Todennäköisyyslaskennan kaavoja (10.2010)

Unionin todennäköisyys

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} P(A_i) - \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} P\left(A_i \cap A_j\right) + - \ldots$$

$$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} (-1)^{i+1} \binom{n}{i} P\left(\bigcap_{j=1}^{i} A_j\right),$$

jos leikkaukset keskenään yhtä todennäköiset

$$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} P(A_i), \text{ jos } A_i : t \text{ toisensa poissulkevat}$$

$$P\left(\bigcup_{i=1}^{n} A_i\right) = 1 - \prod_{i=1}^{n} P\left(A_i^{\,c}\right), \text{ jos } A_i : t \text{ riippumattomat}$$

Leikkauksen todennäköisyys

$P(A \cap B) = P(A)\, P(B \mid A) = P(A \mid B)\, P(B)$

$$P\left(\bigcap_{i=1}^{n} A_i\right) = P(A_1)\, P(A_2 \mid A_1)\, P(A_3 \mid A_1 \cap A_2) \cdots$$

$$P\left(\bigcap_{i=1}^{n} A_i\right) = \prod_{i=1}^{n} P(A_i), \text{ jos } A_i : t \text{ riippumattomat}$$

Ehdollinen todennäköisyys

$P(A \mid B) = P(A \cap B) / P(B)$

$$P(B) = \sum_{i=1}^{n} P(B \mid A_i)\, P(A_i), \text{ jos } A_i : t \text{ muodostavat partition}$$

$P(A_k \mid B) = P(B \mid A_k)\, P(A_k) / P(B)$

Diskreetit satunnaismuuttujat

$F(x) = P(X \le x)$, $p(k) = P(X = k)$, $\sum_{k \in K} p(k) = 1$

$E(X) = \sum_{k \in K} k\, p(k)$, $E[g(X)] = \sum_{k \in K} g(k)\, p(k)$

$$X \ge 0 : E(X) = \sum_{k=0}^{\infty} [1 - F(k)],\ E[X(X-1)] = 2 \sum_{k=1}^{\infty} k[1 - F(k)]$$

$\mathrm{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2 = E[X(X-1)] + \mu - \mu^2$

$$G(z) = E\left(z^X\right),\ p(k) = \frac{G^{(k)}(0)}{k!},\ \begin{cases} E(X) = G'(1) \\ E[X(X-1)] = G''(1) \end{cases}$$

Jatkuvat satunnaismuuttujat

$F(x) = P(X \le x)$, $P(X \in A) = \int_A f(x)\, dx$

$F(x) = \int_{-\infty}^{x} f(t)\, dt$, $f(x) = F'(x)$, $\int_{-\infty}^{\infty} f(x)\, dx = 1$

$E(X) = \int_{-\infty}^{\infty} x\, f(x)\, dx$, $E[g(X)] = \int_{-\infty}^{\infty} g(x)\, f(x)\, dx$

$X \ge 0 : E(X) = \int_0^{\infty} [1 - F(x)]\, dx$, $E\left(X^2\right) = 2 \int_0^{\infty} x[1 - F(x)]\, dx$

$M(t) = E\left(e^{tX}\right)$, $E\left(X^j\right) = M^{(j)}(0)$

Yhteisjakautumat

$F_{X,Y}(x, y) = P(X \le x, Y \le y)$

$P[(X, Y) \in A] = \iint_{(x,y) \in A} f_{X,Y}(x, y)\, dx\, dy$

$F_{X,Y}(x, y) = \int_{-\infty}^{x} \left[\int_{-\infty}^{y} f_{X,Y}(s, t)\, dt\right] ds$, $f_{X,Y}(x, y) = \frac{\partial^2 F_{X,Y}(x,y)}{\partial x\, \partial y}$

$f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x, y)\, dy$, $f_Y(y) = \int_{-\infty}^{\infty} f_{X,Y}(x, y)\, dx$

Yleiset muunnokset

$Y = g(X) : f_Y(y) = f_X\left[g^{-1}(y)\right] \left| \frac{d}{dy} g^{-1}(y) \right|$

$$\begin{cases} Z = g(X, Y) \\ V = h(X, Y) \end{cases},\ \begin{cases} X = r(Z, V) \\ Y = s(Z, V) \end{cases} : \begin{array}{l} f_{Z,V}(z, v) = \\ f_{X,Y}[r(z, v), s(z, v)]\, |J(z, v)| \end{array}$$

Erityiset muunnokset

$f_{X+Y}(z) = \int_{-\infty}^{\infty} f_{X,Y}(z - v, v)\, dv$, $f_{X-Y}(z) = \int_{-\infty}^{\infty} f_{X,Y}(z + v, v)\, dv$

$f_{X\,Y}(z) = \int_{-\infty}^{\infty} f_{X,Y}\left(\frac{z}{v}, v\right) \frac{1}{|v|}\, dv$, $f_{\frac{X}{Y}}(z) = \int_{-\infty}^{\infty} f_{X,Y}(z\, v, v)\, |v|\, dv$

$$Y = \min\{X_1, \ldots, X_n\} : F_Y(y) = 1 - \prod_{i=1}^{n} \left[1 - F_{X_i}(y)\right]$$

$$Z = \max\{X_1, \ldots, X_n\} : F_Z(z) = \prod_{i=1}^{n} F_{X_i}(z)$$

Summat

$\mathrm{Cov}(X, Y) = E[(X - \mu_X)(Y - \mu_Y)] = E(X\,Y) - \mu_X\, \mu_Y$

$$E\left(\sum_{i=1}^{n} X_i\right) = \sum_{i=1}^{n} E(X_i)$$

$$\mathrm{Var}\left(\sum_{i=1}^{n} X_i\right) = \sum_{i=1}^{n} \mathrm{Var}(X_i) + 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \mathrm{Cov}\left(X_i, X_j\right)$$

$$X = \sum_{i=1}^{n} I_{A_i} : E(X) = n\, P(A_i),$$

$$\mathrm{Var}(X) = n\, P(A_i)\, P\left(A_i^{\,c}\right) + n(n-1)\left[P\left(A_i \cap A_j\right) - P(A_i)\, P\left(A_j\right)\right]$$

$P(|X - \mu| \ge t) \le \sigma^2 / t^2$

Ehdolliset jakautumat

$p_{X|Y=l}(k) = p_{X,Y}(k, l) / p_Y(l)$, $E(X \mid Y = l) = \sum_{k \in K} k\, p_{X|Y=l}(k)$

$p_X(k) = \sum_{l \in L} p_{X|Y=l}(k)\, p_Y(l)$, $p_{Y|X=k}(l) = p_{X|Y=l}(k)\, p_Y(l) / p_X(k)$

$f_{X|Y=y}(x) = f_{X,Y}(x, y) / f_Y(y)$, $E(X \mid Y = y) = \int_{-\infty}^{\infty} x\, f_{X|Y=y}(x)\, dx$

$P(a \le X \le b \mid Y = y) = \int_a^b f_{X|Y=y}(x)\, dx$

$f_X(x) = \int_{-\infty}^{\infty} f_{X|Y=y}(x)\, f_Y(y)\, dy$, $f_{Y|X=x}(y) = f_{X|Y=y}(x)\, f_Y(y) / f_X(x)$

$p_X(k) = \int_{-\infty}^{\infty} p_{X|Y=y}(k)\, f_Y(y)\, dy$, $f_{Y|X=k}(y) = p_{X|Y=y}(k)\, f_Y(y) / p_X(k)$

$f_X(x) = \sum_{l \in L} f_{X|Y=l}(x)\, p_Y(l)$, $p_{Y|X=x}(l) = f_{X|Y=l}(x)\, p_Y(l) / f_X(x)$

Kokonaisodotusarvokaava

$$E(X) = E[E(X \mid Y)] = \begin{cases} \sum_{l \in L} E(X \mid Y = l)\, p_Y(l), & Y \text{ diskreetti} \\ \int_{-\infty}^{\infty} E(X \mid Y = y)\, f_Y(y)\, dy, & Y \text{ jatkuva} \end{cases}$$

$\mathrm{Var}(X) = E[\mathrm{Var}(X \mid Y)] + \mathrm{Var}[E(X \mid Y)]$

$$P(A) = E[P(A \mid X)] = \begin{cases} \sum_{k \in K} P(A \mid X = k)\, p_X(k), & X \text{ diskreetti} \\ \int_{-\infty}^{\infty} P(A \mid X = x)\, f_X(x)\, dx, & X \text{ jatkuva} \end{cases}$$

$f_{X|A}(x) = P(A \mid X = x)\, f_X(x) / P(A)$

$$E(X \mid A) = \begin{cases} \sum_k k\, P(X = k \mid A), & X \text{ diskreetti} \\ \int_{\infty}^{\infty} x\, f_{X|A}(x)\, dx, & X \text{ jatkuva} \end{cases}$$

$E(X) = \sum_{i=1}^{n} E(X \mid A_i)\, P(A_i)$

Satunnaisen pituinen summa

$G_{S_N}(z) = G_N[G_X(z)]$, $M_{S_N}(t) = G_N[M_X(t)]$

$E(S_N) = E(N)\, E(X)$, $\mathrm{Var}(S_N) = E(N)\, \mathrm{Var}(X) + \mathrm{Var}(N)\, [E(X)]^2$

Korrelaatio

$\mathrm{Cor}(X, Y) = \mathrm{Cov}(X, Y) \Big/ \sqrt{\mathrm{Var}(X)\, \mathrm{Var}(Y)}$

$l(Y) = \mu_X + \rho\, \frac{\sigma_X}{\sigma_Y}\, (Y - \mu_Y)$

Syntymis-kuolemis – prosessi

$\rho_0 = 1$, $\rho_i = \frac{\lambda_0 \cdots \lambda_{i-1}}{\mu_1 \cdots \mu_i} : \pi_0 = \left(\sum_{i=0}^{\infty} \rho_i\right)^{-1}$, $\pi_i = \rho_i\, \pi_0$

Diskreettejä jakautumia, äärellinen arvojoukko

DU(n): $\frac{1}{n}$, $k = 1, \ldots, n$, $E(X) = \frac{n+1}{2}$, $V(X) = \frac{n^2-1}{12}$, $G(z) = \frac{1}{n}\sum_{k=1}^{n} z^k = \frac{z(1-z^n)}{n(1-z)}$

Ber(p): $p^k q^{1-k}$, $k = 0, 1$, $E(X) = p$, $V(X) = p\,q$, $G(z) = p\,z + q$

Bin(n, p): $\binom{n}{k} p^k q^{n-k}$, $k = 0, \ldots, n$, $E(X) = n\,p$, $V(X) = n\,p\,q$, $G(z) = (p\,z + q)^n$, $X \approx \mathrm{Po}(n\,p)$, $X \sim \mathrm{AsN}(n\,p, n\,p\,q)$

Hyp(N, M, n): $\binom{M}{k}\binom{N-M}{n-k} \Big/ \binom{N}{n}$, $\begin{matrix} 0 \le k \le M \\ 0 \le n-k \le N-M \end{matrix}$, $E(X) = n\,p$, $V(X) = n\,p\,q\,\frac{N-n}{N-1}$, $p = \frac{M}{N}$, $q = 1 - p$, $X \approx \mathrm{Bin}\left(n, \frac{M}{N}\right)$

Multinomi(n, $p_1, \ldots, p_r$): $P(X_1 = k_1, \ldots, X_r = k_r) = \frac{n!}{k_1! \cdots k_r!} {p_1}^{k_1} \cdots {p_r}^{k_r}$, $\sum_{i=1}^{r} p_i = 1$, $\sum_{i=1}^{r} k_i = n$

Multihyp(N, $M_1, \ldots, M_r$, n): $P(X_1 = k_1, \ldots, X_r = k_r) = \binom{M_1}{k_1} \cdots \binom{M_r}{k_r} \Big/ \binom{N}{n}$, $\sum_{i=1}^{r} M_i = N$, $\sum_{i=1}^{r} k_i = n$

Diskreettejä jakautumia, ääretön arvojoukko

Po(λ): $e^{-\lambda}\frac{\lambda^k}{k!}$, $k \ge 0$, $E(X) = \lambda$, $V(X) = \lambda$, $G(z) = e^{\lambda(z-1)}$, $X \sim \mathrm{AsN}(\lambda, \lambda)$

Geom(p): $q^{k-1}\,p$, $k \ge 1$, $F(k) = 1 - q^k$, $E(X) = \frac{1}{p}$, $V(X) = \frac{q}{p^2}$, $G(z) = \frac{p\,z}{1-q\,z}$

ModGeom(p): $q^k\,p$, $k \ge 0$, $F(k) = 1 - q^{k+1}$, $E(X) = \frac{q}{p}$, $V(X) = \frac{q}{p^2}$, $G(z) = \frac{p}{1-q\,z}$

Negbin(n, p): $\binom{k-1}{n-1} p^n q^{k-n}$, $k \ge n$, $E(X) = \frac{n}{p}$, $V(X) = \frac{n\,q}{p^2}$, $G(z) = \left(\frac{p\,z}{1-q\,z}\right)^n$

ModNegbin(n, p): $\binom{k+n-1}{n-1} p^n q^k$, $k \ge 0$, $E(X) = \frac{n\,q}{p}$, $V(X) = \frac{n\,q}{p^2}$, $G(z) = \left(\frac{p}{1-q\,z}\right)^n$

Normaalijakautuma

$N(0, 1)$: $\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$, $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}t^2}\,dt$, $E(X) = 0$, $V(X) = 1$, $M(t) = e^{\frac{1}{2}t^2}$

$N\left(\mu, \sigma^2\right)$: $f(x) = \frac{1}{\sigma}\phi\left(\frac{x-\mu}{\sigma}\right)$, $F(x) = \Phi\left(\frac{x-\mu}{\sigma}\right)$, $E(X) = \mu$, $V(X) = \sigma^2$, $M(t) = e^{\mu\,t + \frac{1}{2}\sigma^2 t^2}$

$P(\mu - [1.96, 2.58, 3.29]\,\sigma < X < \mu + [1.96, 2.58, 3.29]\,\sigma) = [0.95, 0.99, 0.999]$

$S_n \sim \mathrm{AsN}\left(n\,\mu, n\,\sigma^2\right)$, $\overline{X}_n \sim \mathrm{AsN}\left(\mu, \frac{\sigma^2}{n}\right)$

Muita yleisimpiä jatkuvia jakautumia

$U(a, b)$: $\frac{1}{b-a}$, $a < x < b$, $F(x) = \frac{x-a}{b-a}$, $E(X) = \frac{a+b}{2}$, $V(X) = \frac{(b-a)^2}{12}$, $M(t) = \frac{e^{b\,t} - e^{a\,t}}{(b-a)\,t}$

Exp(λ): $\lambda\,e^{-\lambda\,x}$, $x > 0$, $F(x) = 1 - e^{-\lambda\,x}$, $E(X) = \frac{1}{\lambda}$, $V(X) = \frac{1}{\lambda^2}$, $M(t) = \frac{\lambda}{\lambda - t}$

Gamma(α, λ): $\frac{\lambda^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\lambda\,x}$, $x > 0$, $E(X) = \frac{\alpha}{\lambda}$, $V(X) = \frac{\alpha}{\lambda^2}$, $M(t) = \left(\frac{\lambda}{\lambda - t}\right)^\alpha$

Erlang(n, λ): $\frac{\lambda^n}{(n-1)!} x^{n-1} e^{-\lambda\,x}$, $x > 0$, $F(x) = 1 - e^{-\lambda\,x} \sum_{i=0}^{n-1} \frac{(\lambda\,x)^i}{i!}$, $E(X) = \frac{n}{\lambda}$, $V(X) = \frac{n}{\lambda^2}$, $M(t) = \left(\frac{\lambda}{\lambda - t}\right)^n$

Beta(α, β): $\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\,\Gamma(\beta)} x^{\alpha-1} (1-x)^{\beta-1}$, $0 < x < 1$, $E(X) = \frac{\alpha}{\alpha+\beta}$, $V(X) = \frac{\alpha\,\beta}{(\alpha+\beta)^2\,(\alpha+\beta+1)}$

Muita jatkuvia jakautumia, $x > 0$ ellei toisin mainita

LogN$\left(\mu, \sigma^2\right)$: $\frac{1}{\sigma\,x}\phi\left(\frac{\ln(x)-\mu}{\sigma}\right)$, $F(x) = \Phi\left(\frac{\ln(x)-\mu}{\sigma}\right)$, $E(X) = e^{\mu + \frac{1}{2}\sigma^2}$, $E\left(X^2\right) = e^{2\,\mu + 2\,\sigma^2}$

Weibull(α, λ): $\alpha\,\lambda\,x^{\alpha-1} e^{-\lambda\,x^\alpha}$, $F(x) = 1 - e^{-\lambda\,x^\alpha}$, $E(X) = \left(\frac{1}{\lambda}\right)^{\frac{1}{\alpha}} \Gamma\left(1 + \frac{1}{\alpha}\right)$, $E\left(X^2\right) = \left(\frac{1}{\lambda}\right)^{\frac{2}{\alpha}} \Gamma\left(1 + \frac{2}{\alpha}\right)$

Hypoexp($\vec{\lambda}$): $\sum_{i=1}^{n} \alpha_i\,\lambda_i\,e^{-\lambda_i\,x}$, $\lambda_i \ne \lambda_j$, $\alpha_i = \prod_{j=i, j\ne i}^{n} \frac{\lambda_j}{\lambda_j - \lambda_i}$, $F(x) = \sum_{i=1}^{n} \alpha_i\left(1 - e^{-\lambda_i\,x}\right)$, $E(X) = \sum_{i=1}^{n} \frac{1}{\lambda_i}$, $V(X) = \sum_{i=1}^{n} \frac{1}{{\lambda_i}^2}$

Hyperexp$\left(\vec{\lambda}, \vec{\alpha}\right)$: $\sum_{i=1}^{n} \alpha_i\,\lambda_i\,e^{-\lambda_i\,x}$, $\sum_{i=1}^{n} \alpha_i = 1$, $F(x) = \sum_{i=1}^{n} \alpha_i\left(1 - e^{-\lambda_i\,x}\right)$, $E(X) = \sum_{i=1}^{n} \frac{\alpha_i}{\lambda_i}$, $E\left(X^2\right) = 2\sum_{i=1}^{n} \frac{\alpha_i}{{\lambda_i}^2}$

Pareto(α, λ): $\frac{\lambda}{\alpha}\left(\frac{\alpha}{x}\right)^{\lambda+1}$, $x > \alpha$, $F(x) = 1 - \left(\frac{\alpha}{x}\right)^\lambda$, $E(X) = \frac{\alpha\,\lambda}{\lambda - 1}$ $(\lambda > 1)$, $V(X) = \frac{\lambda\,\alpha^2}{(\lambda-1)^2\,(\lambda-2)}$ $(\lambda > 2)$

Muita jatkuvia jakautumia, $x \in \mathbb{R}$

Laplace(μ, λ): $\frac{1}{2} \lambda e^{-\lambda|x-\mu|}$, $F(x) = \begin{cases} \frac{1}{2} e^{-\lambda(\mu-x)} & x < \mu \\ 1 - \frac{1}{2} e^{-\lambda(x-\mu)} & x \geq \mu \end{cases}$, $E(X) = \mu$, $V(X) = \frac{2}{\lambda^2}$, $M(t) = \frac{\lambda^2 e^{\mu t}}{\lambda^2 - t^2}$

Logistic(μ, σ^2): $\frac{a e^{-a y}}{\sigma(1+e^{-a y})^2}$, $y = \frac{x-\mu}{\sigma}$, $a = \frac{\pi}{\sqrt{3}}$, $F(x) = \frac{1}{1+e^{-a y}}$, $E(X) = \mu$, $V(X) = \sigma^2$

Cauchy(a, b): $\frac{1}{\pi b}\left[1 + \left(\frac{x-a}{b}\right)^2\right]^{-1}$, $F(x) = \frac{1}{2} + \frac{1}{\pi} \arctan\left(\frac{x-a}{b}\right)$

$N\left(\mu_1, \mu_2, \sigma_1{}^2, \sigma_2{}^2, \rho\right)$: $E(X \mid Y = y) = \mu_1 + \rho \frac{\sigma_1}{\sigma_2} (y - \mu_2)$, $V(X \mid Y = y) = \sigma_1{}^2\left(1 - \rho^2\right)$

Tilastollisia jakautumia

$t(n)$: $\frac{\Gamma\left(\frac{n+1}{2}\right)}{\sqrt{n \pi}\, \Gamma\left(\frac{n}{2}\right)} \left(1 + \frac{x^2}{n}\right)^{-\frac{n+1}{2}}$, $x \in \mathbb{R}$, $E(X) = 0$ $(n \geq 2)$, $V(X) = \frac{n}{n-2}$ $(n \geq 3)$

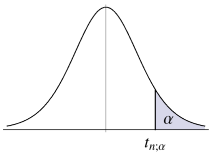
$\chi^2(n)$: $\left[2^{\frac{n}{2}} \Gamma\left(\frac{n}{2}\right)\right]^{-1} x^{\frac{n}{2}-1} e^{-\frac{x}{2}}$, $x > 0$, $E(X) = n$, $V(X) = 2n$, $M(t) = \left(\frac{1}{1-2t}\right)^{\frac{n}{2}}$

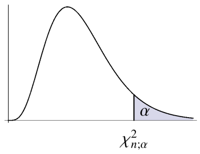
$F(m, n)$: $\frac{\Gamma\left(\frac{m+n}{2}\right)}{\Gamma\left(\frac{m}{2}\right)\Gamma\left(\frac{n}{2}\right)} \sqrt{\frac{m^m n^n x^{m-2}}{(n+m x)^{m+n}}}$, $x > 0$, $E(X) = \frac{n}{n-2}$ $(n \geq 3)$, $V(X) = \frac{2 n^2(m+n-2)}{m(n-2)^2 (n-4)}$ $(n \geq 5)$

$N(0, 1)$-jakautuman todennäköisyyksiä $P(X \leq x) = \alpha$

Esim. $P(X \leq 1.23) = 0.8907$

x	0	1	2	3	4	5	6	7	8	9	
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359	**0.0**
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753	**0.1**
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141	**0.2**
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517	**0.3**
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879	**0.4**
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224	**0.5**
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549	**0.6**
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852	**0.7**
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133	**0.8**
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389	**0.9**
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621	**1.0**
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830	**1.1**
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015	**1.2**
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177	**1.3**
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319	**1.4**
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441	**1.5**
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545	**1.6**
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633	**1.7**
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706	**1.8**
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767	**1.9**
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817	**2.0**
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857	**2.1**
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890	**2.2**
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916	**2.3**
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936	**2.4**
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952	**2.5**
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964	**2.6**
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974	**2.7**
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981	**2.8**
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986	**2.9**
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990	**3.0**
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993	**3.1**
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995	**3.2**
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997	**3.3**
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998	**3.4**
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	**3.5**
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	**3.6**
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	**3.7**
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	**3.8**
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	**3.9**
	0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	





$t(n)$ -jakautuman kriittisiä arvoja $t_{n;\alpha}$: $P\left(X > t_{n;\alpha}\right) = \alpha$

Esim. $t_{10;\,0.05} = 1.812$: jos $X \sim t(10)$, niin $P(X > 1.812) = 0.05$

	α				
n	**0.1**	**0.05**	**0.025**	**0.01**	**0.005**
1	3.078	6.314	12.71	31.82	63.66
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
40	1.303	1.684	2.021	2.423	2.704
50	1.299	1.676	2.009	2.403	2.678
60	1.296	1.671	2.000	2.390	2.660
70	1.294	1.667	1.994	2.381	2.648
80	1.292	1.664	1.990	2.374	2.639
90	1.291	1.662	1.987	2.368	2.632
100	1.290	1.660	1.984	2.364	2.626
150	1.287	1.655	1.976	2.351	2.609
200	1.286	1.653	1.972	2.345	2.601
∞	1.282	1.645	1.960	2.326	2.576

$\chi^2(n)$ -jakautuman kriittisiä arvoja $\chi^2_{n;\alpha}$: $P\left(X > \chi^2_{n;\alpha}\right) = \alpha$

Esim. $\chi^2_{10;\,0.05} = 18.31$: jos $X \sim \chi^2(10)$, niin $P(X > 18.31) = 0.05$

	α							
n	**0.995**	**0.99**	**0.975**	**0.95**	**0.05**	**0.025**	**0.01**	**0.005**
1	0.00003927	0.0001571	0.0009821	0.003932	3.841	5.024	6.635	7.879
2	0.01003	0.02010	0.05064	0.1026	5.991	7.378	9.210	10.60
3	0.07172	0.1148	0.2158	0.3518	7.815	9.348	11.34	12.84
4	0.2070	0.2971	0.4844	0.7107	9.488	11.14	13.28	14.86
5	0.4117	0.5543	0.8312	1.145	11.07	12.83	15.09	16.75
6	0.6757	0.8721	1.237	1.635	12.59	14.45	16.81	18.55
7	0.9893	1.239	1.690	2.167	14.07	16.01	18.48	20.28
8	1.344	1.646	2.180	2.733	15.51	17.53	20.09	21.95
9	1.735	2.088	2.700	3.325	16.92	19.02	21.67	23.59
10	2.156	2.558	3.247	3.940	18.31	20.48	23.21	25.19
11	2.603	3.053	3.816	4.575	19.68	21.92	24.72	26.76
12	3.074	3.571	4.404	5.226	21.03	23.34	26.22	28.30
13	3.565	4.107	5.009	5.892	22.36	24.74	27.69	29.82
14	4.075	4.660	5.629	6.571	23.68	26.12	29.14	31.32
15	4.601	5.229	6.262	7.261	25.00	27.49	30.58	32.80
16	5.142	5.812	6.908	7.962	26.30	28.85	32.00	34.27
17	5.697	6.408	7.564	8.672	27.59	30.19	33.41	35.72
18	6.265	7.015	8.231	9.390	28.87	31.53	34.81	37.16
19	6.844	7.633	8.907	10.12	30.14	32.85	36.19	38.58
20	7.434	8.260	9.591	10.85	31.41	34.17	37.57	40.00
21	8.034	8.897	10.28	11.59	32.67	35.48	38.93	41.40
22	8.643	9.542	10.98	12.34	33.92	36.78	40.29	42.80
23	9.260	10.20	11.69	13.09	35.17	38.08	41.64	44.18
24	9.886	10.86	12.40	13.85	36.42	39.36	42.98	45.56
25	10.52	11.52	13.12	14.61	37.65	40.65	44.31	46.93
26	11.16	12.20	13.84	15.38	38.89	41.92	45.64	48.29
27	11.81	12.88	14.57	16.15	40.11	43.19	46.96	49.64
28	12.46	13.56	15.31	16.93	41.34	44.46	48.28	50.99
29	13.12	14.26	16.05	17.71	42.56	45.72	49.59	52.34
30	13.79	14.95	16.79	18.49	43.77	46.98	50.89	53.67
31	14.46	15.66	17.54	19.28	44.99	48.23	52.19	55.00
32	15.13	16.36	18.29	20.07	46.19	49.48	53.49	56.33
33	15.82	17.07	19.05	20.87	47.40	50.73	54.78	57.65
34	16.50	17.79	19.81	21.66	48.60	51.97	56.06	58.96
35	17.19	18.51	20.57	22.47	49.80	53.20	57.34	60.27
36	17.89	19.23	21.34	23.27	51.00	54.44	58.62	61.58
37	18.59	19.96	22.11	24.07	52.19	55.67	59.89	62.88
38	19.29	20.69	22.88	24.88	53.38	56.90	61.16	64.18
39	20.00	21.43	23.65	25.70	54.57	58.12	62.43	65.48
40	20.71	22.16	24.43	26.51	55.76	59.34	63.69	66.77

Luottamusvälit

$$P\left(\overline{X} - 1.96\,\frac{S}{\sqrt{n}} < \mu < \overline{X} + 1.96\,\frac{S}{\sqrt{n}}\right) \simeq 0.95$$

$$P\left(\overline{X} - t_{n-1;\,\alpha/2}\,\frac{S}{\sqrt{n}} < \mu < \overline{X} + t_{n-1;\,\alpha/2}\,\frac{S}{\sqrt{n}}\right) = 1 - \alpha$$

$$P\left(\frac{(n-1)\,S^2}{\chi^2_{n-1;\,\alpha/2}} < \sigma^2 < \frac{(n-1)\,S^2}{\chi^2_{n-1;\,1-\alpha/2}}\right) = 1 - \alpha$$

$$P\left(\hat{p} - 1.96\,\sqrt{\hat{p}\,\hat{q}/n} < p < \hat{p} + 1.96\,\sqrt{\hat{p}\,\hat{q}/n}\right) \simeq 0.95$$

Hypoteesien testaus

$$H_0 : \mu = \mu_0 : T = \left(\overline{X} - \mu_0\right) \Big/ \left(S \Big/ \sqrt{n}\right) \sim t(n-1)$$

$$H_0 : p = p_0 : Z = \left(\hat{p} - p_0\right) \Big/ \sqrt{p_0\,q_0/n} \sim N(0,\,1)$$

$$H_0 : P(X = x_i) = p_i : Q = \sum_{i=1}^{k} \frac{\left(N_i - n\,p_i\right)^2}{n\,p_i} \sim \chi^2(k - m - 1)$$