(1) $\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta,$ $\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

(2) $\tan(\alpha \pm \beta) = (\tan\alpha \pm \tan\beta)/(1 \mp \tan\alpha\tan\beta)$

(3) $\sin\alpha + \sin\beta = 2\sin\dfrac{\alpha+\beta}{2}\cos\dfrac{\alpha-\beta}{2},$ $\sin\alpha - \sin\beta = 2\sin\dfrac{\alpha-\beta}{2}\cos\dfrac{\alpha+\beta}{2}$

(4) $\cos\alpha + \cos\beta = 2\cos\dfrac{\alpha+\beta}{2}\cos\dfrac{\alpha-\beta}{2},$ $\cos\alpha - \cos\beta = -2\sin\dfrac{\alpha+\beta}{2}\sin\dfrac{\alpha-\beta}{2}$

(5) $\sin\alpha\sin\beta = \dfrac{1}{2}\cos(\alpha-\beta) - \dfrac{1}{2}\cos(\alpha+\beta),$ $\cos\alpha\cos\beta = \dfrac{1}{2}\cos(\alpha-\beta) + \dfrac{1}{2}\cos(\alpha+\beta)$

(6) $\sin\alpha\cos\beta = \dfrac{1}{2}\sin(\alpha+\beta) + \dfrac{1}{2}\sin(\alpha-\beta)$

(7) $1 + \tan^2\alpha = \dfrac{1}{\cos^2\alpha},$ $1 + \cot^2\alpha = \dfrac{1}{\sin^2\alpha},$ $1 + \cos\alpha = 2\cos^2\dfrac{\alpha}{2},$ $1 - \cos\alpha = 2\sin^2\dfrac{\alpha}{2}$

(8) $\sin 2\alpha = 2\sin\alpha\cos\alpha,$ $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha,$ $\tan 2\alpha = \dfrac{2\tan\alpha}{1-\tan^2\alpha}$

(9) $\sin\alpha = \dfrac{2t}{1+t^2},\ \cos\alpha = \dfrac{1-t^2}{1+t^2},\ \tan\alpha = \dfrac{2t}{1-t^2},$ missä $t = \tan\dfrac{\alpha}{2},$ $\tan\dfrac{\alpha}{2} = \dfrac{\sin\alpha}{1+\cos\alpha} = \dfrac{1-\cos\alpha}{\sin\alpha}$

(10) $\sinh x = \dfrac{1}{2}(e^x - e^{-x}),\ \ \cosh x = \dfrac{1}{2}(e^x + e^{-x}),\ \operatorname{arsinh} x = \ln(x + \sqrt{x^2+1}),\ \operatorname{arcosh} x = \ln(x + \sqrt{x^2-1})$

(11) $\cosh^2 x - \sinh^2 x = 1,$ $\sinh 2x = 2\sinh x\cosh x,$ $\cosh 2x = \cosh^2 x + \sinh^2 x$

Sarjat

(12) $f(x) = \displaystyle\sum_{k=0}^{n-1}\dfrac{f^{(k)}(x_0)}{k!}(x-x_0)^k + \dfrac{f^{(n)}(\xi)}{n!}(x-x_0)^n$

(13) $(1+x)^\alpha = 1 + \alpha x + \dfrac{\alpha(\alpha-1)}{2!}x^2 + \dfrac{\alpha(\alpha-1)(\alpha-2)}{3!}x^3 + \cdots,$ $|x| < 1$

(14) $\sqrt{1+x} = 1 + \dfrac{1}{2}x - \dfrac{1}{2\cdot4}x^2 + \dfrac{1\cdot3}{2\cdot4\cdot6}x^3 - \dfrac{1\cdot3\cdot5}{2\cdot4\cdot6\cdot8}x^4 + \cdots,$ $|x| \le 1$

(15) $\dfrac{1}{\sqrt{1+x}} = 1 - \dfrac{1}{2}x + \dfrac{1\cdot3}{2\cdot4}x^2 - \dfrac{1\cdot3\cdot5}{2\cdot4\cdot6}x^3 + \dfrac{1\cdot3\cdot5\cdot7}{2\cdot4\cdot6\cdot8}x^4 - \cdots,$ $-1 < x \le 1$

(16) $e^x = 1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \cdots$

(17) $\ln(1+x) = x - \dfrac{1}{2}x^2 + \dfrac{1}{3}x^3 - \dfrac{1}{4}x^4 + \cdots,$ $-1 < x \le 1$

(18) $\sin x = x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!} - \dfrac{x^7}{7!} + \cdots,$ $\cos x = 1 - \dfrac{x^2}{2!} + \dfrac{x^4}{4!} - \dfrac{x^6}{6!} + \cdots$

(19) $\tan x = x + \dfrac{1}{3}x^3 + \dfrac{2}{15}x^5 + \dfrac{17}{315}x^7 + \cdots,$ $|x| < \dfrac{\pi}{2}$

(20) $\arcsin x = x + \dfrac{1}{2}\cdot\dfrac{x^3}{3} + \dfrac{1\cdot3}{2\cdot4}\cdot\dfrac{x^5}{5} + \dfrac{1\cdot3\cdot5}{2\cdot4\cdot6}\cdot\dfrac{x^7}{7} + \cdots,$ $|x| \le 1$

(21) $\arctan x = x - \dfrac{x^3}{3} + \dfrac{x^5}{5} - \dfrac{x^7}{7} + \cdots,$ $|x| \le 1$

(22) $\dfrac{1}{2}\ln\dfrac{1+x}{1-x} = x + \dfrac{x^3}{3} + \dfrac{x^5}{5} + \dfrac{x^7}{7} + \cdots,$ $|x| < 1$

(23) $(x+y)^n = \displaystyle\sum_{k=0}^{n}\binom{n}{k}x^k y^{n-k},$ $\displaystyle\sum_{i=1}^{n} i = \dfrac{1}{2}n(n+1),$ $\displaystyle\sum_{i=1}^{n} i^2 = \dfrac{1}{6}n(n+1)(2n+1)$

(24) $\dfrac{1}{(1-x)^n} = \displaystyle\sum_{k=0}^{\infty}\binom{n+k-1}{k}x^k,$ $|x| < 1, n \ge 1.$

Integrointi

(25) $\quad \displaystyle\int Q(x, \sqrt{x^2 + a^2})dx : \text{ sij. } x = at, \, a\tan t, \, a\sinh t, \ldots \qquad \displaystyle\int Q(x, \sqrt[n]{\dfrac{ax+b}{cx+d}})dx : \text{ sij. } t = \sqrt[n]{\dfrac{ax+b}{cx+d}}$

(26) $\quad \displaystyle\int Q(x, \sqrt{x^2 - a^2})dx : \text{ sij. } x = at, \, a/t, \, a\cosh t, \ldots \qquad \displaystyle\int Q(x, \sqrt{a^2 - x^2})dx : \text{ sij. } x = at, \, a\sin t, \ldots$

(27) $\quad \displaystyle\int Q(\tan x)dx : \qquad \text{sij. } t = \tan x, \, t = \cot x, \ldots \qquad \displaystyle\int Q(\sin x, \cos x)dx : \quad \text{sij. } t = \tan\dfrac{x}{2}, \ldots$

(28) $\quad \displaystyle\int \dfrac{dx}{a^2 - x^2} = \dfrac{1}{2a}\ln\left|\dfrac{a+x}{a-x}\right|, \qquad\qquad\qquad \displaystyle\int \dfrac{dx}{a^2 + x^2} = \dfrac{1}{a}\arctan\dfrac{x}{a}$

(29) $\quad \displaystyle\int \dfrac{dx}{(a^2 \pm x^2)^{n+1}} = \dfrac{x}{2na^2(a^2 \pm x^2)^n} + \dfrac{2n-1}{2na^2}\int \dfrac{dx}{(a^2 \pm x^2)^n}$

(30) $\quad \displaystyle\int \dfrac{dx}{\sqrt{a^2 - x^2}} = \arcsin\dfrac{x}{|a|}, \qquad\qquad\qquad \displaystyle\int \dfrac{dx}{\sqrt{x^2 + a}} = \ln\left|x + \sqrt{x^2 + a}\right|$

(31) $\quad \displaystyle\int \sqrt{a^2 - x^2}\,dx = \dfrac{x}{2}\sqrt{a^2 - x^2} + \dfrac{a^2}{2}\arcsin\dfrac{x}{|a|}$

(32) $\quad \displaystyle\int \sqrt{x^2 + a}\,dx = \dfrac{x}{2}\sqrt{x^2 + a} + \dfrac{a}{2}\ln\left|x + \sqrt{x^2 + a}\right|$

(33) $\quad \displaystyle\int \cos^2 x\,dx = \dfrac{1}{2}x + \dfrac{1}{4}\sin 2x, \qquad\qquad\qquad \displaystyle\int \sin^2 x\,dx = \dfrac{1}{2}x - \dfrac{1}{4}\sin 2x$

(34) $\quad \displaystyle\int \dfrac{dx}{\cos x} = \ln\left|\tan(\dfrac{x}{2} + \dfrac{\pi}{4})\right|, \qquad\qquad\qquad \displaystyle\int \dfrac{dx}{\sin x} = \ln\left|\tan\dfrac{x}{2}\right|$

(35) $\quad \displaystyle\int \dfrac{dx}{\cosh x} = 2\arctan e^x, \qquad\qquad\qquad \displaystyle\int \dfrac{dx}{\sinh x} = \ln\left|\tanh\dfrac{x}{2}\right|$

(36) $\quad \displaystyle\int e^{ax}\sin bx\,dx = \dfrac{a\sin bx - b\cos bx}{a^2 + b^2}e^{ax}, \qquad\qquad \displaystyle\int e^{ax}\cos bx\,dx = \dfrac{a\cos bx + b\sin bx}{a^2 + b^2}e^{ax}$

(37) $\quad \displaystyle\int_0^{\pi/2} \sin^{2k} x\,dx = \int_0^{\pi/2} \cos^{2k} x\,dx = \dfrac{1}{2}\cdot\dfrac{3}{4}\cdots\dfrac{2k-1}{2k}\cdot\dfrac{\pi}{2}$

(38) $\quad \displaystyle\int_0^{\pi/2} \sin^{2k+1} x\,dx = \int_0^{\pi/2} \cos^{2k+1} x\,dx = \dfrac{2}{3}\cdot\dfrac{4}{5}\cdots\dfrac{2k}{2k+1}$

(39) $\quad$ Kaaren pituus $\displaystyle\int \sqrt{1 + y'^2}dx = \int \sqrt{x'^2 + y'^2}dt = \int \sqrt{r^2 + r'^2}d\varphi$

(40) $\quad$ Pyöräyspinnan ala $2\pi \displaystyle\int |y|\sqrt{1 + y'^2}dx$

(41) $\quad \displaystyle\int xe^{-ax}dx = -\dfrac{e^{-ax}}{a}\left(x + \dfrac{1}{a}\right), \qquad\qquad \displaystyle\int x^2 e^{-ax}dx = -\dfrac{e^{-ax}}{a}\left(\left(x + \dfrac{1}{a}\right)^2 + \dfrac{1}{a^2}\right)$

(42) $\quad \displaystyle\int_0^\infty x^n e^{-ax}dx = \dfrac{n!}{a^{n+1}}, \quad n = 0, 1, 2, \ldots, \qquad \displaystyle\int_{-\infty}^\infty e^{-ax^2}dx = \sqrt{\dfrac{\pi}{a}}$

(43) $\quad \displaystyle\int_{-\infty}^\infty x^{2n} e^{-ax^2}dx = \dfrac{(n - \frac{1}{2})(n - \frac{3}{2})\cdots\frac{1}{2}}{a^n}\sqrt{\dfrac{\pi}{a}}, \quad n = 1, 2, \ldots$

(44) $\quad \dfrac{\partial}{\partial z}\displaystyle\int_{\alpha(z)}^{\beta(z)} f(x, z)dx = \int_{\alpha(z)}^{\beta(z)} \dfrac{\partial f}{\partial z}dx + f(\beta(z), z)\dfrac{\partial\beta}{\partial z} - f(\alpha(z), z)\dfrac{\partial\alpha}{\partial z}$