Laplace-muunnoksia

Taulukossa on merkitty $\mathcal{L}[f(t)](s) = F(s)$, $\mathcal{L}[g(t)](s) = G(s)$, $\operatorname{erf}(t) = \frac{2}{\sqrt{\pi}} \int_0^t e^{-u^2}\, du$,
$f(0+) = \lim_{x \to 0+} f(x)$ ja $\gamma = -\int_0^\infty \ln t\, e^{-t}\, dt = 0,577215665\ldots$ (Eulerin vakio).
$\Gamma(x) = \int_0^\infty t^{x-1} e^{-t}\, dt$, $\Gamma(x+1) = x\Gamma(x)$, $\Gamma(1) = 1$, $\Gamma(\frac{1}{2}) = \sqrt{\pi}$.

	Funktio	Laplace-muunnos
1	$f(t)$	$F(s) = \int_0^\infty f(t) e^{-st}\, dt$
2	$af(t) + bg(t)$	$aF(s) + bG(s)$
3	$f(\alpha t)$ kun $\alpha > 0$	$\frac{1}{\alpha} F(\frac{s}{\alpha})$
4	$f^{(n)}(t)$	$s^n F(s) - f(0+)s^{n-1} - f'(0+)s^{n-2} - \ldots - f^{(n-1)}(0+)$
5	$(-1)^n t^n f(t)$	$F^{(n)}(s)$
6	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\, dq$
7	$\int_0^t f(u)\, du$	$\frac{F(s)}{s}$
8	$f(t-\tau)$ kun $\tau > 0$	$e^{-s\tau} F(s)$
9	$e^{\lambda t} f(t)$	$F(s - \lambda)$
10	$H(t) = \begin{cases} 0 & \text{kun } t < 0 \\ \frac{1}{2} & \text{kun } t = 0 \\ 1 & \text{kun } t > 0 \end{cases}$	$\frac{1}{s}$
11	t^n kun $n \in \mathbb{N}$	$\frac{n!}{s^{n+1}}$
12	t^a (kun $\operatorname{Re} a > -1$)	$\frac{\Gamma(a+1)}{s^{a+1}}$
13	$e^{\lambda t}$	$\frac{1}{s - \lambda}$
14	$\sin \omega t$	$\frac{\omega}{s^2 + \omega^2}$
15	$\cos \omega t$	$\frac{s}{s^2 + \omega^2}$
16	$\sinh \omega t$	$\frac{\omega}{s^2 - \omega^2}$
17	$\cosh \omega t$	$\frac{s}{s^2 - \omega^2}$
18	$t \sin \omega t$	$\frac{2\omega s}{(s^2 + \omega^2)^2}$
19	$t \cos \omega t$	$\frac{s^2 - \omega^2}{(s^2 + \omega^2)^2}$
20	$\frac{e^{-bt} - e^{-at}}{a - b}$	$\frac{1}{(s+a)(s+b)}$
21	$\frac{be^{-bt} - ae^{-at}}{b - a}$	$\frac{s}{(s+a)(s+b)}$
22	$\frac{t^{n-1}}{(n-1)!} e^{-at}$	$\frac{1}{(s+a)^n}$
23	$\ln t$	$-\frac{\ln s}{s} - \frac{\gamma}{s}$
24	$(f * g)(t) = \int_0^t f(u) g(t-u)\, du$	$F(s)G(s)$
25	$\operatorname{erf}(\sqrt{t})$	$\frac{1}{s\sqrt{s+1}}$

Fourier-muunnoksia

Taulukossa on merkitty $\mathcal{F}[f(x)](y) = F(y)$ ja $\mathcal{F}[g(x)](y) = G(y)$, jolloin

$$f(x) = \int_{-\infty}^{\infty} F(y) e^{2\pi i x y}\, dy.$$

- Esitys trigonometristen funktioiden integraalina:

$$f(x) = \int_{0}^{\infty} (A(y)\cos(2\pi x y) + B(y)\sin(2\pi x y))\, dy,$$

  missä $A(y) = F(y) + F(-y)$ ja $B(y) = i(F(y) - F(-y))$.

- Reaaliarvoiselle funktiolle $f(x)$ pätee $F(-y) = \overline{F(y)}$.

- Parsevalin kaava:

$$\int_{-\infty}^{\infty} f(x)\overline{g(x)}\, dx = \int_{-\infty}^{\infty} F(y)\overline{G(y)}\, dy.$$

	Funktio	Fourier-muunnos
1	$f(x)$	$F(y) = \int_{-\infty}^{\infty} f(x) e^{-2\pi i x y}\, dx$
2	$af(x) + bg(x)$	$aF(y) + bG(y)$
3	$F(x)$	$f(-y)$
4	$f(\alpha x) \quad \text{kun } \alpha > 0$	$\frac{1}{\alpha} F(\frac{y}{\alpha})$
5	$f^{(n)}(x)$	$(2\pi i y)^n F(y)$
6	$\int_{-\infty}^{x} f(t)\, dt$	$\frac{1}{2\pi i y} F(y) + \frac{\delta(y)}{2} F(0)$
7	$f(x - x_0)$	$F(y) e^{-2\pi i x_0 y}$
8	$f(x) e^{2\pi i y_0 x}$	$F(y - y_0)$
9	$e^{2\pi i x y_0}$	$\delta(y - y_0)$
10	$\sin 2\pi x y_0$	$\frac{1}{2i}\delta(y - y_0) - \frac{1}{2i}\delta(y + y_0)$
11	$\cos 2\pi x y_0$	$\frac{1}{2}\delta(y - y_0) + \frac{1}{2}\delta(y + y_0)$
12	$H(x) = \begin{cases} 0 & \text{kun } x < 0 \\ \frac{1}{2} & \text{kun } x = 0 \\ 1 & \text{kun } x > 0 \end{cases}$	$\frac{1}{2\pi i y} + \frac{\delta(y)}{2}$
13	$\operatorname{sgn}(x) = \begin{cases} -1 & \text{kun } x < 0 \\ 0 & \text{kun } x = 0 \\ 1 & \text{kun } x > 0 \end{cases}$	$\frac{1}{\pi i y}$
14	$\Pi(x) = \begin{cases} 0 & \text{kun } \lvert x\rvert > \frac{1}{2} \\ \frac{1}{2} & \text{kun } \lvert x\rvert = \frac{1}{2} \\ 1 & \text{kun } \lvert x\rvert < \frac{1}{2} \end{cases}$	$\operatorname{sinc} y = \begin{cases} 1 & \text{jos } y = 0 \\ \frac{\sin \pi y}{\pi y} & \text{jos } y \neq 0 \end{cases}$
15	$e^{-\pi x^2}$	$e^{-\pi y^2}$
16	$(f * g)(x) = \int_{-\infty}^{\infty} f(t) g(x - t)\, dt$	$F(y)G(y)$