Collection of abstracts of the Analytic Number Theory Workshop

May 26-30, 2014

Turku

VORONIN UNIVERSALITY OF SOME DOUBLE DIRICHLET SERIES

Johan Andersson, *Stockholm University*

We discuss Voronin universality in two complex variables and compare our previous results on universality of Euler-Zagier type multiple zeta-functions with new results on universality for certain double Dirichlet series of Weyl group multiple Dirichlet series type. While for the Euler-Zagier multiple zeta case we require both variables to remain between the half line and the 1-line in the critical strip, added freedom in the proof in the double Dirichlet series case allows us to prove universality when the second variable may lie in the half plane $\text{Re}(s) > 1/2$. In particular this shows that for any $1/2 < \sigma < 1$ and $N > 1$ there exists some w with $\text{Re}(w) = \sigma$ such that $Z(w, s)$ has $\gg T$ zeroes for $\{s \in \mathbb{C} : |\text{Im}(s)| \leq T, \text{Re}(s) > N\}$, where $Z(w, s)$ denote the double Dirichlet series studied by Siegel and Goldfeld-Hoffstein.

SUP NORMS OF AUTOMORPHIC FORMS IN THE CUSP

Farrell Brumley, *Université Paris 13*

There has been a lot of recent activity on the problem of estimating the sup norm of L^2 normalized automorphic forms. Most attention has been devoted to the compact setting, where there are firm conjectures to work with. In recent preprint, N. Templier and I have explored the question of what happens without the hypothesis of compacity. Cusp forms on non-compact finite volume locally symmetric spaces decay rapidly at infinity, but before doing so, they evince a sort of automorphic Gibbs phenomenon, attaining their largest value before dying. We are able to quantify this for $SL_n(Z)$ and find bounds on a different scale in comparison with the compact setting.

ON TWO PROBLEMS CONCERNING THE LAURENT STIELTJEST COEFFICIENTS OF DIRICHLET L-SERIES

Sumaia Saad Eddin, *Laboratoire Paul Painlevé, Université des Sciences et Technologie de Lille*

The Laurent-Stieltjes constants $\gamma_n(\chi)$ are, up to a trivial coefficient, the coefficients of the Laurent expansion of the usual Dirichlet L-series : when χ is a non-principal, $(-1)^n\gamma_n(\chi)$ is simply the value of the n-th derivative of $L(z, \chi)$ at $z = 1$.

The interest in these constants has a long history (started by Stieltjes in 1885). Among the applications, let us cite: determining zero-free regions for Dirichlet L-functions near the real axis in the critical strip $0 \leq \Re(z) \leq 1$, computing the values of the Riemann and Hurwitz zeta functions in the complex plane and studying the class number of the quadratic field, etc. In this talk, I will give explicit upper bounds for the Laurent-Stieltjes constants in the following two cases:

- The character χ is fixed and the order n goes to infinity.
- The order n is 0 and the modulus q goes to infinity.

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LOW-LYING ZEROS OF ELLIPTIC CURVE L-FUNCTIONS: BEYOND THE RATIOS CONJECTURE

Daniel Fiorilli, *University of Michigan*

We study the 1-level density of low-lying zeros of the L-functions attached to the family of quadratic twists E_d of a given elliptic curve E defined over $\mathbb{Q}$. For test functions whose Fourier transforms have sufficiently restricted support, we obtain an error term that is significantly sharper than the square-root cancellation predicted by the L-functions ratios conjecture of Conrey, Farmer and Zirnbauer.

DISPROOF OF A CONJECTURE BY RADEMACHER ON PARTIAL FRACTIONS

Stefan Gerhold,*Vienna University of Technology*

In his book Topics in Analytic Number Theory, Rademacher considered the generating function of partitions into at most N parts, and conjectured certain limits for the coefficients of its partial fraction decomposition. We carry out an asymptotic analysis that disproves this conjecture, thus confirming recent observations of Sills and Zeilberger (Journal of Difference Equations and Applications 19, 2013), who gave strong numerical evidence against the conjecture.

ON THE SUP-NORM PROBLEM FOR ARITHMETIC HYPERBOLIC 3-MANIFOLDS

***Gergely Harcos**, Rényi Institute*

The systematic study of the sup-norm problem for Hecke-Maass forms on arithmetic hyperbolic manifolds was initiated by Iwaniec and Sarnak 20 years ago. The talk will discuss new upper bounds in the 3-dimensional case, which are as strong as the best corresponding results in the 2-dimensional case. The proof is based on a novel combination of diophantine and geometric arguments in a noncommutative setting. Joint work with Valentin Blomer and Djordje Milicevic.

INVERSE QUESTIONS FOR THE LARGE SIEVE

***Adam Harper**, University of Cambridge*
joint work with Ben Green

Suppose A is a subset of the natural numbers less than N, and A occupies at most $(p+1)/2$ residue classes modulo all odd primes p. Then the large sieve implies that $|A| \ll \sqrt{N}$, which is sharp because of the example where A is a set of squares. The "inverse conjecture for the large sieve" proposes, amongst other things, that the only examples of sets A where the $\sqrt{N}$ bound is sharp are sets that "look a lot like the squares".

In this talk I will try to explain some recent results towards the inverse conjecture, that give (at least) a small power saving over the $\sqrt{N}$ bound under extra assumptions on the structure of the residue classes occupied by A.

A SIDEWAYS APPROACH TO THE CIRCLE PROBLEM

***Martin Huxley**, University of Cardiff*

The average of the sum-of-two-squares function $r(n)$ can be read as counting integer points in a circle with centre at the origin. Moving the centre, or changing the shape, hasn't helped yet, but it leads to some interesting problems.

ON EQUIDISTRIBUTION OF SIGNS AND THE SATO-TATE CONJECTURE

Ilker Inam,*Bilecik Seyh Edebali University*
Joint work with Gabor Wiese and Sara Arias-de-Reyna

Let f be a cusp form of weight $k+1/2$ and at most quadratic nebentype character whose Fourier coefficients $a(n)$ are all real. The aim of this talk, to give a proof of an equidistribution conjecture of Bruinier and Kohnen for the signs of $a(n)$ for certain subfamilies of coefficients that are accessible via the Shimura lift by using the Sato-Tate equidistribution theorem for integral weight modular forms. Firstly, an unconditional proof is given for the family $\{a(tp^2)\}_p$ where t is a squarefree number and p runs through the primes. In this case, the result is in terms of the natural density. To prove it for the family $\{a(tn^2)\}_n$ where t is a squarefree number and n runs through all natural numbers, we assume the existence of a suitable error term for the convergence of the Sato-Tate distribution, which is like in the prime number theorem. In this case, the results are again in terms of the natural density.

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ZETA MOMENTS IN SHORT INTERVALS

Aleksandar Ivić, *University of Belgrade*

Moments of $|\zeta(1/2+it)|$ in short intervals are investigated, and old and new results are presented. Connections with $\Delta(x)$, the error term in the classical Dirichlet divisor problem are pointed out. This involves the function, introduced by M. Jutila,

$$E^*(T) := E(T) - 2\pi\Delta^*(T/(2\pi)),$$

where $E(T)$ is the error term in the mean square formula for $|\zeta(1/2+it)|$, and

$$\begin{aligned}\Delta^*(x) :&= -\Delta(x) + 2\Delta(2x) - \frac{1}{2}\Delta(4x)\\ &= \frac{1}{2}\sum_{n\le 4x}(-1)^n d(n) - x(\log x + 2\gamma - 1).\end{aligned}$$

It turns out that $2\pi\Delta^*(T/(2\pi))$ is in some sense the appropriate analogue of $E(T)$, due to F.V. Atkinson's classic formula (1949) for $E(T)$.

A RESONANCE ESTIMATE AND RELATED Ω-RESULT

Jesse Jääsaari, *University of Helsinki*
joint work with Anne-Maria Ernvall-Hytönen and Esa Vesalainen

We consider short exponential sums weighted by Fourier coefficients of SL$(n, \mathbb{Z})$ Maass wave forms. We show that in a particular case we get a sharp bound for the sum. As an application we prove a lower bound for the sum of Fourier coefficients $A(m, 1, ..., 1)$ over a very short interval.

RIEMANN'S ZETA-FUNCTION AND THE DIVISOR FUNCTION

Matti Jutila, *University of Turku*

Connections between Riemann's zeta-function and the divisor function $d(n)$ are considered in the light of sum formulae of the Voronoi type and Atkinson's formula for the mean square of the zeta-function.

NODAL LENGTH STATISTICS FOR ARITHMETIC RANDOM WAVES

Pär Kurlberg, *KTH*

Using spectral multiplicities of the Laplacian acting on the standard two-torus, we endow each eigenspace with a Gaussian probability measure. This induces a notion of a random eigenfunction on the torus, and we study the statistics of nodal lengths of the eigenfunctions in the high energy limit. In particular, we determine the variance for a generic sequence of energy levels, and also find that the variance can be different for certain "degenerate" subsequences. (These degenerate subsequences are closely related to circles on which lattice points are very badly distributed.)

SHIFTED CONVOLUTION OF CUSP-FORMS WITH θ-SERIES

Guanshi Lü, *Shandong University*

In this talk, we introduce a simple approach to improve a recent result due to Luo, concerning a shifted convolution sum involving the Fourier coefficients of cusp forms with those of theta series.

EIGENVALUES AND EIGENVECTORS OF SOME HANKEL MATRICES RELATED TO THE ZETA AND L-FUNCTIONS

Yu. V. Matiyasevich, *St.Petersburg Department of V. A. Steklov Institute of Mathematics of Russian Academy of Sciences*
http://logic.pdmi.ras.ru/~yumat/personaljournal/zetahiddenlife

In 2007 the speaker ([1, 2]) reformulated the Riemann Hypothesis as statements about the eigenvalues of certain Hankel matrices, entries of which are defined via the Taylor series coefficients of the zeta function. Numerical calculations revealed some very interesting visual patterns in the behaviour of the eigenvalues and allowed the speaker to state a number of new conjectures related to the Riemann Hypothesis.

Recently computations has been extended to Dirichlet L-functions and were performed on more powerful computers. This led to new conjectures about the finer structure of the eigenvalues and eigenvectors, about non-evident relations among Taylor coefficients of the zeta and L-functions and to conjectures that are (formally) stronger than RH (partly this is presented in [3]).

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ON SINGULAR LCM MATRICES AND HONG'S CONJECTURES

Mika Mattila, *University of Tampere*
joint work with Pentti Haukkanen and Jori Mäntysalo

The invertibility of LCM matrices and their Hadamard powers have been studied a lot over the years by many authors. Bourque and Ligh conjectured in 1992 that the LCM matrix $[S] = [[x_i, x_j]]$ on any GCD closed set

$S = \{x_1, x_2, \ldots, x_n\}$ is invertible, but in 1997 this was proven false by Haukkanen et al. However, currently there are many open conjectures concerning LCM matrices and their real Hadamard powers presented by Hong. In this presentation we utilize lattice-theoretic structures and the Möbius function to explain the singularity of classical LCM matrices and their Hadamard powers. At the same time we end up disproving some of Hong's conjectures. We apply the mathematics software Sage to show that every 8-element GCD closed set S, for which the LCM matrix [S] is singular, has the same semilattice structure. We also construct a GCD closed set S of odd numbers such that the LCM matrix [S] is singular. Elementary mathematical analysis is applied to prove that for most semilattice structures there exist a set $S = \{x_1, x_2, \ldots, x_n\}$ of positive integers and a real number $\alpha > 0$ such that S possesses this structure and the power LCM matrix $[[x_i, x_j]^\alpha]$ is singular.

SMALL GAPS BETWEEN PRIMES

***James Maynard**, Université de Montréal*

We will introduce a refinement of the 'GPY sieve method' for studying these problems. This refinement will allow us to show (amongst other things) that $\liminf_n(p_{n+m} - p_n) < \infty$ for any integer m, and so there are infinitely many bounded length intervals containing m primes.

UNIFORM AUTOMORPHIC EXTENSION OF LINNIK'S PRIME NUMBER THEOREM

Yoichi Motohashi

An automorphic extension of the Linnik's least prime number theorem is to be presented. By 'automorphic extension' I mean that I deal with Fourier coefficients of Maass cusp forms (or rather the coefficients of Rankin L-functions). The result is [completely] uniform with respect to the underlying cusp forms and the moduli. The argument is an extension of my (ancient) proof of Linnik's theorem; it relies on a form of the large sieve plus the theory of symmetric product L-functions.

EXTREME VALUES OF L-FUNCTIONS FROM THE SELBERG CLASS

Łukasz Pańkowski, *Adam Mickiewicz University*

We prove estimates for extreme values of L-functions from the Selberg class under assumption of the corresponding analogue of the Riemann hypothesis. The method of proof combines Montgomery's approach with an effective version of Kronecker's diophantine approximation theorem due to Weber.

COUNTING HYPERBOLIC LATTICE POINTS FOR THE MODULAR GROUP

Morten Risager, *University of Copenhagen*

The hyperbolic lattice point counting problem concerns estimating the number of point of the orbit Gz in a hyperbolic ball of radius X. We discuss the classical and interesting case of G equal to the full modular group. The correct main term is known since Huber (at least) and has order X. Selberg proved an error term of order $O(X^{2/3})$ which has not been improved for any group. We show how making a spacial average allows us to improve the error-term (on average). The proof uses surprisingly strong arithmetic input like Lindelöf on average and bounds on spectral exponential sums.

We also discuss what should be the correct error term, and discuss a conjecture on a spectral exponential sum which implies the optimal bound.

LOW-LYING ZEROS OF DEDEKIND ZETA FUNCTIONS

Anders Södergren, *University of Copenhagen*
joint work with Arul Shankar and Nicolas Templier

In this talk we discuss the distribution of low-lying zeros of certain families of Dedekind zeta functions. In particular, we present a general setup for the study of low-lying zeros of families of Dedekind zeta functions of S_n-number fields and we illustrate our method by discussing several explicit examples.

On the Zeros of the k-th Derivative of the Riemann Zeta Function under the Riemann Hypothesis

Ade Irma Suriajaya, *Nagoya University*

The number of zeros and the distribution of the real part of non-real zeros of the derivatives of the Riemann zeta function have been investigated by Berndt, Levinson, Montgomery, and Akatsuka. Berndt, Levinson, and Montgomery investigated the general case, meanwhile Akatsuka gave sharper estimates for the first derivative of the Riemann zeta function under the truth of the Riemann hypothesis. In this talk, we shall introduce the generalization of the results of Akatsuka to the k-th derivative (for positive integer k) of the Riemann zeta function, that is, we give sharper estimates than those of Berndt, Levinson, and Montgomery for all derivatives of the Riemann zeta function under the assumption of the Riemann hypothesis.

Short linear exponential sums related to Maass forms

Esa V. Vesalainen, *University of Helsinki*
joint work with Jesse Jääsaari

In this talk we shall discuss the problem of estimating short linear exponential sums involving the Fourier coefficients of a Maass form. More precisely, if $t(n)$ are the Fourier coefficients of a Maass form for $\mathrm{SL}(2,\mathbb{Z})$, we are interested in exponential sums of the form

$$\sum_{M\leqslant n\leqslant M+\Delta} t(n)\, e(n\alpha),$$

where M is a large real number, $\Delta \in [1, M]$, and the estimates are to be uniform in $\alpha \in \mathbb{R}$.

On probability measures arising from lattice points on circles

Igor Wigman, *King's College London*
joint work with Pär Kurlberg

A circle, centered at the origin and with radius chosen so that it has non-empty intersection with the standard integer lattice, gives rise to a probability measure on the unit circle in a natural way. Such measures, and their weak limits, are said to be attainable from lattice points on circles.

We investigate the set of attainable measures and show that it contains all extreme points, in the sense of convex geometry, of the set of all probability measures that are invariant under some natural symmetries. Further, the set of attainable measures is closed under convolution, yet there exist symmetric probability measures that are not attainable. To prove this we study the geometry of projections onto a finite number of Fourier coefficients, and fully characterize a neighbourhood of a certain symmetrized delta measure. Outside this neighbourhood, the attainable measure can have quite complicated "fractal" singularities. Our results imply that these arise from prime powers - singularities do not appear for circles of radius $\sqrt{n}$ when n is square free.

CARMICHAEL NUMBERS: ELLIPTIC AND ARITHMETICAL

***Thomas Wright**, Wofford University*

In this talk, we show that there are infinitely many Carmichael numbers in arithmetic progressions. We also show how this result helps to establish the infinitude of an elliptic curve analogue of Carmichael numbers known as "elliptic Carmichael numbers."