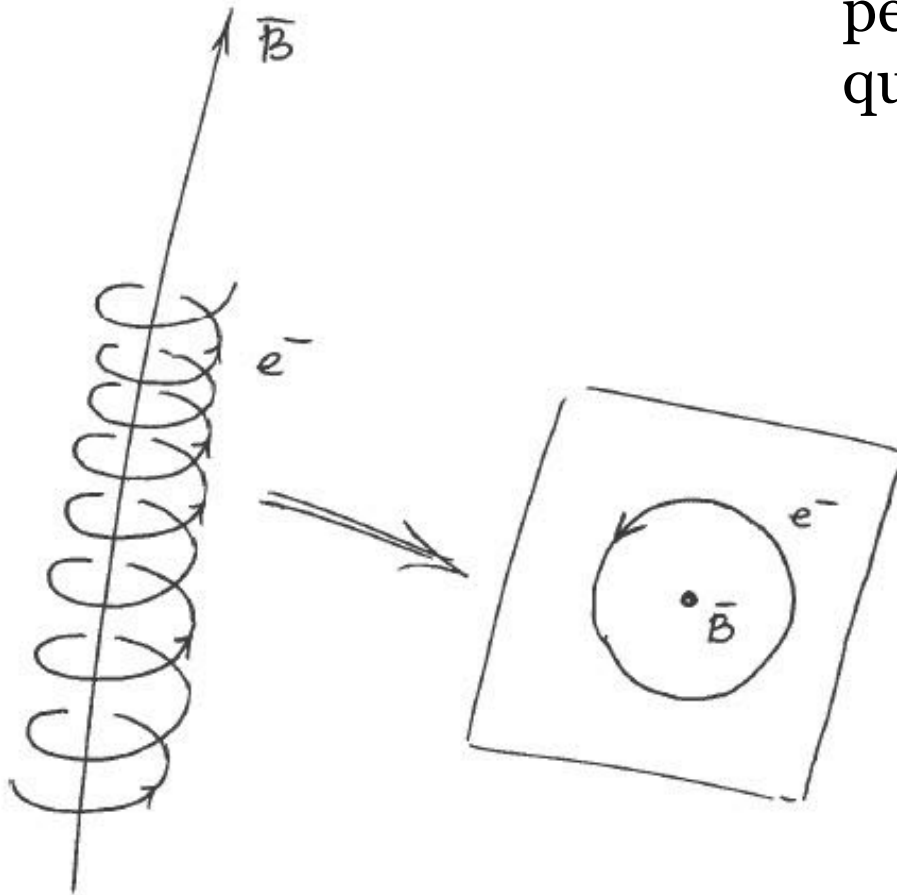
X-ray pulsars



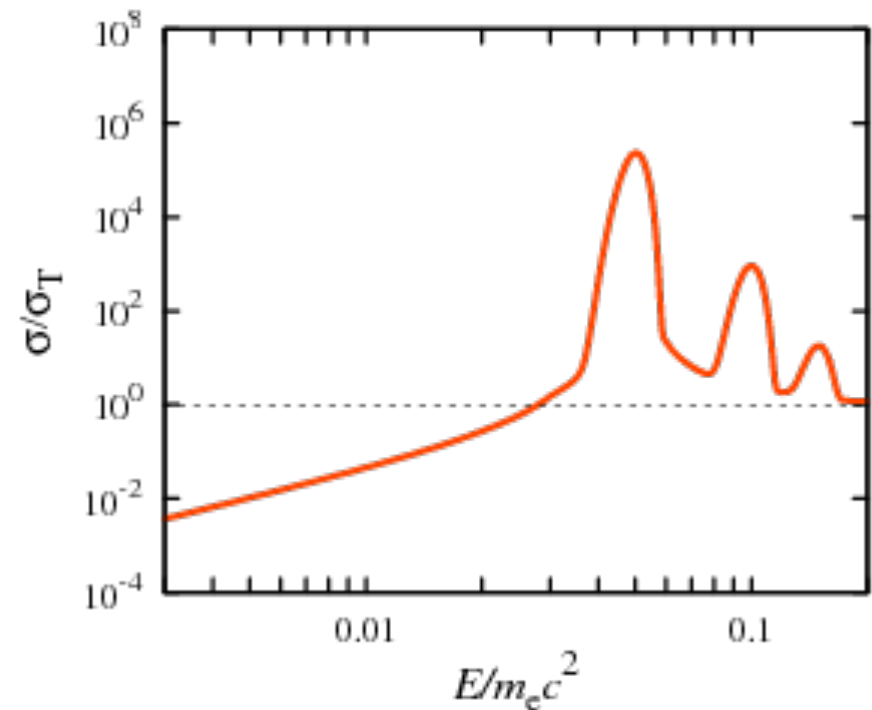
- Currently about 100 X-ray pulsars are known.
- Usually in massive binary systems with O-B optical companions. Only a few are known in low-mass X-ray binaries (e.g., Her X-1).
- Orbital periods from ~1 day to ~1 year.

Ultra strong magnetic field

The motion of the electron
perpendicularly to the B field is
quantized in the Landau levels

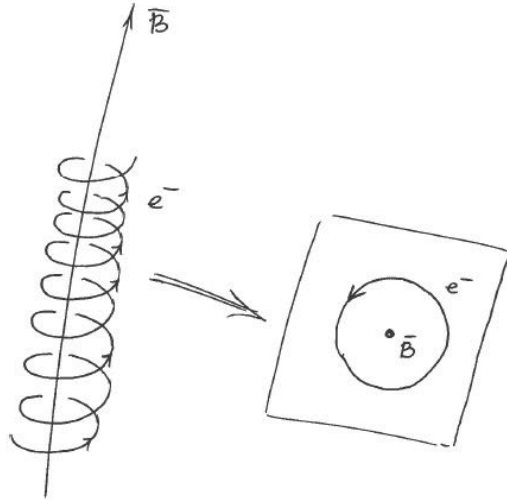


Scattering cross section



Magnetic field of the neutron star

Lorentz force =
centripetal force



$$\frac{|q|}{c} V B = \frac{m V^2}{r}$$

$$r = \frac{cmV}{|q|B}$$

$$\frac{2\pi r}{V} = \frac{2\pi}{\omega} \quad \omega = \frac{V}{r}$$

Cyclotron frequency

$$\omega_c = \frac{eB}{m_e c}$$

Cyclotron energy, keV

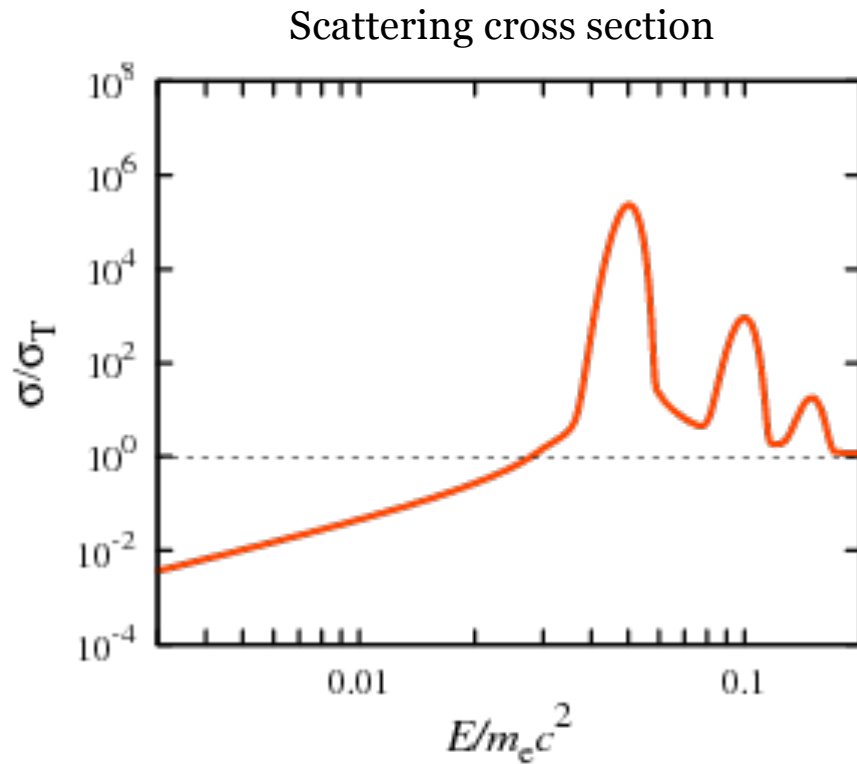
$$E_{\text{cyc}} = \hbar \omega_c \quad \hbar \omega_n = n \hbar \omega_c$$

Magnetic field of NS, 10^{12} Gauss

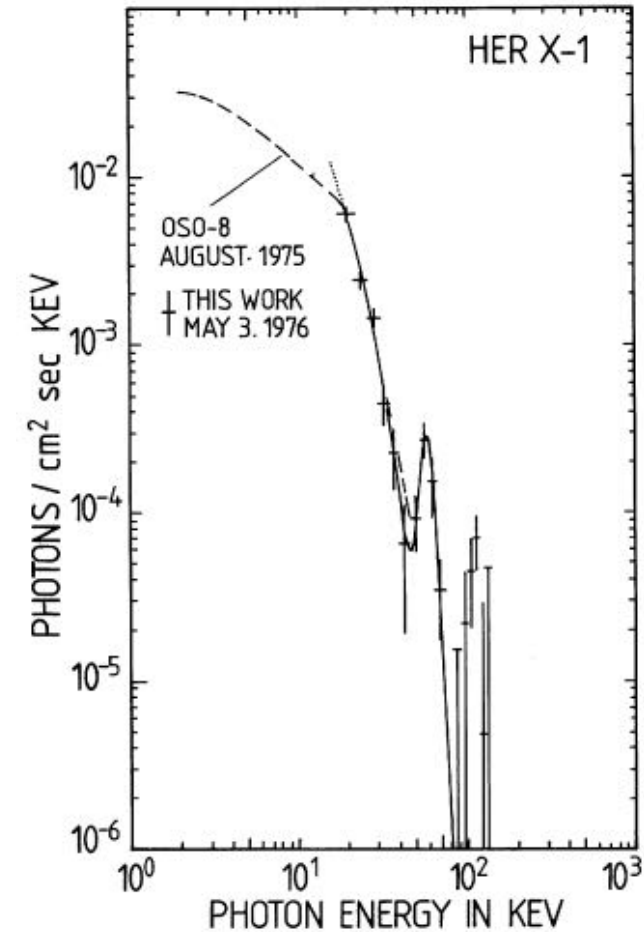
$$B_{12} = \frac{E_{\text{cyc}}}{11.2}$$

Cyclotron absorption line

Truemper et al. (1978)

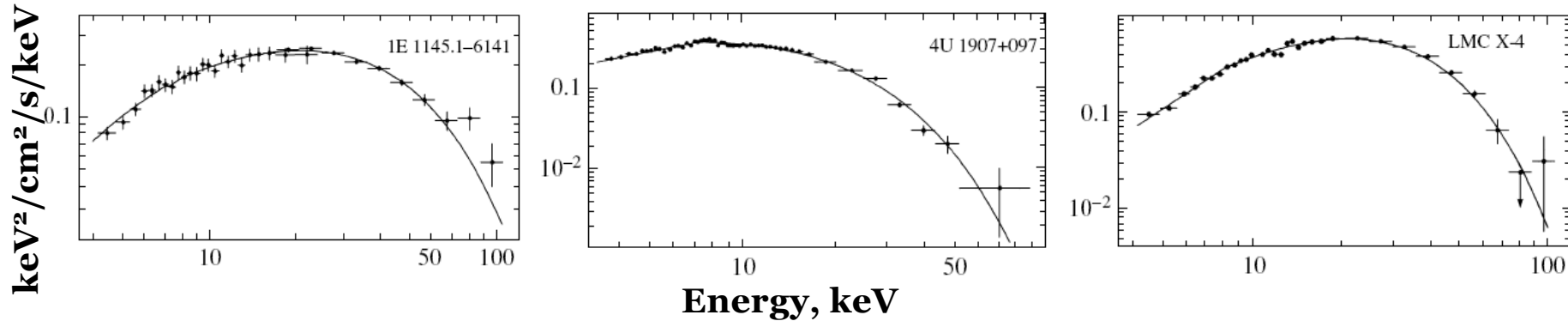


Cyclotron lines directly probe the magnetic fields of neutron stars!

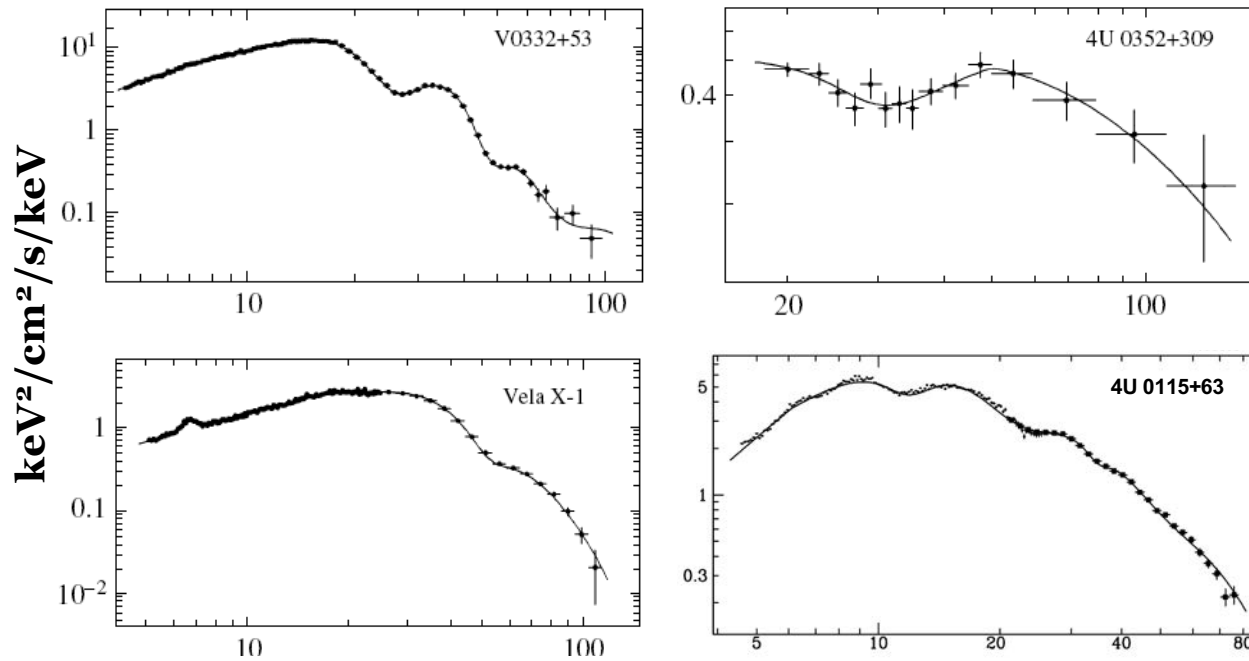


$$B_{12} = \frac{E_{\text{cyc}}}{11.2} \quad E_{\text{cyc}} \text{ in units of keV, } B_{12} \text{ in units of } 10^{12} \text{ G}$$

Spectra of X-ray pulsars



With cyclotron absorption line

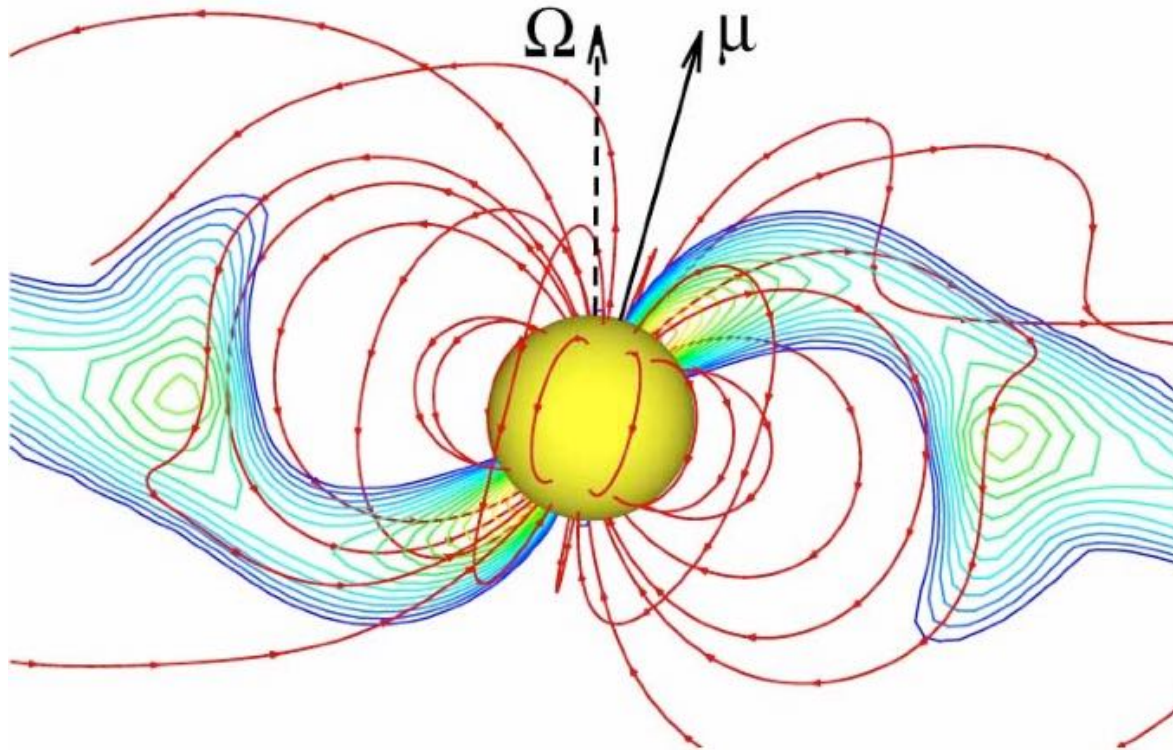


<i>Name</i>	<i>E_{cyc}, keV</i>
4U 0115+63	~11, 21, 34, 45
V 0332+53	~28, 55
A 0535+26	~45, 100
Hercules X-1	~38
Vela X-1	~22, 56
GX 301-2	~50
GX 1+4	~34
Cen X-3	~31
OA0 1657-415	~78
EXO 2030+375	~63

Energy, keV

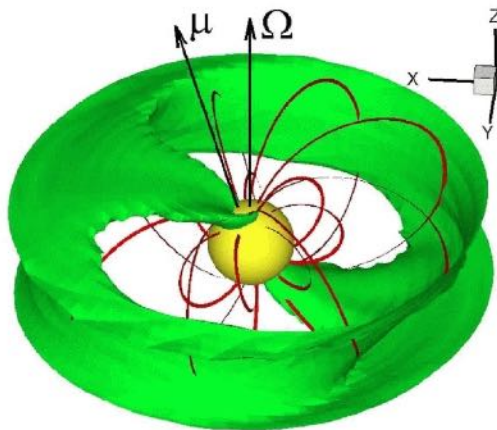
$$E_{\text{cyc}} = 11.6 B_{12} \text{ keV}$$

X-ray pulsars

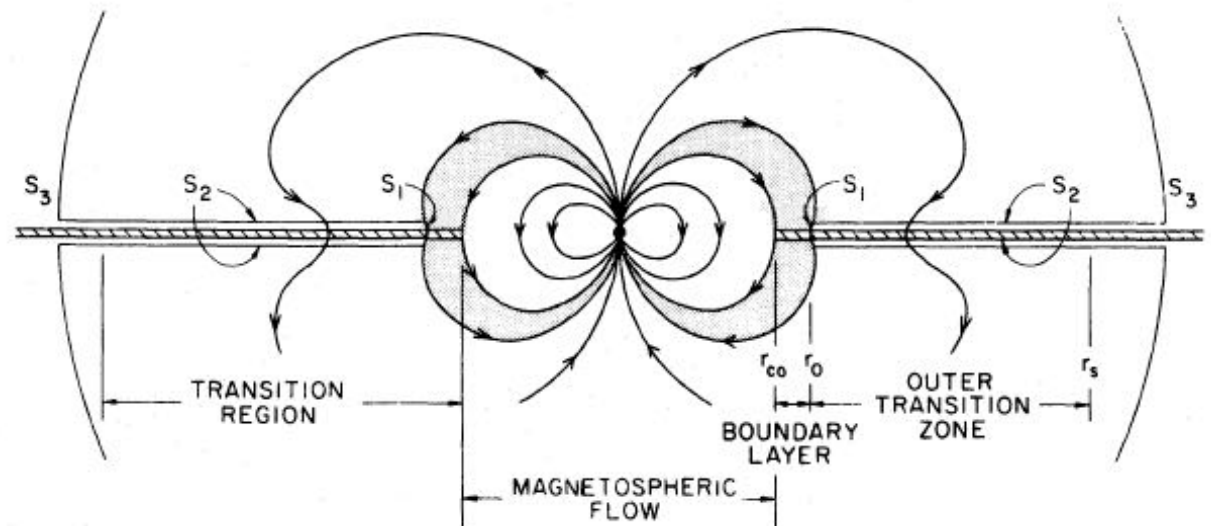


There are two main characteristic radii related to the accretion onto the magnetized NS:

- Magnetospheric radius
- Corotation radius



Romanova et al. 2009



Magnetospheric radius (Alfven radius)

Magnetic pressure equals the matter pressure

$$\frac{B^2}{8\pi} = \rho V^2$$

Free fall velocity

$$\frac{GMm}{r} = \frac{1}{2}mV^2 \quad V = \sqrt{\frac{2GM}{r}}$$

Accretion rate; continuity equation

$$\dot{M} = 4\pi r^2 \rho V \quad \rho = \frac{\dot{M}}{4\pi r^2 V}$$

Magnetic field strength in the dipole configuration

$$B = B_0 \left(\frac{R_{ns}}{r} \right)^3 \quad (/2) \quad \text{for dipole magnetic field}$$

$$\frac{B_0^2 R_{ns}^6}{r^6 8\pi} = \frac{\dot{M}}{4\pi r^2} \sqrt{\frac{2GM}{r}}$$

Magnetospheric radius (Alfven radius)

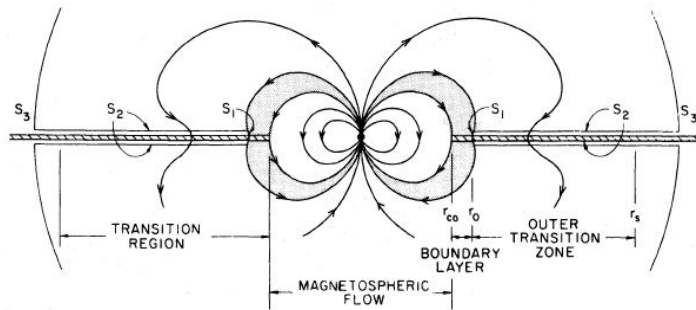
$$\frac{B_0^2 R_{ns}^6}{r^6 8\pi} = \frac{\dot{M}}{4\pi r^2} \sqrt{\frac{2GM}{r}}$$

$$\frac{B_0^4 R_{ns}^{12}}{r^8} = \frac{8GM\dot{M}^2}{r}$$

Magnetospheric radius

$$r_A = \left(\frac{B_0^4 R_{ns}^{12}}{8GM\dot{M}^2} \right)^{1/7}$$

$$r_m = \xi r_A \quad \xi \sim 0.5$$



$$R_m \simeq 1.8 \times 10^8 \xi B_{12}^{4/7} \dot{M}_{17}^{-2/7} M_{\odot}^{-1/7} R_6^{12/7} \text{ cm}$$

$$R_m \simeq 2.5 \times 10^8 \xi M_{1.4}^{1/7} R_6^{10/7} B_{12}^{4/7} L_{37}^{-2/7} \text{ cm}$$

Corotation radius

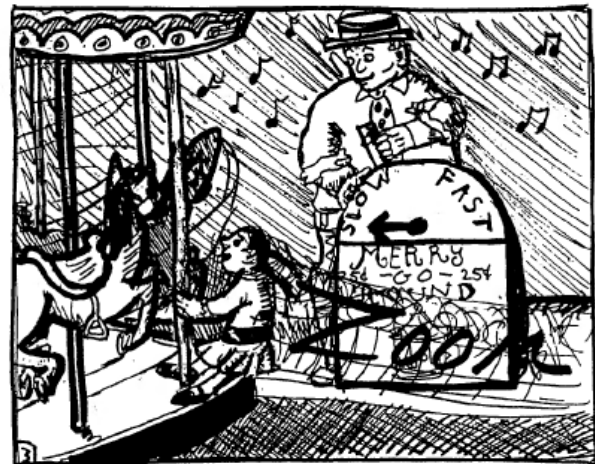
Radius where velocity of the magnetic field lines is equal to the Keplerian one

$$\omega R = \sqrt{\frac{GM}{R}}$$

$$R_c = \left(\frac{GM}{\omega^2} \right)^{1/3}$$

$$R_c = \left(\frac{GMP^2}{4\pi^2} \right)^{1/3} \simeq 1.68 \times 10^8 M_{1.4}^{1/3} P^{2/3} \text{ cm}$$

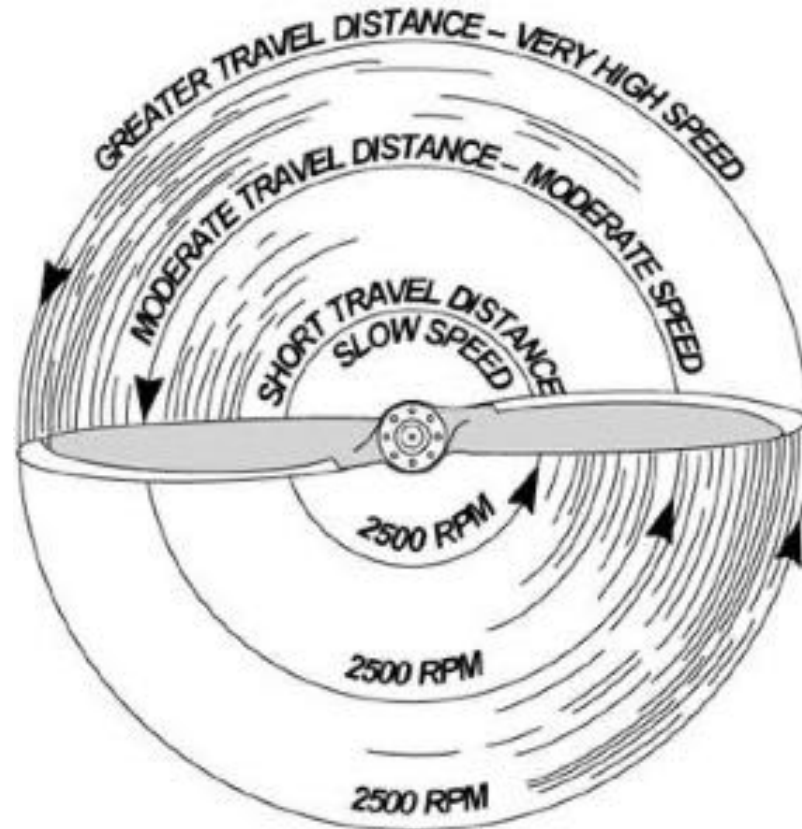
“Propeller” effect



Patterson, 1994

$R_m < R_c$ - accretion is possible

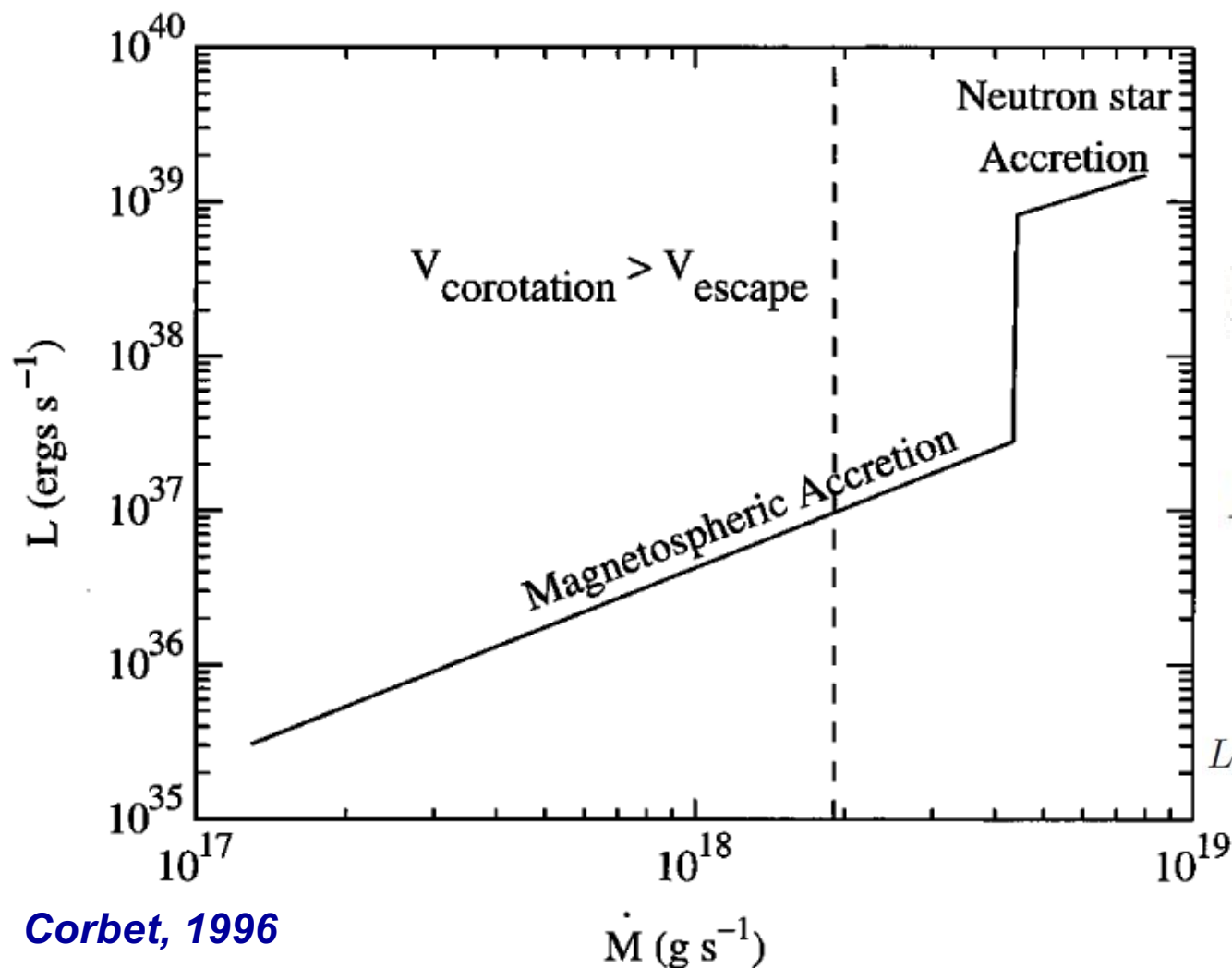
$R_m > R_c$ - accretion is prohibited due to centrifugal barrier



“Propeller effect”

Illarionov & Sunyaev, 1975

“Propeller” effect



Corbet, 1996

$$R_c = R_m$$

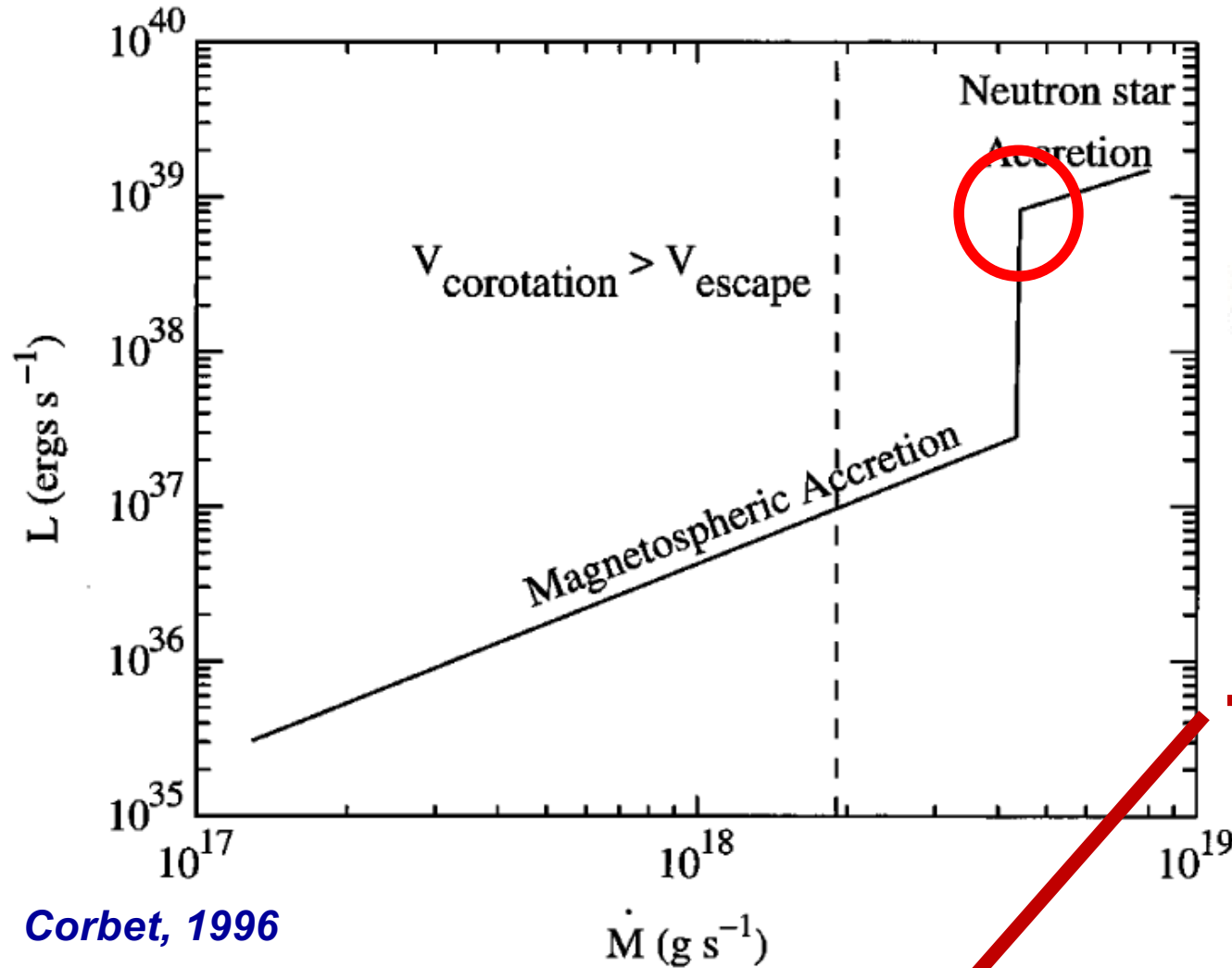
$$L_{\text{lim}}(R) \simeq \frac{GM\dot{M}_{\text{lim}}}{R}$$

$$L_{\text{lim}}(R_c) = \frac{GM\dot{M}_{\text{lim}}}{R_c}$$

$$L_{\text{lim}}(R_c) = \frac{GM\dot{M}_{\text{lim}}}{R_c} = L_{\text{lim}}(R) \frac{R}{R_c}$$

$$r_A = \left(\frac{B_0^4 R_{ns}^{12}}{8GM\dot{M}^2} \right)^{1/7}$$

“Propeller” effect



Corbet, 1996

$$R_c = R_m$$

$$L_{\text{lim}}(R) \simeq \frac{GM\dot{M}_{\text{lim}}}{R}$$

$$r_{\text{co}} = \left(\frac{GM}{\Omega^2} \right)^{1/3}$$

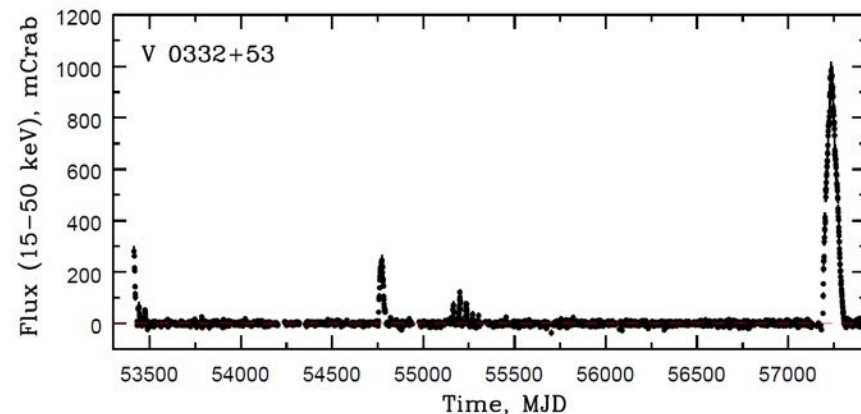
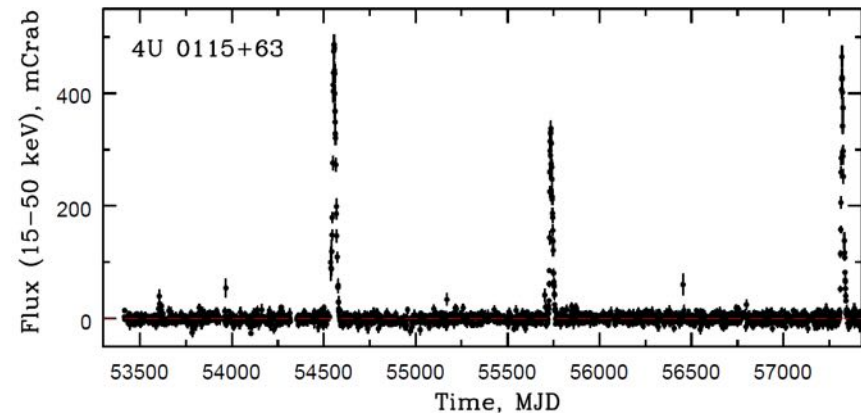
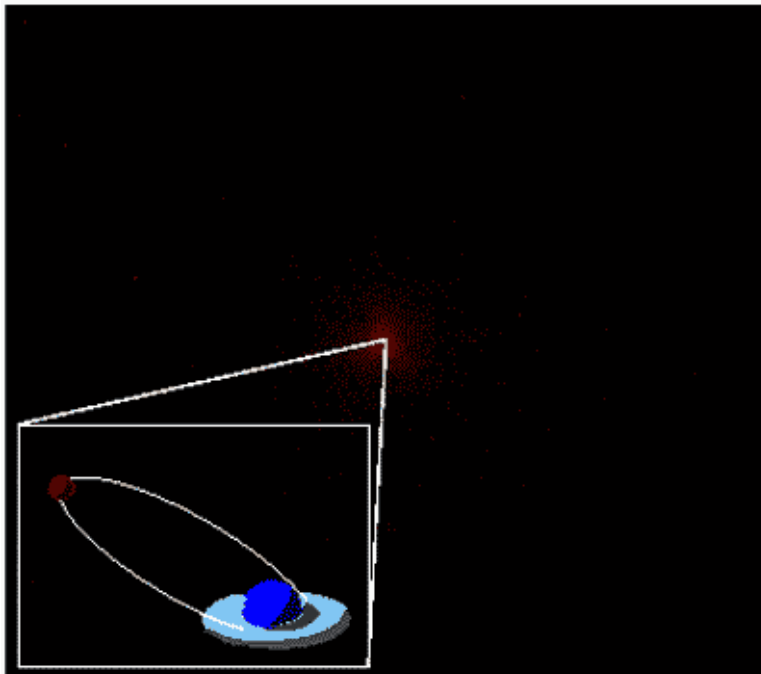
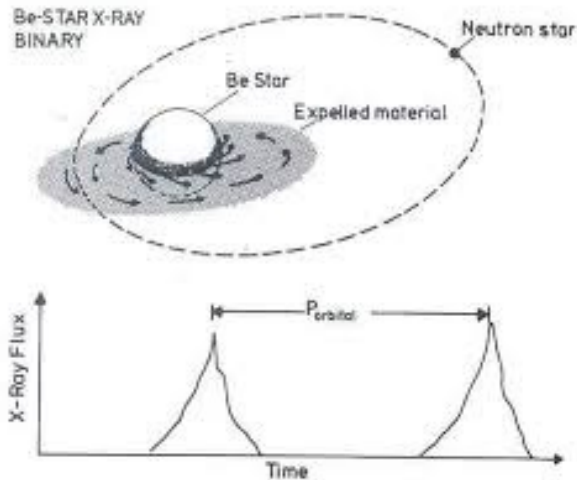
$$r_A = \left(\frac{B_0^4 R_{ns}^{12}}{8GM\dot{M}^2} \right)^{1/7}$$

$$r_m = \xi r_A$$

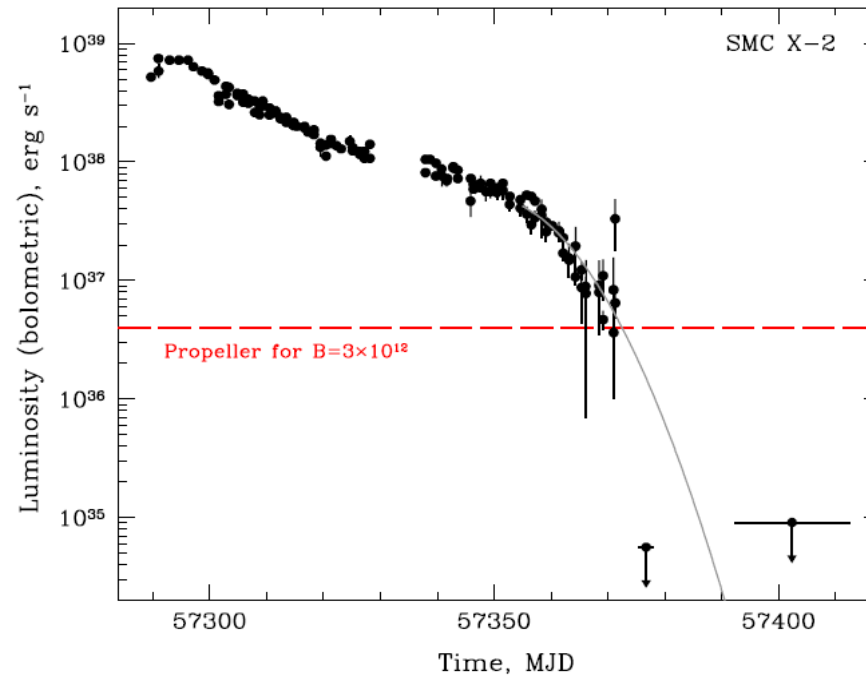
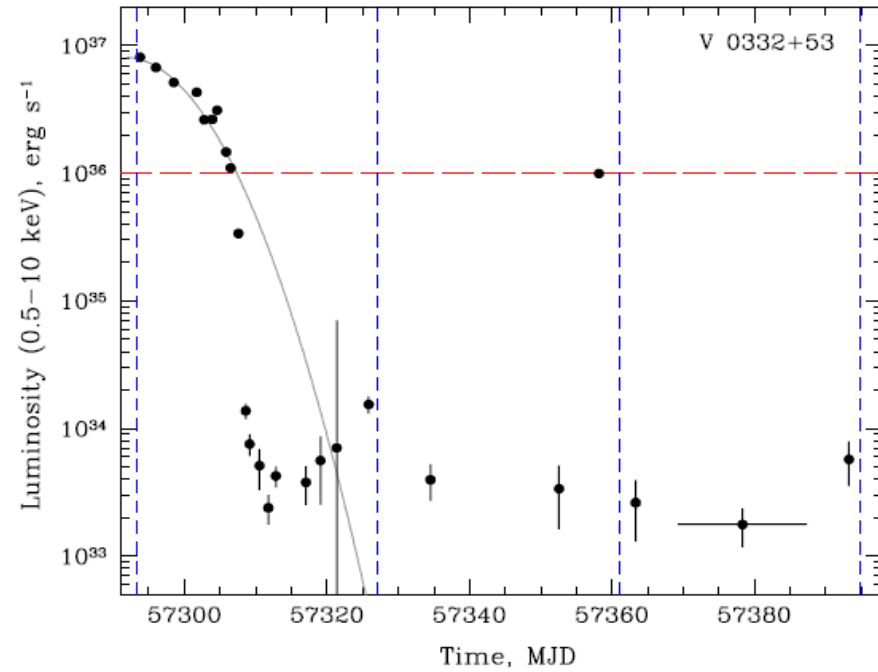
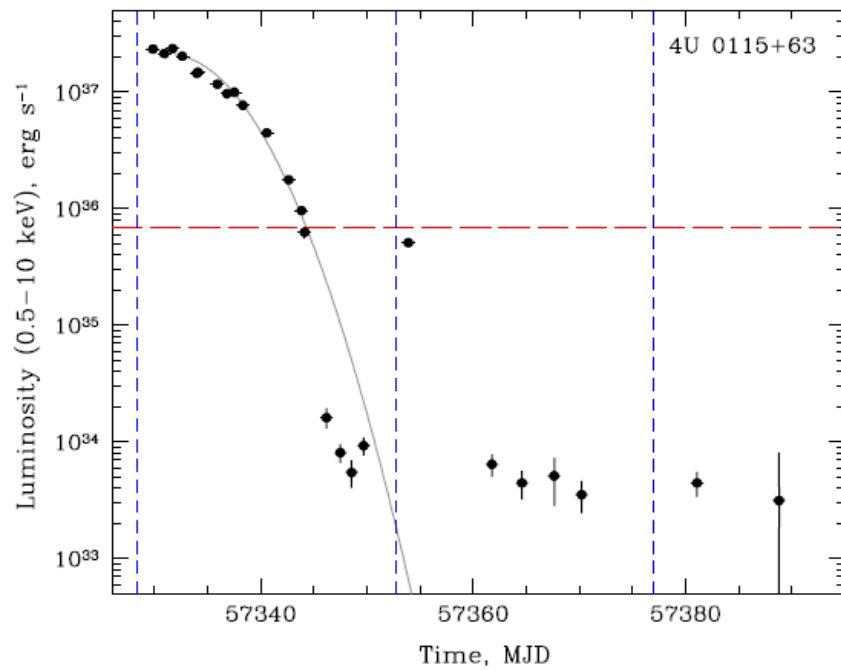
$$L_{\text{lim}}(R) \simeq \frac{GM\dot{M}_{\text{lim}}}{R} \simeq 4 \times 10^{37} \xi^{7/2} B_{12}^2 P^{-7/3} M_{1.4}^{-2/3} R_6^5 \text{ erg s}^{-1}$$

Transient X-ray pulsars

Neutron star in binary system with Be optical companion demonstrates periodic episodes of accretion due to interaction with decretion disc of the companion.

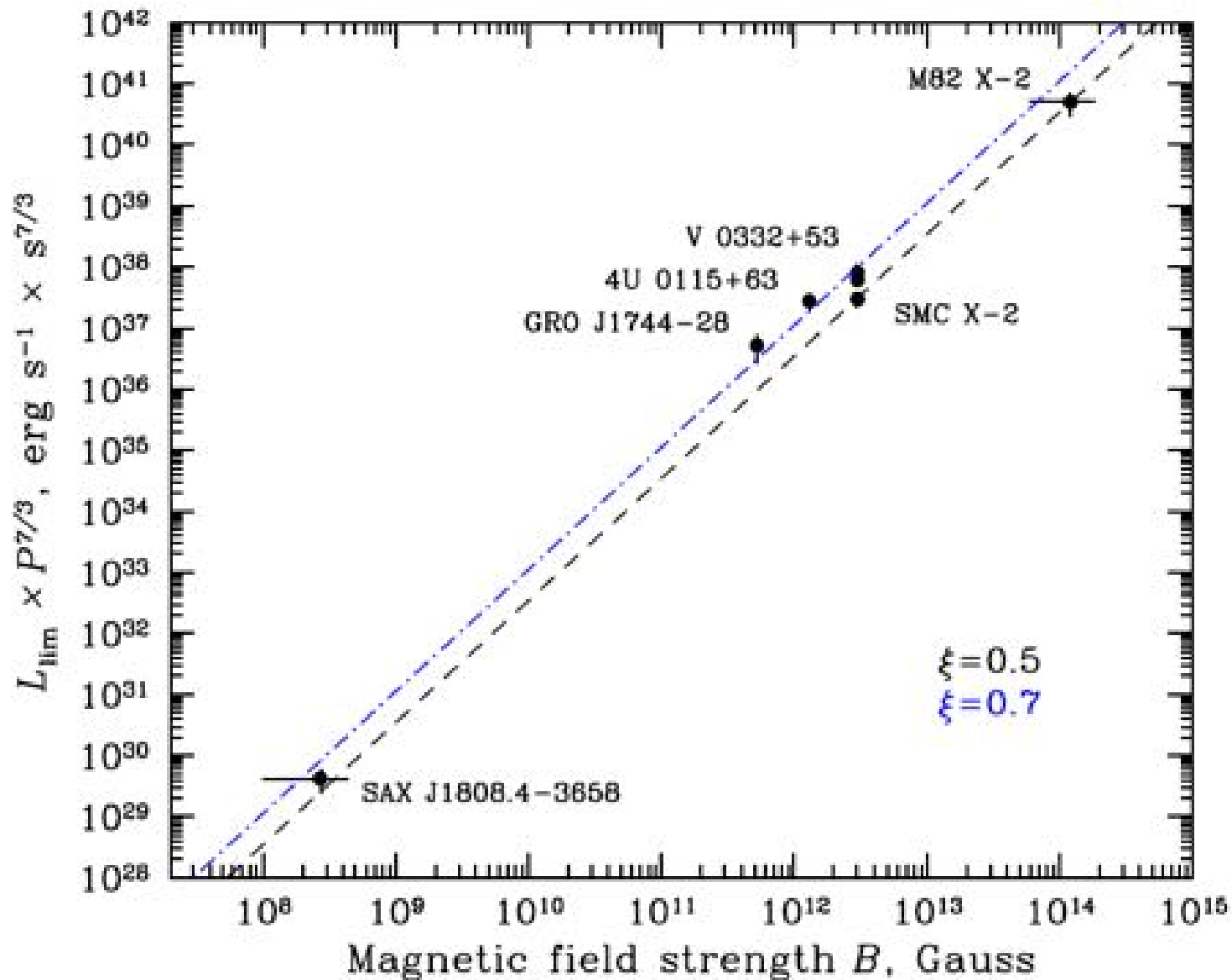


Temporal analysis



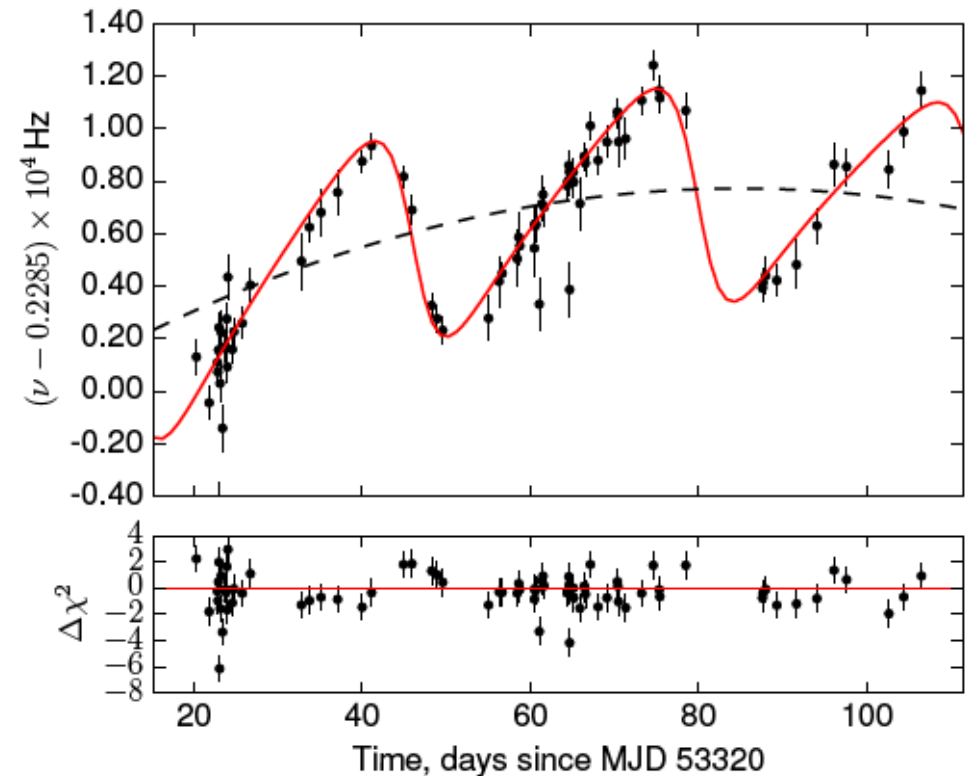
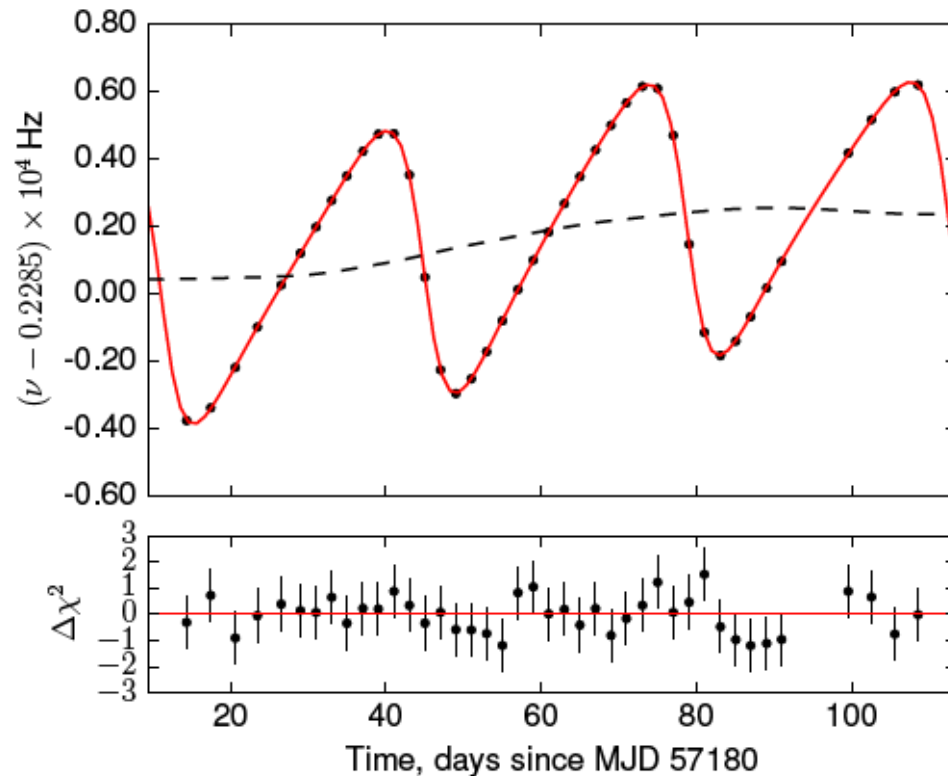
Tsygankov et al. (2016)

“Propeller” effect $\rightarrow B$ value



$$L_{\text{lim}}(R) \simeq \frac{GM\dot{M}_{\text{lim}}}{R} \simeq 4 \times 10^{37} \xi^{7/2} B_{12}^2 P^{-7/3} M_{1.4}^{-2/3} R_6^5 \text{ erg s}^{-1}$$

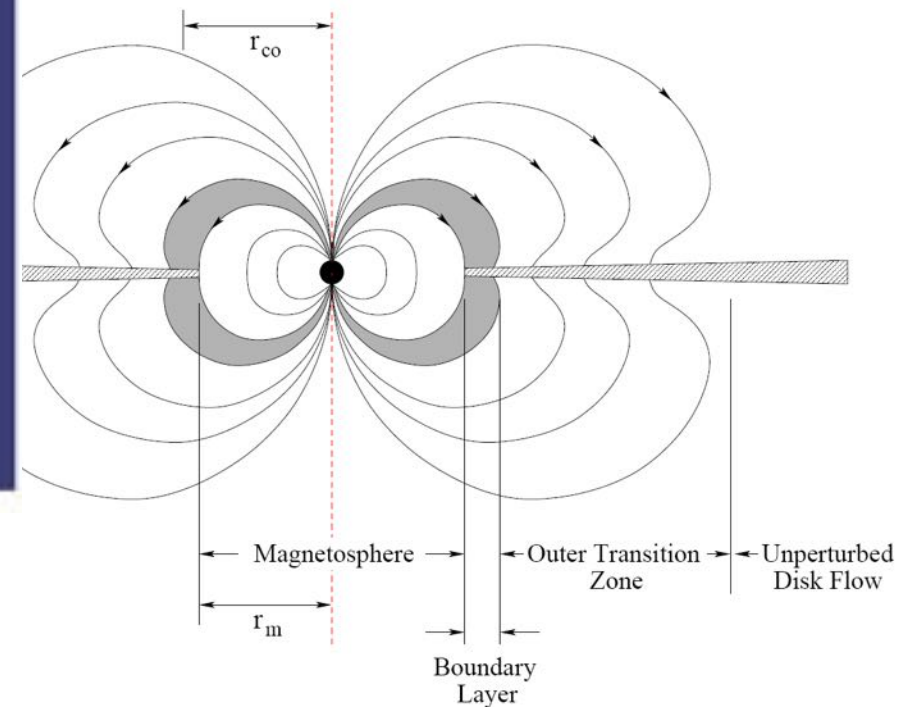
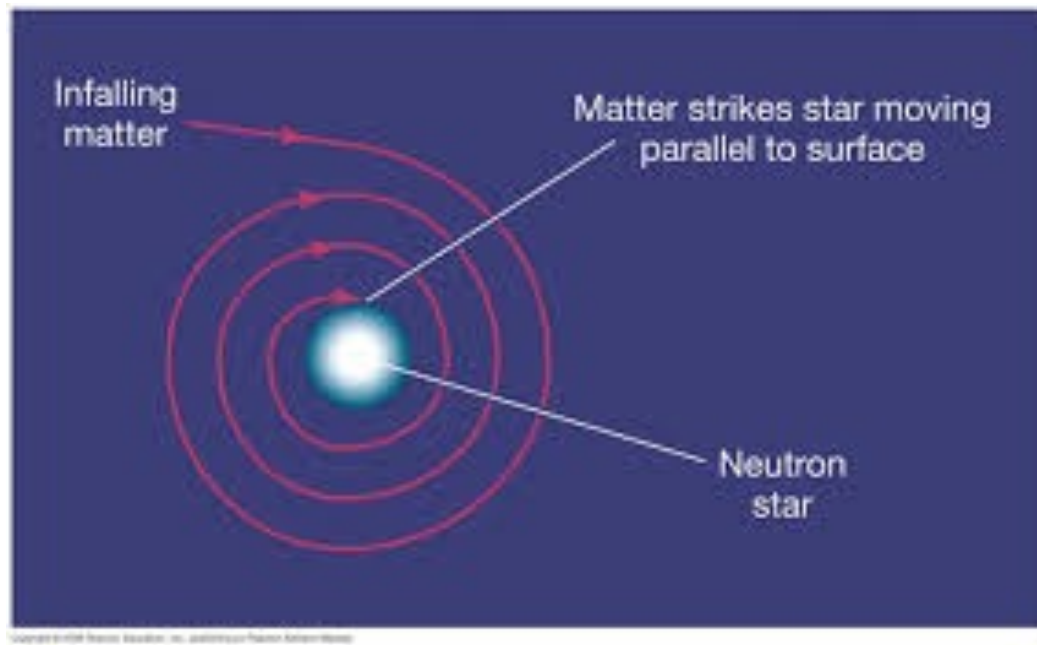
Pulse period evolution



Pulse period of the transient X-ray pulsar V 0332+53 during bright outbursts in 2015 and 2004.

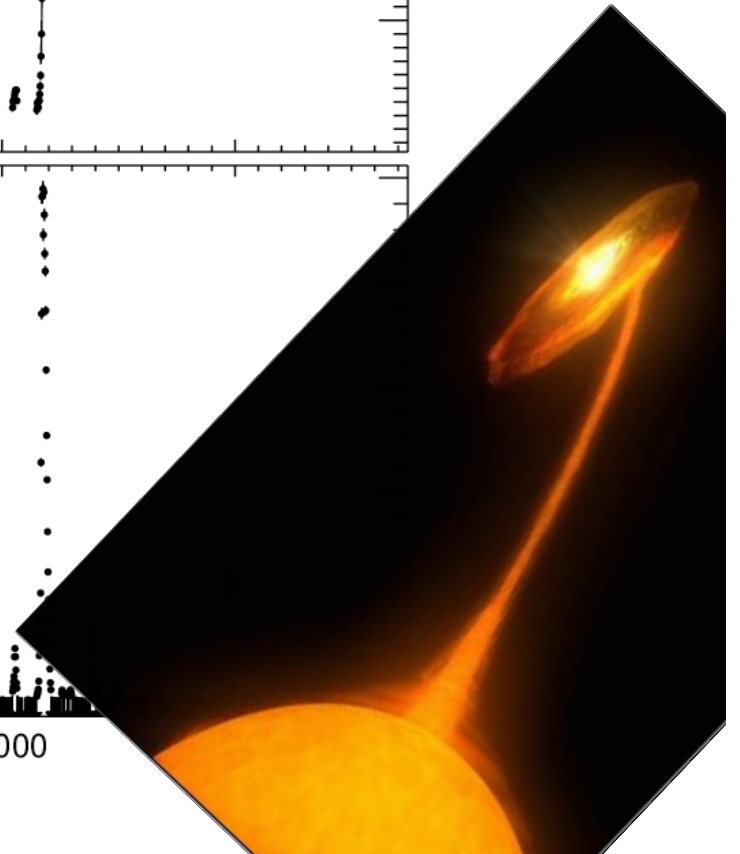
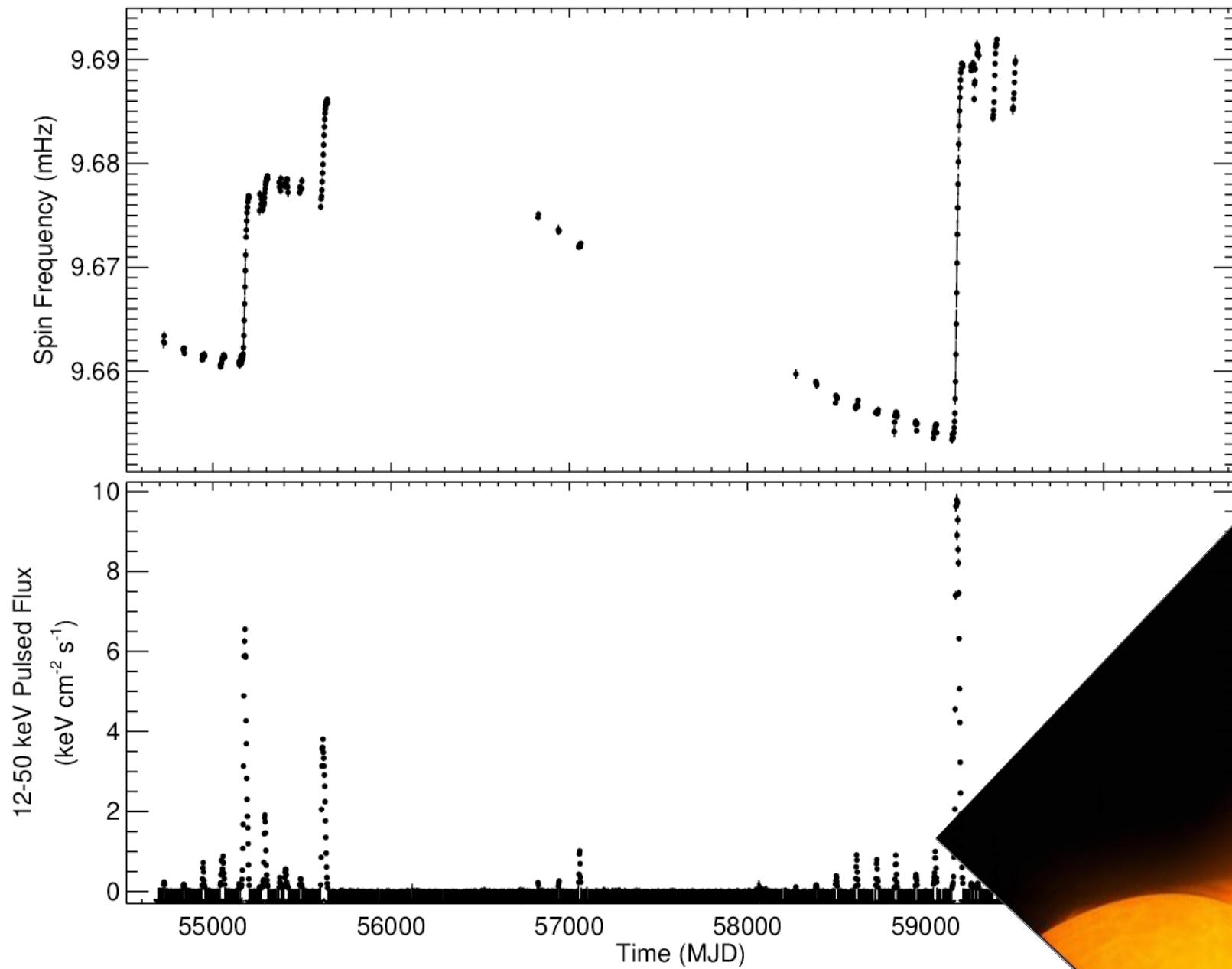
Pulse period evolution

Matter moving in the accretion disk carries a significant angular momentum. When matter interacts with the NS magnetic field, this angular momentum accelerates rotation of the star.

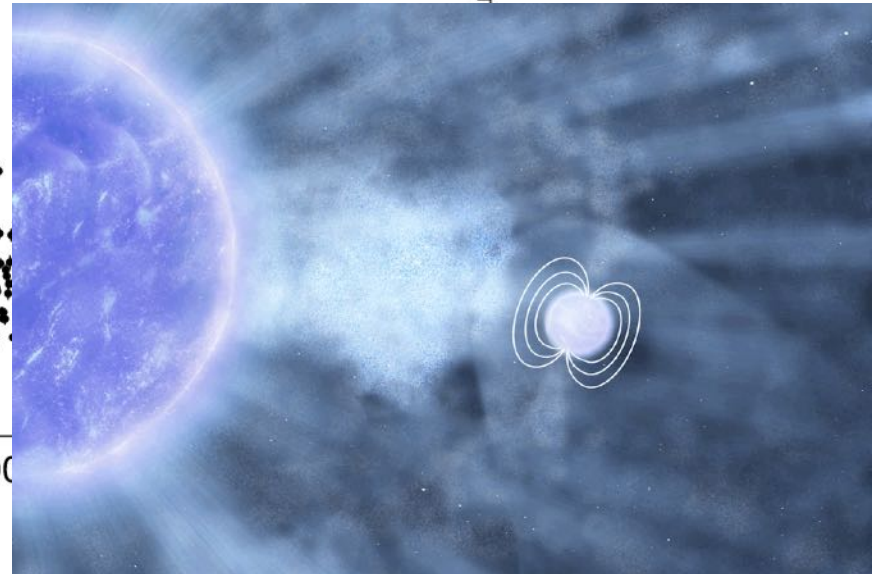
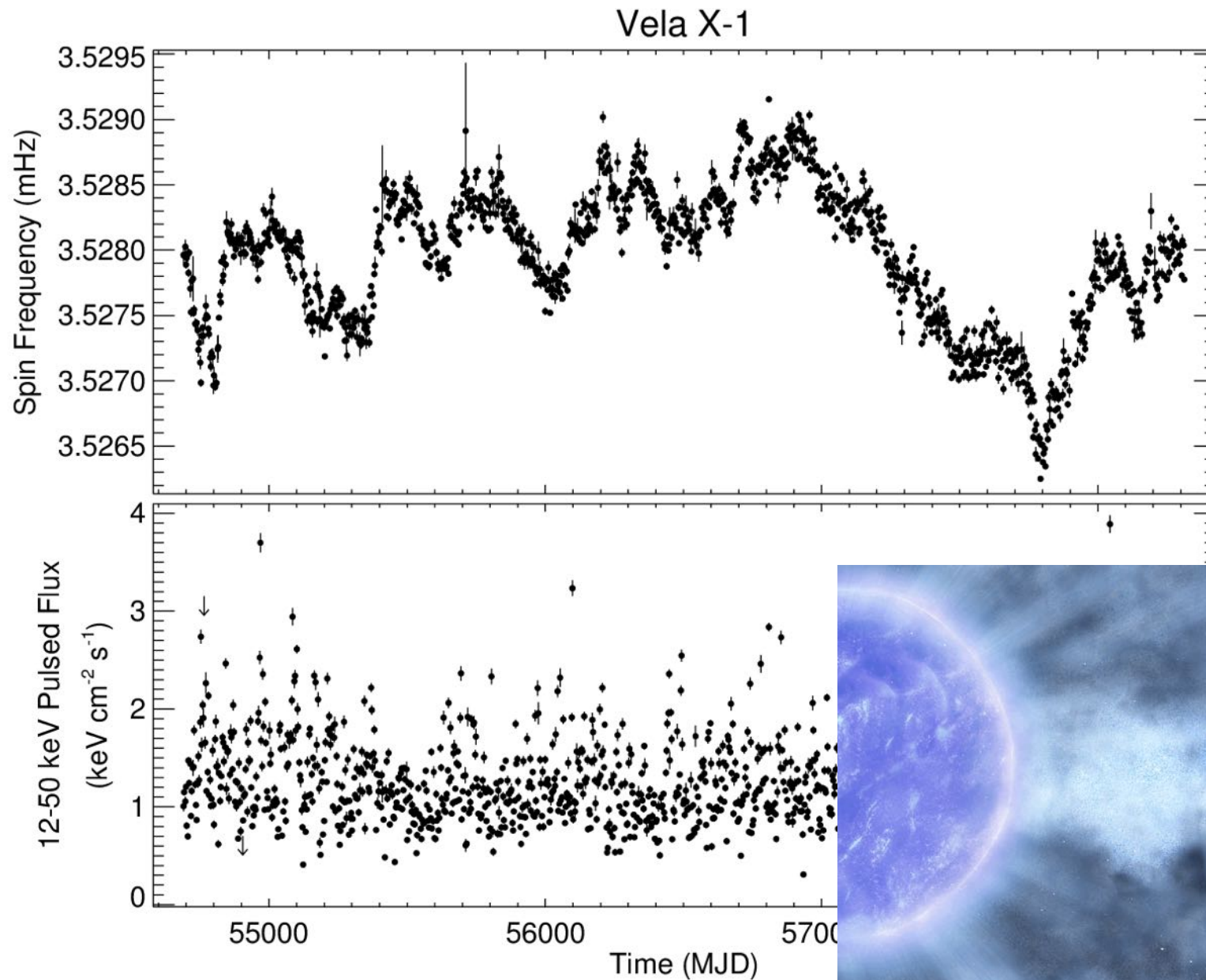


Pulse period evolution

A 0535+26



Pulse period evolution



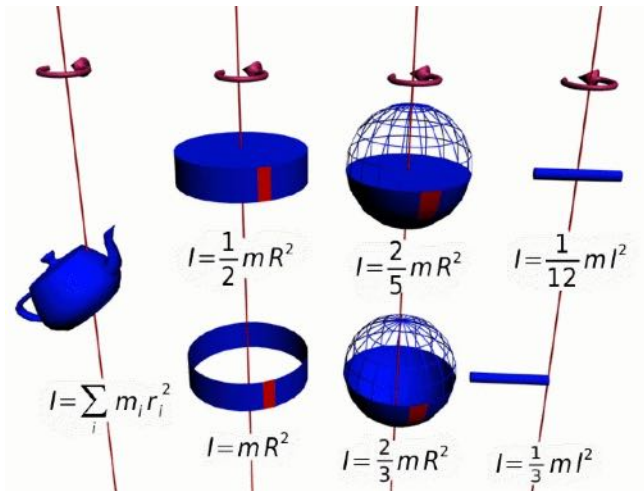
Pulse period evolution

Angular moment of matter in the accretion disc

$$L = I \times \omega = r^2 \times m \times \omega$$

Specific angular momentum

$$l = \frac{L}{m} = r \times V \quad \omega = \frac{V}{r}$$



$$l(r) = \sqrt{GM r} \quad V_K = \sqrt{\frac{GM}{r}}$$

I – moment of inertia

Torque (moment of force)

$$N_{\text{acc}} = \frac{dL}{dt} = \frac{dI\omega}{dt} = I \frac{d\omega}{dt}$$

Pulse period evolution

$$\omega = \frac{2\pi}{P} \qquad N_{\text{acc}} = \frac{dL}{dt} = \frac{dI\omega}{dt} = I \frac{d\omega}{dt}$$

$$N_{\text{acc}} = I \frac{d\omega}{dt} = -2\pi I \frac{\dot{P}}{P^2}$$

$$N_{\text{acc}} \approx \dot{M} l(r_{\text{m}}) \approx \dot{M} \sqrt{GM r_{\text{m}}}$$

$$-\dot{P} = \frac{N_{\text{acc}} P^2}{2\pi I} \approx \frac{\dot{M} \sqrt{GM r_{\text{m}}} P^2}{2\pi I}$$

$$R_{\text{m}} \simeq 2.5 \times 10^8 \, \xi \, M_{1.4}^{1/7} \, R_6^{10/7} \, B_{12}^{4/7} \, L_{37}^{-2/7} \, \text{cm}$$

Pulse period evolution

Prediction of the Ghosh and Lamb (1979) theory is a relation between the spin-up rate, the period, luminosity and magnetic moment:

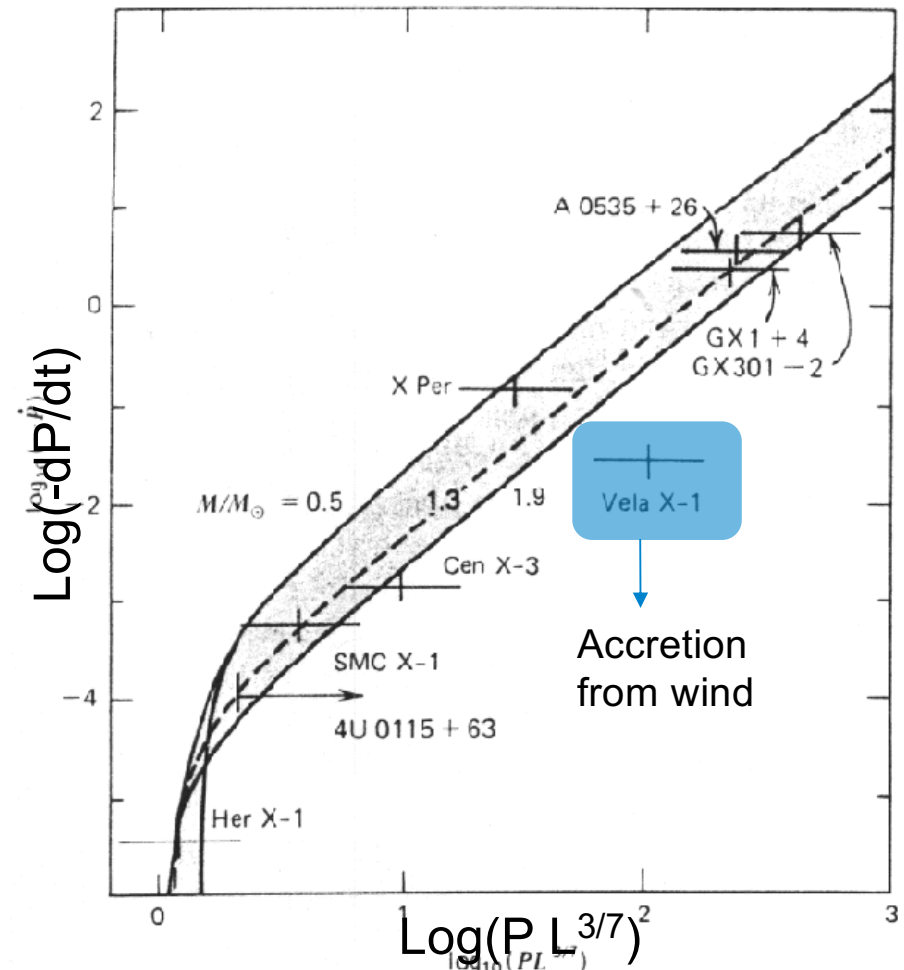
$$-\dot{P} = 5.0 \times 10^{-5} \mu_{30}^{2/7} n(\omega_s) S_1(M) (PL_{37}^{3/7})^2 \text{ s yr}^{-1}$$

$\mu = B_0 R^3 / 2$ – magnetic moment

$$-\dot{P} = f(PL^{3/7})$$

for given mass M and magnetic moment μ .

But if μ is unknown?

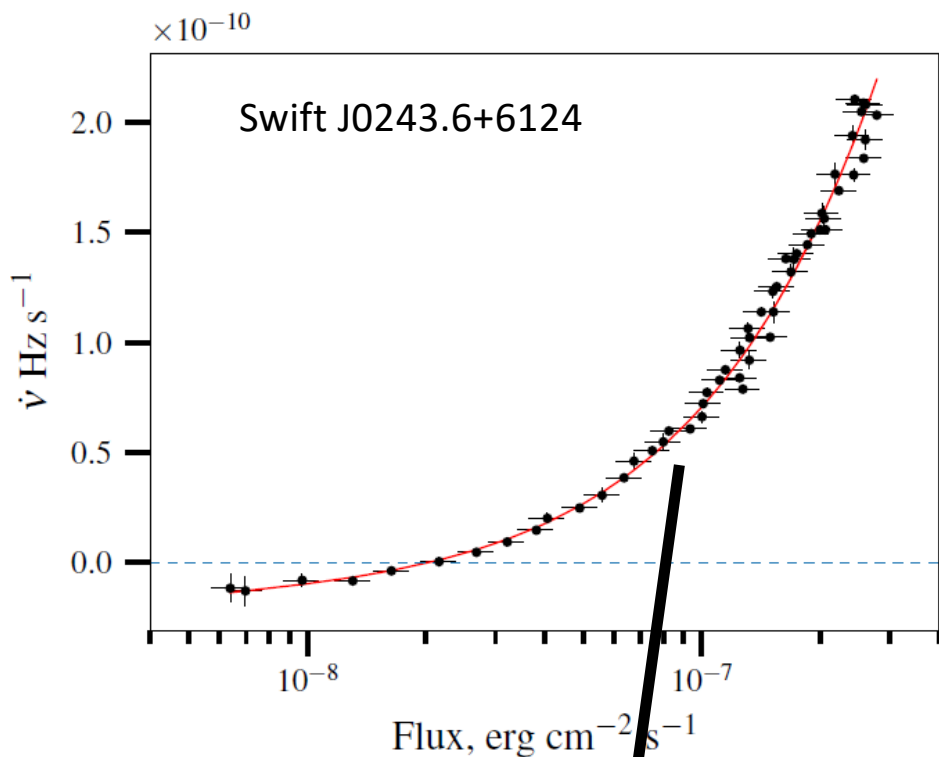


Shapiro, Teukolsky (1983)

Pulse period evolution

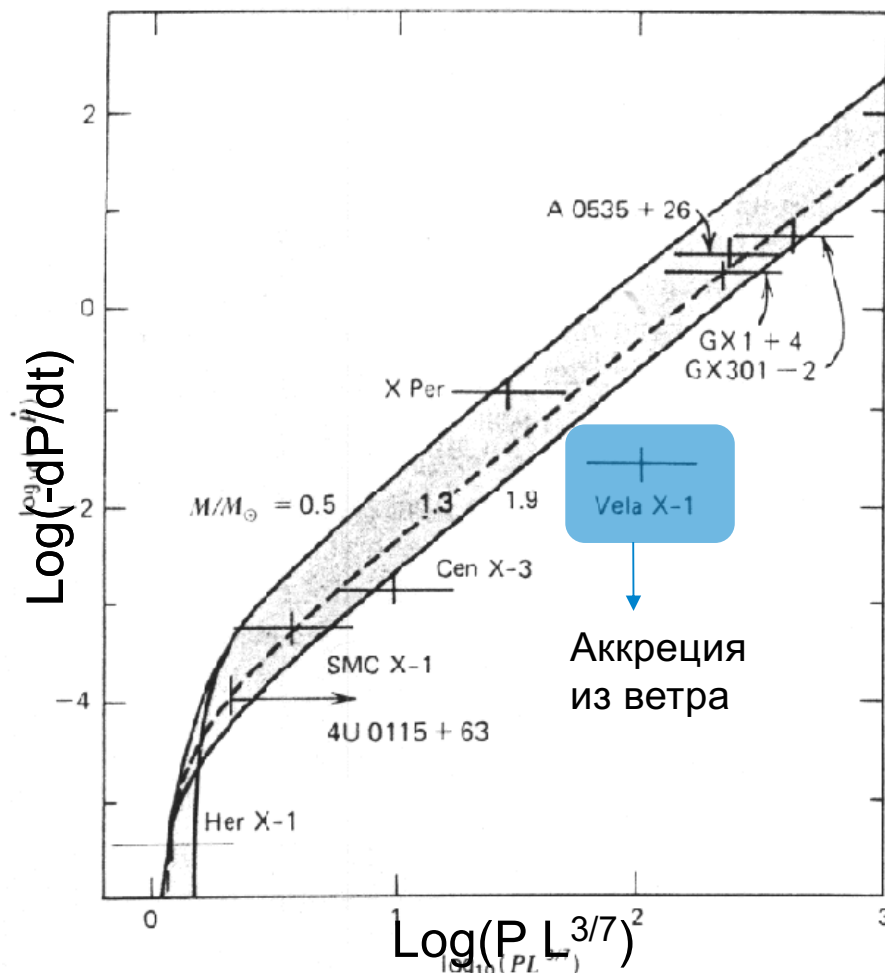
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$$-\dot{P} = 5.0 \times 10^{-5} \mu_{30}^{2/7} n(\omega_s) S_1(M) (PL_{37}^{3/7})^2 \text{ s yr}^{-1}$$



Doroshenko et al. (2018)

$B \sim 10^{13} \text{ G}$, $d \sim 7 \text{ kpc}$



Shapiro, Teukolsky (1983)

Spin-up time

$$-\dot{P} = \frac{N_{\text{acc}} P^2}{2\pi I} \approx \frac{\dot{M} \sqrt{GM r_m} P^2}{2\pi I}$$

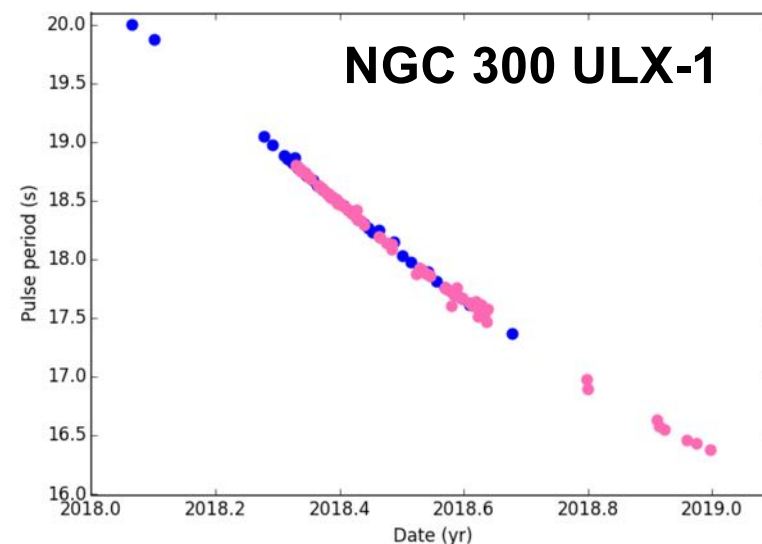
$$\boxed{\frac{|\dot{P}|}{P} \propto PL^{6/7}}$$

Spin-up time ($\ll$ pulsar age)

$$t_s = \frac{P}{|\dot{P}|} = 2^{17/14} \pi I P^{-1} \dot{M}^{-6/7} \mu^{-2/7} (GM)^{-3/7} =$$

$$2 \times 10^5 \text{ yr} \left(\frac{10^{-10} M_{\text{sun}} \text{ yr}^{-1}}{\dot{M}} \right) \left(\frac{1 \text{ s}}{P} \right)^{4/3} \left(\frac{R_{\text{co}}}{R_m} \right)^{1/2}$$

Note that about a Gy is needed to reach a ms period. For ULX pulsars (with $\dot{M} \sim 10^{-6} M_{\odot}$), the spin-up time is $\lesssim 100$ yr.



Spin equilibrium

If magnetospheric radius is smaller than corotational radius, then the accreting matter rotates faster than the star, which is spun-up as a result (and vice versa). The spin equilibrium is achieved when $R_m \approx R_{co}$:

$$L = \eta \dot{M} c^2, \text{ define } m = M / M_o, \dot{m} = \frac{\eta \dot{M} c^2}{L_{Edd}} = \frac{\eta \dot{M} c^2}{1.3 \cdot 10^{38} m}$$

$$1.5 \cdot 10^8 B_{12}^{4/7} R_6^{12/7} \dot{m}^{-2/7} m^{-3/7} = 1.5 \cdot 10^8 P_s^{2/3} m^{1/3}$$

$$P_s = B_{12}^{6/7} R_6^{18/7} \dot{m}^{-3/7} m^{-8/7} \text{ s}$$

For $\dot{m} = 1, R_6 = 1, m = 1.4$

$$P_{s,\min} = 0.7 \text{ s } B_{12}^{6/7} = 1.8 \text{ ms } B_9^{6/7}$$

- "spin-up" line

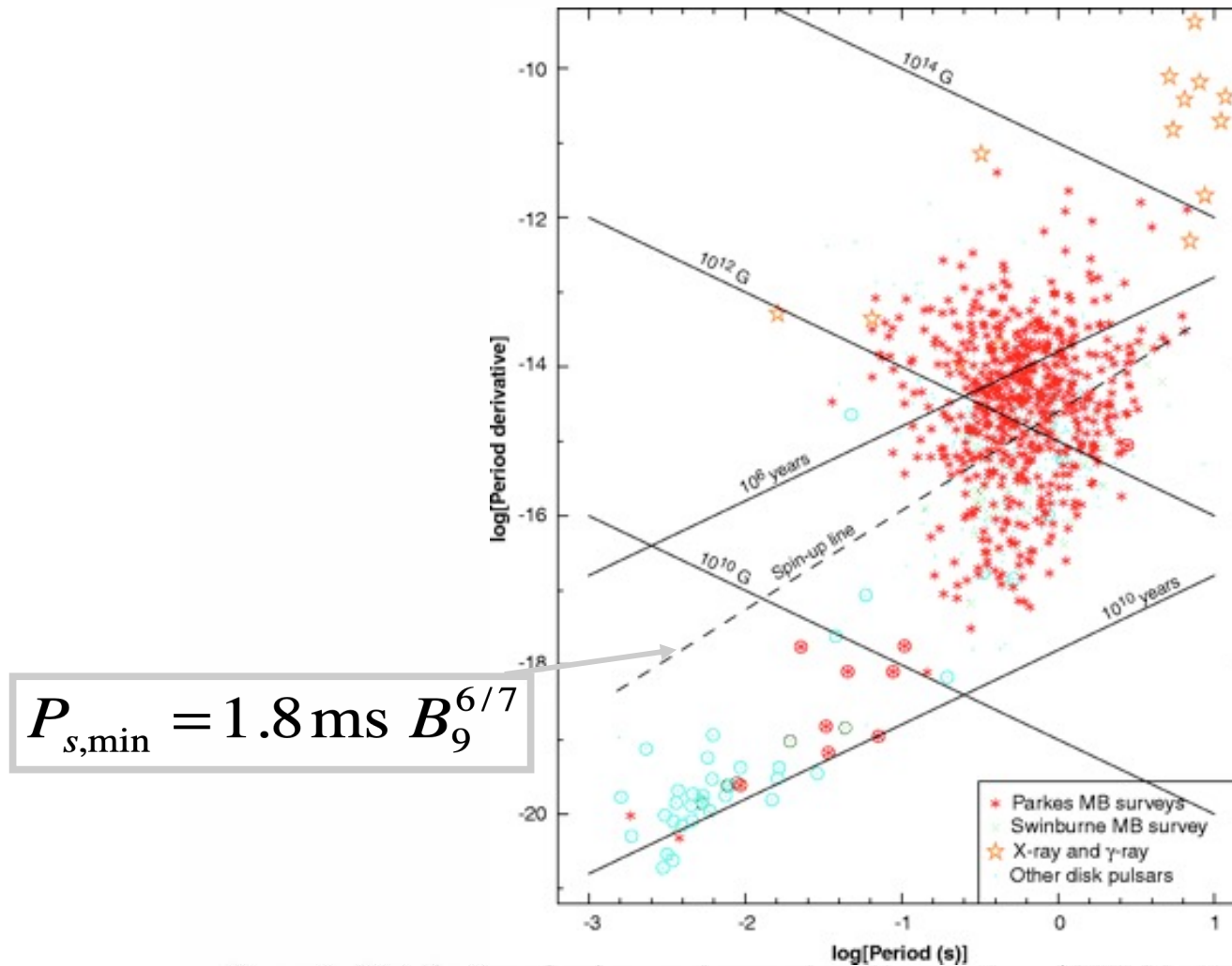


Figure 5: Distribution of pulsars and anomalous X-ray pulsars (AXPs) in the $P - \dot{P}$ plane. Binary systems are indicated by a circle around the point. Lines of constant pulsar characteristic age, $\tau_c = P/(2\dot{P})$ and surface dipole magnetic field strength, $B_s \propto (P\dot{P})^{1/2}$ are indicated. The spin-up line, representing the minimum period attainable by accretion from a binary companion, is also shown.

$$B \approx \sqrt{3c^3 I P \dot{P} / 2\pi^2 R^6} \text{ Gauss} \approx 6 \times 10^{19} \text{ G} \sqrt{P \dot{P}}$$

Eddington luminosity (Eddington limit)

radiation pressure = gravitational pull

At this point accretion stops, effectively imposing a ‘limit’ on the luminosity of a given body.

$$\frac{L\sigma_T}{4\pi r^2 c} = G \frac{Mm}{r^2}$$

So the Eddington luminosity (for pure H plasma) is:

$$L_{\text{Edd}} = \frac{4\pi G M m_p c}{\sigma_T} \approx 1.3 \times 10^{38} \frac{M}{M_{\odot}} \text{ erg s}^{-1}$$

Ultraluminous X-ray sources (ULXs)

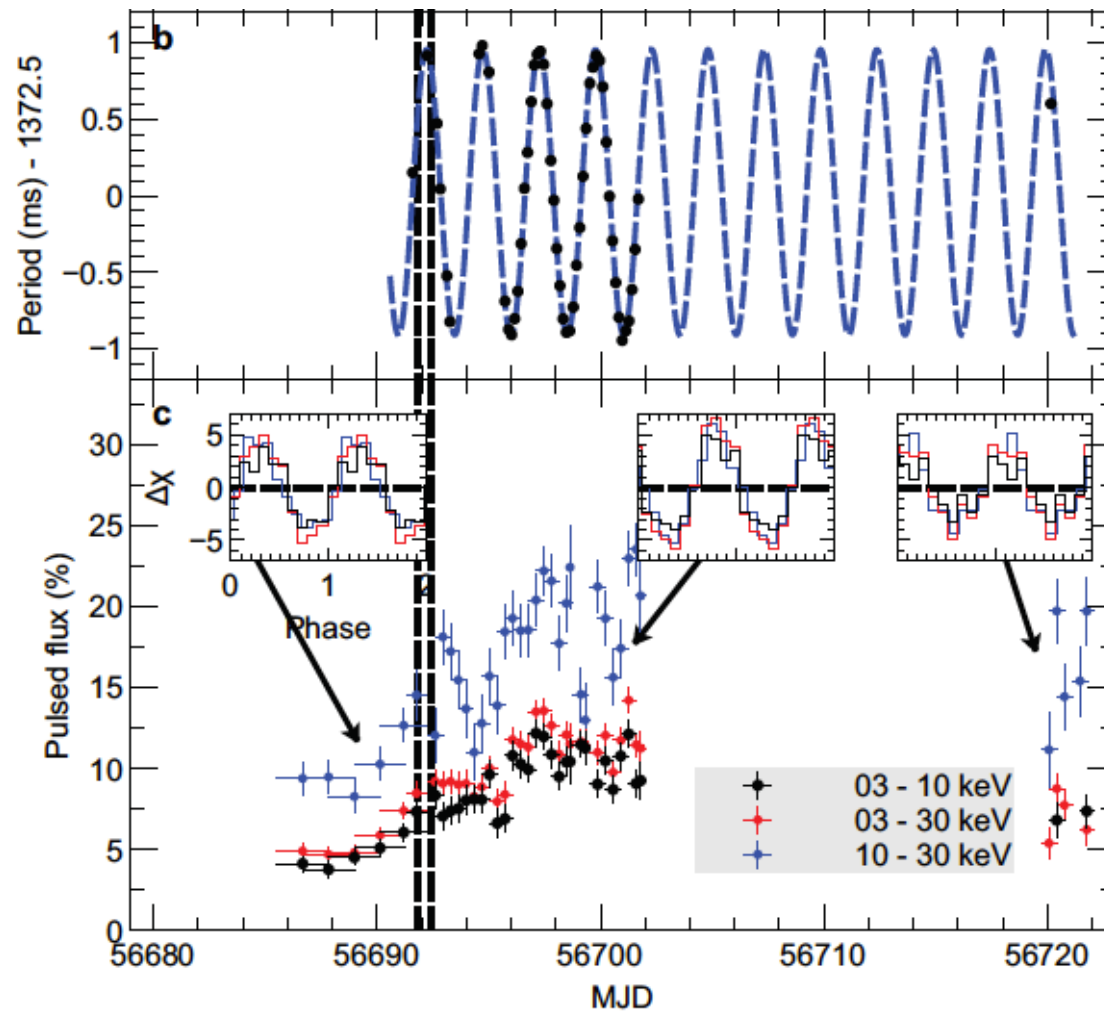
- Brightest extra-nuclear X-ray sources, with $L_X > 10^{39} \text{ erg s}^{-1}$
- Now around 500 candidates are known (Walton et al. 2011)

$10^{39} \text{ erg s}^{-1}$ is $\sim$ Eddington limit
for a $10 M_{\text{sun}}$ black hole

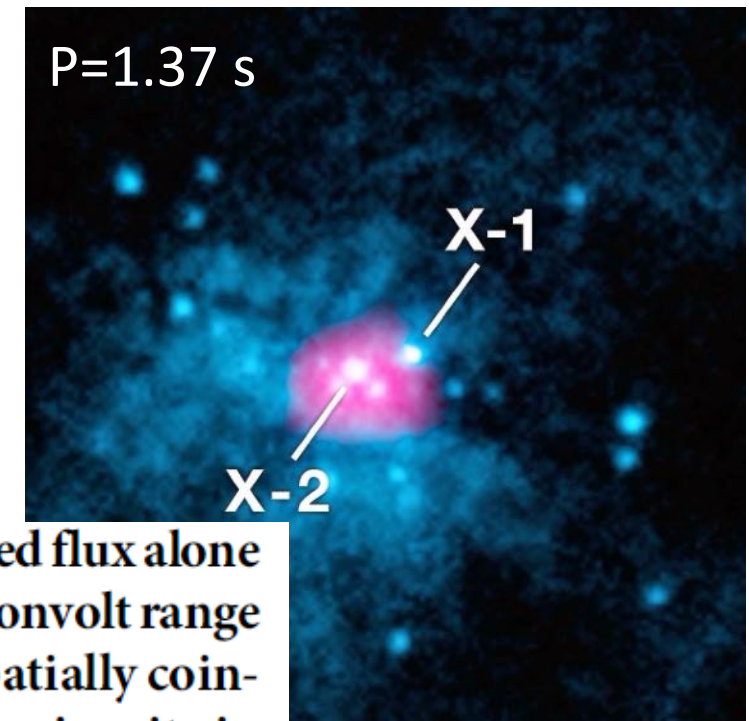


- ➔ **ULX:** either (1) a new class of intermediate-mass black holes with $M \sim 10^2 - 10^4 M_{\text{sun}}$ (IMBHs; e.g. Colbert & Mushotzky 1999) or (2) stellar mass black holes that exceed the Eddington limit, i.e. accrete in the most extreme environments (e.g. King 2001, Poutanen et al. 2007).

Discovery of pulsations from ULX in M82 galaxy



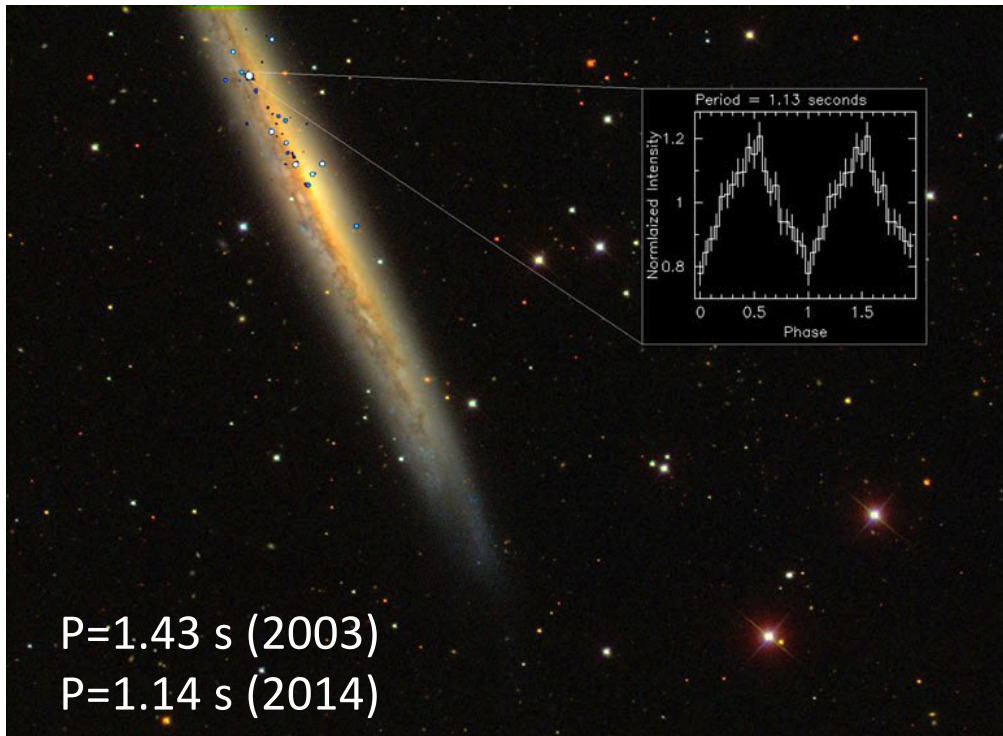
$L \sim 100 L_{\text{Edd}}$ for $1.4 M_{\text{sun}}$
mass object!



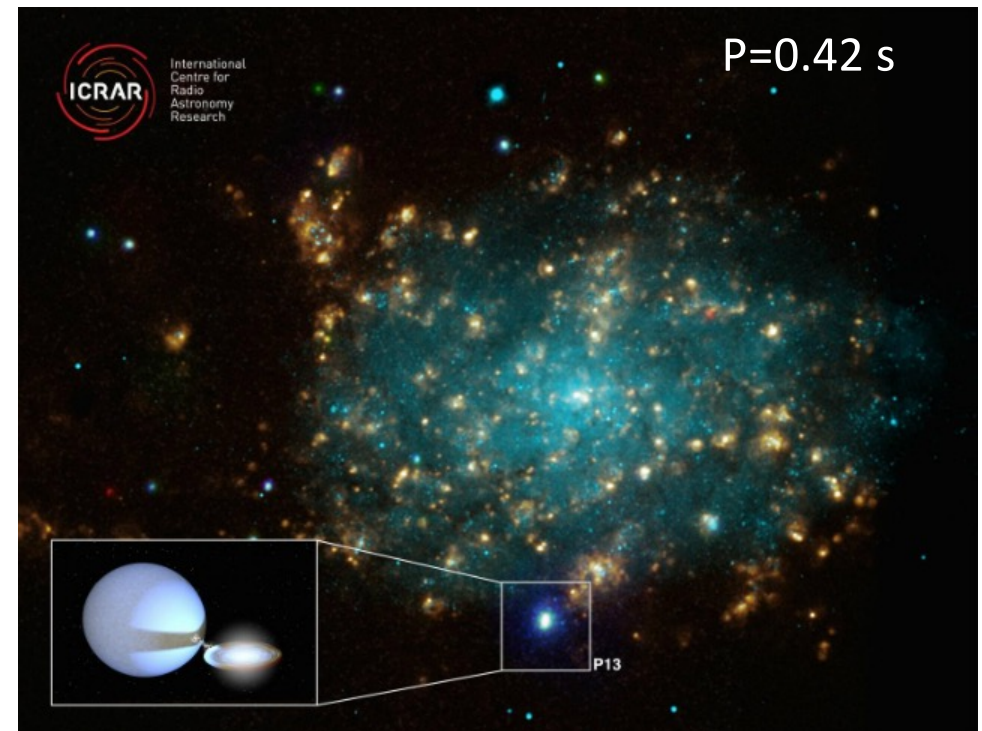
and the modulation arises from its binary orbit. The pulsed flux alone corresponds to an X-ray luminosity in the 3–30 kiloelectronvolt range of 4.9×10^{39} ergs per second. The pulsating source is spatially coincident with a variable source⁴ that can reach an X-ray luminosity in the 0.3–10 kiloelectronvolt range of 1.8×10^{40} ergs per second¹. This

Bachetti+ (2014)

More pulsating ULXs: NGC7793 P13 and ULX-1 NGC5907



$$L \sim 10^{41} \text{ erg s}^{-1}$$

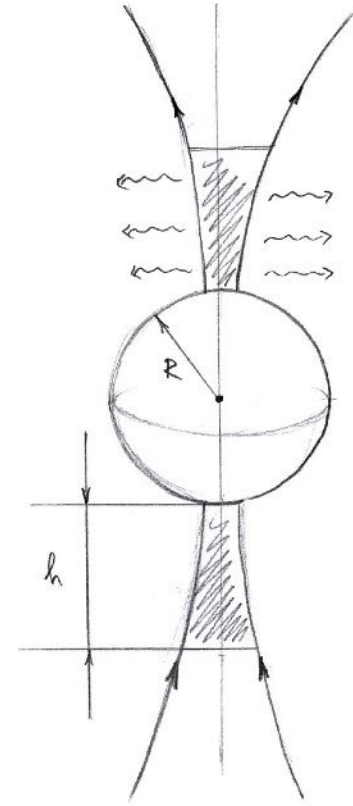
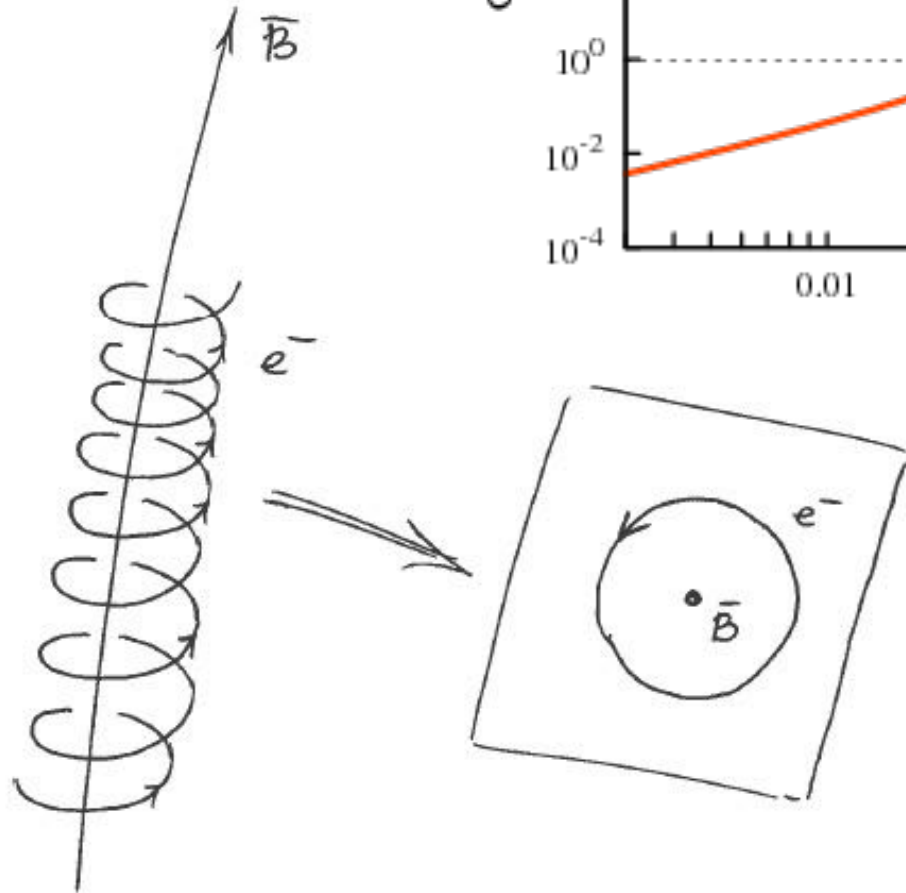
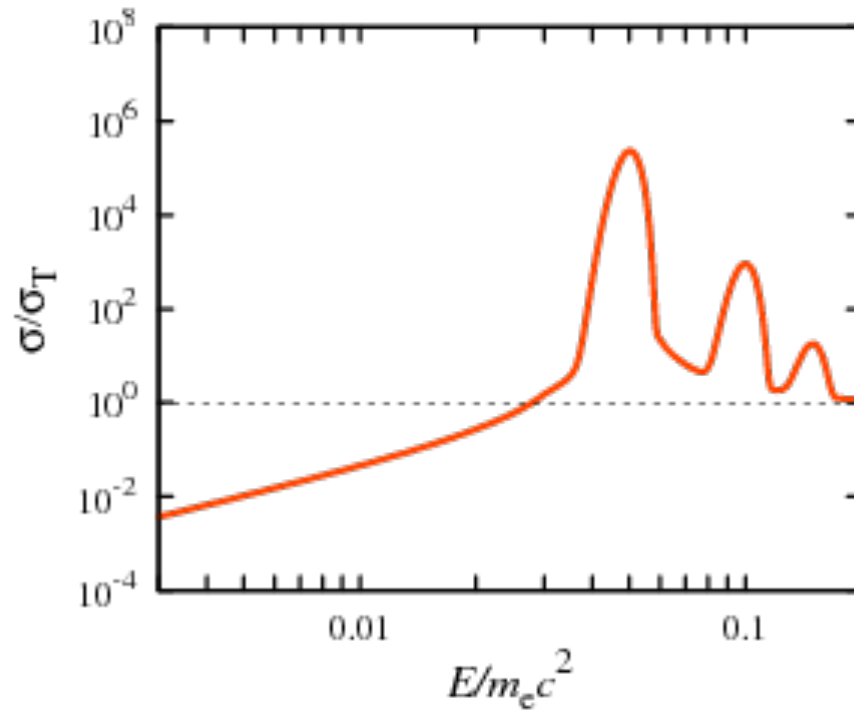


$$L \sim 5 \times 10^{39} \text{ erg s}^{-1}$$

Israel+, MNRAS 2017; Fuerst+, ApJ, 2017,
Israel+, Science, 2017

Maximal luminosity of X-ray pulsar

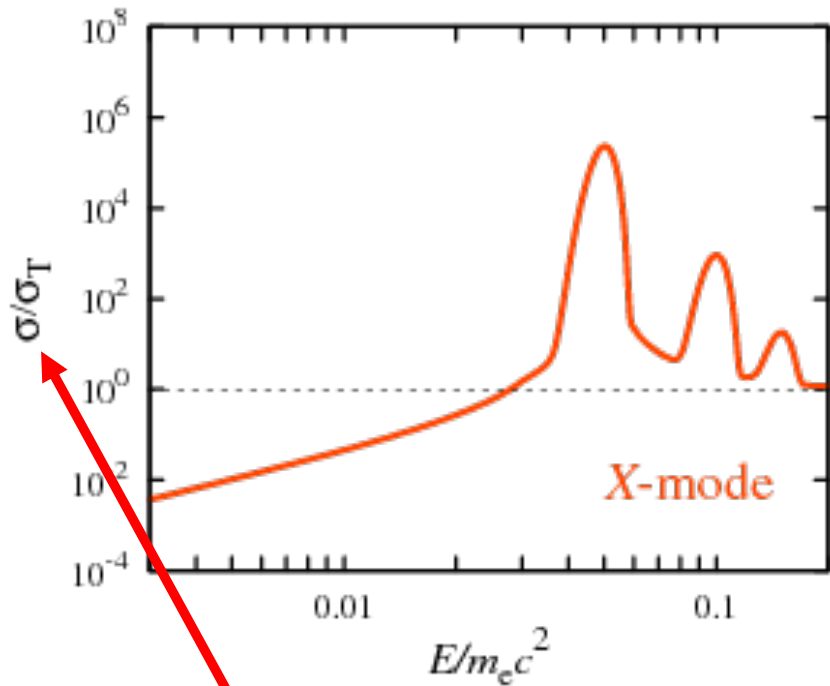
$$\frac{\sigma_{\perp}}{\sigma_T} \approx \left(\frac{E_{\gamma}}{E_{\text{cyc}}} \right)^2$$



$$L_{\text{Edd}} = \frac{4\pi GMm_p c}{\sigma_T}$$

$$\approx 1.3 \times 10^{38} \frac{M}{M_{\odot}} \text{ erg s}^{-1}$$

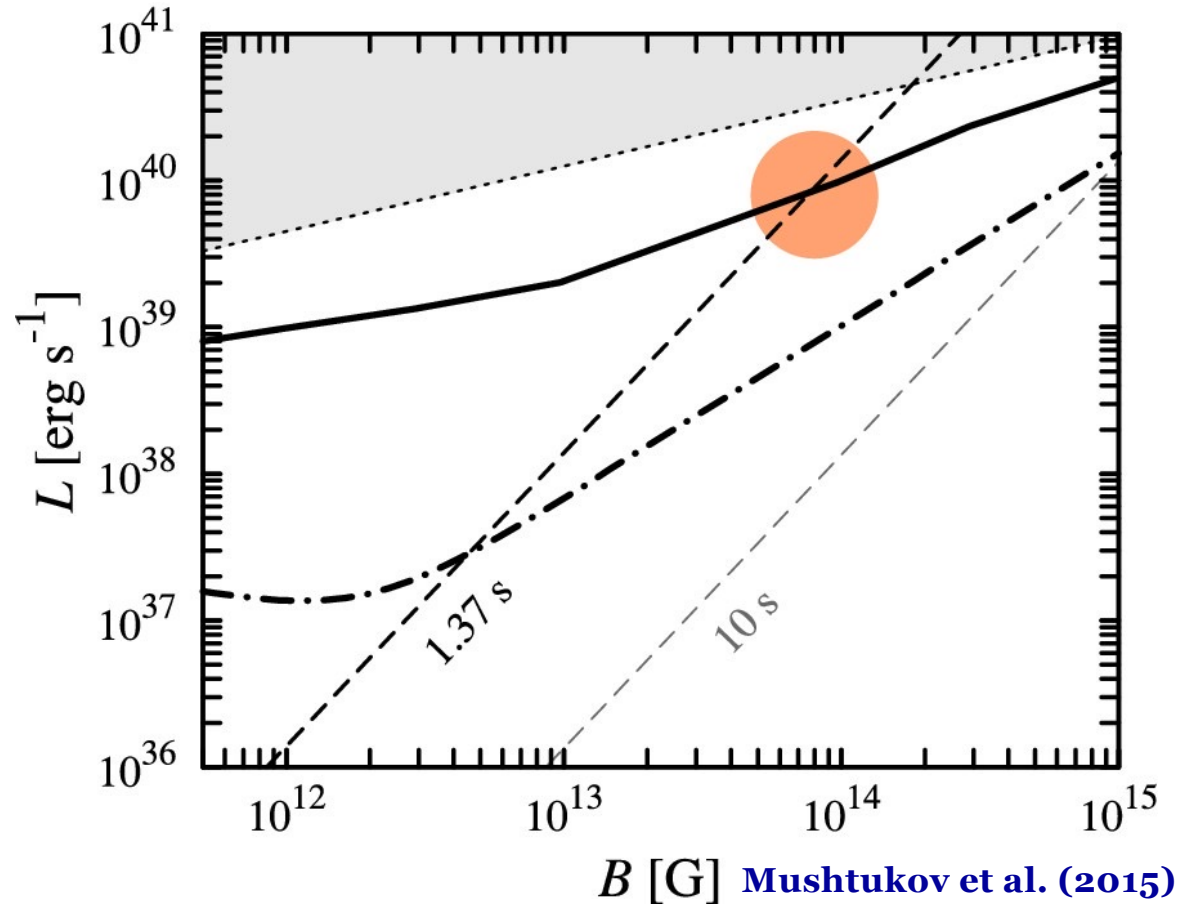
Maximal luminosity of X-ray pulsar



$$L_{\text{Edd}} = \frac{4\pi GMm_p c}{\sigma_T}$$

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$$\frac{\sigma_\perp}{\sigma_T} \approx \left(\frac{E_\gamma}{E_{\text{cyc}}} \right)^2$$

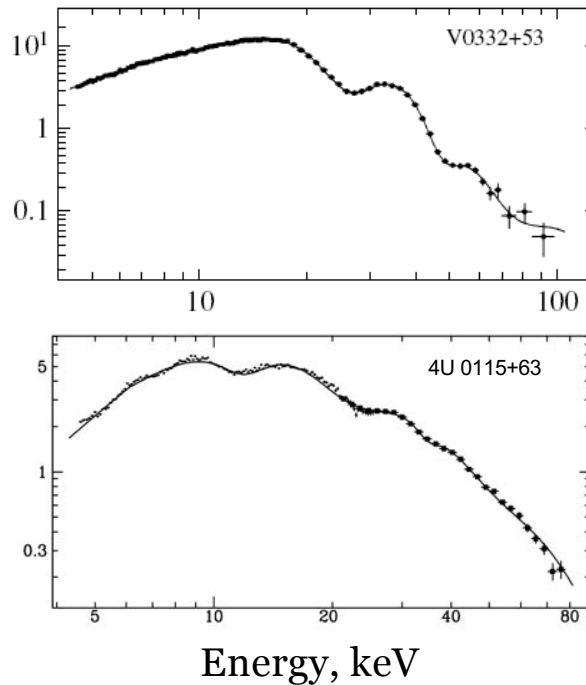


In the interval $10^{13} < B < 10^{15}$ G it can be approximated as $L_{39}^{**} \approx 0.35 B_{12}^{3/4}$



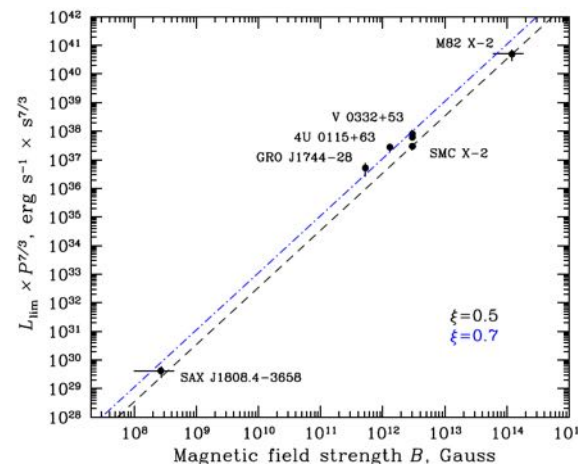
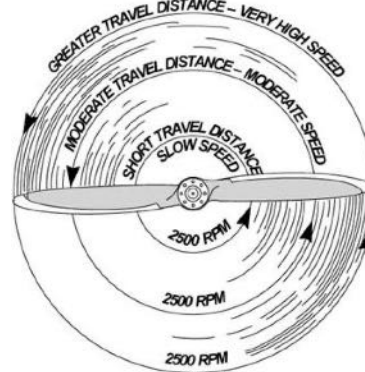
Neutron stars: the strongest and the brightest magnets in the Universe

*Cyclotron
absorption lines*



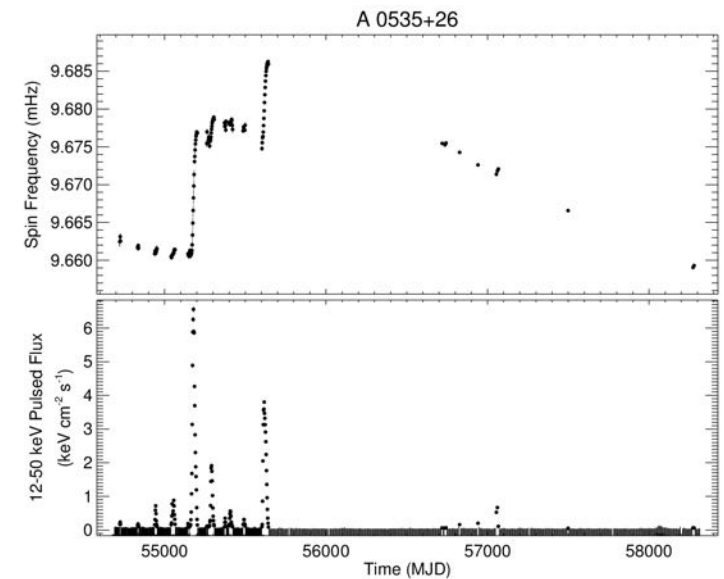
$$B_{12} = \frac{E_{\text{cyc}}}{11.2}$$

*“Propeller”
effect*



$$L_{\text{lim}}(R) \simeq \frac{GM\dot{M}_{\text{lim}}}{R} \simeq 4 \times 10^{37} \xi^{7/2} B_{12}^2 P^{-7/3} M_{1.4}^{-2/3} R_6^5 \text{ erg s}^{-1}$$

*Pulse period
evolution*



$$-\dot{P} = \frac{N_{\text{acc}} P^2}{2\pi I} \approx \frac{\dot{M} \sqrt{GM r_m} P^2}{2\pi I}$$