WORKSHOP
ON MODERN TRENDS
IN CLASSICAL ANALYSIS AND APPLICATIONS

The 1^{st} Chinese-Finnish Seminar

August 17-18, 2012
University of Turku, Finland

Program

.

Friday, August 17

(All lectures take place at Lecture Hall XI, Natural Science Building I)

9:30	Matti Vuorinen	Opening Remarks
9:45–10:30	Wang, Xiantao	On the quasisymmetry of quasiconformal mappings and its applications
10:30–10:45		**Coffee**
10:45–11:10	Riku Klen	Properties of hyperbolic type balls
11:15–11:40	Antti Rasila	Properties of quasihyperbolic balls and geodesics on Banach spaces
11:40–13:30		**Lunch**
13:30–13:55	Huang, Manzi	The quasi-invariance of uniform domains in real Banach spaces
14:00–14:25	Vesa Ala-Mattila	Geometric characterizations for Patterson-Sullivan measures of geometrically finite Kleinian groups
14:30–14:55	Wang, Gendi	The visual angle metric and Möbius transformations
15:00–15:30		**Coffee**
15:30–15:55	Sami Hokuni	Local convexity properties of balls in the triangular ratio metric
16:00–16:25	Yang, Xuxin	On Existence of Solutions for Differential Equations with Impulses
18:00		**Dinner**

Saturday, August 18

(Excursion by ship to Maarianhamina ca 7.30-19.00)

8:00		**Breakfast on the sea**
9:30–10:00	Tommi Sottinen	Yukawa Potential, Harmonic Measure and Killing Brownian Motion
10:10–10:40	Zhang, Xiaohui	Inequalities for the generalized trigonometric and hyperbolic functions
10:45–	Matti Vuorinen	
14:00		**Lunch**
15.30–15:55	Li, Yaxiang	Two properties of John domains in real Banach spaces

Geometric characterizations for Patterson-Sullivan measures of geometrically finite Kleinian groups

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Abstract. Let G be a non-elementary geometrically finite Kleinian group and μ a Patterson-Sullivan measure of G. Dennis Sullivan showed in [S] that one can sometimes use the standard covering construction or the standard packing construction to construct a measure ν such that $\mu = c\nu$ for some constant $c > 0$; sometimes neither of the standard constructions can be used to construct ν. We will discuss the main result of [A-M] which shows that if the covering and packing constructions are modified in a suitable way, one can always use either one of them to construct a suitable measure ν.

References

[A-M] V. Ala-Mattila: *Geometric characterizations for Patterson-Sullivan measures of geometrically finite Kleinian groups*, Ann. Acad. Sci. Fenn. Math. Diss. 157 (2011), 120 pp. (PDF)

[S] D. Sullivan: *Entropy, Hausdorff measures old and new, and limit sets of geometrically finite Kleinian groups*, Acta Math. 153 (1984), no. 3-4, 259-277.

Local convexity properties of balls in the triangular ratio metric

Sami Hokuni*, Riku Klén, Matti Vuorinen

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Abstract.

In this presentation triangular ratio metric is studied in three different domains. Especially convexity of balls in these domains is discussed. The first domain is $\mathbb{R}^n \setminus \{0\}$, where strict bound between convex and nonconvex balls is shown. The second domain is the upper half plane $\mathbb{H}^2$, where it is shown that all triangular ratio metric balls are Euclidean circles and therefore convex.

The third domain is the upper half plane with one point removed. Results in two previous domains are combined to give one upper bound for convexity of balls in this last domain. However, this upper bound is not strict in a general case, which is shown by an example.

References

[HKV] Sami Hokuni, Riku Klen and Matti Vuorinen: *Local convexity properties of balls in the triangular ratio metric.* 2012, under preparation.

[0]*represents the speaker in the case of multiple authors.

The quasi-invariance of uniform domains in real Banach spaces

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Abstract. Suppose that E and E' denote real Banach spaces with dimension at least 2, that $D \subset E$ and $D' \subset E'$ are domains, and that $f : D \to D'$ is a FQC mapping and that D' is uniform. Then the image $f(D_1)$ of every uniform subdomain D_1 in D under f is still uniform.

Acknowledgements. The author is supported by the Academy Finland Project 2600066611 (M. Vuorinen)

References

[HVW] M. Huang, Y. Li, M. Vuorinen, and X. Wang : *The quasi-invariance of uniform domains in Banach spaces,* Preprint.

Properties of hyperbolic type balls

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Abstract. The hyperbolic geometry has many applications in mathematics and physics. However, the hyperbolic metric is not sufficient in higher dimensions due to the absence of a useful counterpart of the Riemann mapping theorem. Various generalizations of the hyperbolic metric have been introduced to overcome this problem and we call these metrics hyperbolic type metrics.

We consider properties of hyperbolic type balls, especially balls defined by the Apollonain and the Seittenranta's metrics, in subdomains of $\mathbb{R}^n$. The talk is based on [1].

References

[1] R. Klén: *Local convexity properties of Apollonian and Seittenranta's metric balls* Manuscript, arXiv:1204.0329, 2012.

Two properties of John domains in real Banach spaces

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Abstract. Suppose that E and E' denote real Banach spaces with equal dimension at least 2. The main aim of this paper is to show that: (1). A domain D is a John domain if and only if $D \setminus P_D$ is a John domain, where P_D denotes a set in D, in which the quasihyperbolic distance between each pair of points is at least $c > 0$. (2). A c-John domain $D \subset E$ can be written as the union of domain D_1, D_2, ... such that for each j,

1. $\overline{D}_j$ is contained in D_{j+1}, and
2. D_j is a c_1-John domain with $c_1 = c_1(c)$.

Acknowledgements. The author is supported by the Academy Finland Project 2600066611 (M. Vuorinen)

References

[LVW] Y. Li, M. Vuorinen, and X. Wang : *Two properties of John domains in real Banach spaces,* Preprint.

Properties of quasihyperbolic balls and geodesics on Banach spaces

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Abstract. We deal with Banach manifolds, which are obtained by defining a conformal metric on non-trivial subdomains of a given Banach space. An example of such metric is the quasihyperbolic metric on a domain of a Banach space. It is obtained from the norm-induced metric by adding a weight, which depends only on distance to the boundary of the domain. We present results on convexity and starlikeness of quasihyperbolic and distance ratio metric balls on Banach spaces. In particular, problems related to these metrics on a punctured Banach space are considered [RT1]. We also discuss our recent work on existence and smoothness of quasihyperbolic geodesics on Banach spaces [RT2].

This presentation is based on joint work (in process) with Jarno Talponen.

References

[RT1] Antti Rasila and Jarno Talponen: *Convexity properties of quasihyperbolic balls on Banach spaces.* Ann. Acad. Sci. Fenn. Math. 37 (2012), 215-228.

[RT2] Antti Rasila and Jarno Talponen: *Quasihyperbolic geodesics on Banach spaces.* Manuscript.

Yukawa Potential, Harmonic Measure and Killing Brownian Motion

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Tommi Sottinen[*]

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Abstract. We define the panharmonic measure, which is a generalization of the harmonic measure to solutions of the Yukawa partial differential equation. We show that this quantity has some of the important properties of the classical harmonic measure. In particular, there is a natural stochastic definition of the panharmonic measure in terms of the Brownian walk that is killed with independent exponential stopping time.

References

[D] R.J. Duffin: *Yukawan potential theory,* J. Math. Anal. Appl. 35 (1971), 105–130.

[K] O. Kallenberg: *Foundations of Modern Probability*, Springer, 1997.

[RS] A. Rasila and T. Sottinen: *Yukawa Potential, Harmonic Measure and Killing Brownian Motion,* Manuscript in preparation 2012 .

The visual angle metric and Möbius transformations

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Abstract. A new similarity-invariant metric v_G, defined on a domain $G \subsetneq \mathbb{R}^n$ whose boundary is not a proper subset of a line, is introduced. We find sharp bounds for v_G in terms of the hyperbolic metric in the unit ball and the upper half space. We also obtain sharp Lipschitz constants w. r. t. v_G under the Möbius transformations from the upper half plane onto itself or onto the unit disk as well as from the unit ball onto itself. Furthermore, we show that bilipschitz maps from the unit ball onto itself w. r. t. v_G are quasiconformal and bilipschitz w. r. t. the hyperbolic metric.

References

[KLVW] R. Klén, H. Lindén, M. Vuorinen, G. Wang: *The visual angle metric and Möbius transformations.* arXiv:1208.2871v1 [math.MG]. (PDF)

On the quasisymmetry of quasiconformal mappings and its applications

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(joint with M. Huang, S. Ponnusamy and A. Rasila)

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Abstract. Suppose that D is a proper domain in $\mathbb{R}^n$ and that f is a quasiconformal mapping from D onto a John domain D' in $\mathbb{R}^n$. First, we show that if D and D' are bounded, and D is a broad domain, then for an arcwise connected subset A in D, $f(A)$ is LLC_2 with respect to $\delta_{D'}$ in D' if and only if the restriction $f|_A : A \to f(A)$ is quasisymmetric in the metrics δ_D and $\delta_{D'}$. This result implies that the answer to one of the open problems raised by Heinonen from 1989 is affirmative under the additional assumption "A being arcwise connected". As an interesting consequence of this result, we establish nine equivalent conditions for the domain quasiconformally equivalent to a uniform domain to be John. This result is indeed a generalization of the main result of Heinonen in 1989. Based on the equivalent conditions to be John, we prove that if f is a quasiconformal mapping from a bounded uniform domain onto a bounded John domain, then f is uniformly Hölder continuous, which is a generalization of a result of Näkki and Palka in 1980.

Acknowledgement. The author is supported by the Academy Finland Project 2600066611 (M. Vuorinen)

References

[1] M. Huang, S. Ponnusamy, A. Rasila, and X. Wang: *On the quasisymmetry of quasiconformal mappings and its applications*, Preprint.

On Existence of Solutions for Differential Equations with Impulses

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Abstract. Many evolution processes are characterized by the fact that they experience abrupt changes in their state. These processes are subject to short-time perturbations whose duration is negligible in comparison with the duration of the process. Consequently, it is natural to assume that these perturbations act instantaneously, that is, in the form of impulses. Differential equations involving impulse effects, appear as a natural description of observed evolution phenomena of several real world problems. We consider two examples of differential equations with impulses. The Duffing equation is a non-linear second-order ODE which is used in physics to model oscillators. It is a well-known example of a dynamical system exhibiting chaotic behavior. We discuss the existence of solutions to the Duffing-type equation given by W.Y. Ding,

$$x'' + g(x) = f(x,t), \tag{1}$$

with impulses. We obtain the sufficient condition of existence of infinitely many solutions, by means of the Poincaré-Birkhoff fixed point theorem. We also consider the one-dimensional Wave Equation with impulses

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}, \qquad c^2 = \frac{T}{\rho} > 0, \tag{2}$$

which arises from the standard mathematical model for a vibrating string.

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References

[1] Xuxin Yang, Weibing Wang and Jianhua Shen: *Existence of Periodic Solutions for the Duffing Equation with Impulses* Mathematical Problems in Engineering 2012 (2012), Article ID 903653, doi:10.1155/2012/903653.

[2] Xuxin Yang, Antti Rasila and Weibing Wang: *On Solutions of the Wave Equation with Impulses in Time.* Manuscript.

Inequalities for the generalized trigonometric and hyperbolic functions

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Abstract. The generalized trigonometric functions occur as an eigenfunction of the Dirichlet problem for the one-dimensional $p-$Laplacian. The generalized hyperbolic functions are defined similarly. Some classical inequalities for trigonometric and hyperbolic functions, such as Lazarević's inequality, Huygen's inequality, wilker's inequality, and Cuza-Huygens type inequality, are generalized to the case of generalized functions.

References

[1] R. Klén, M. Vuorinen, and X. Zhang: *Inequalities for the generzlized trignometric and hyperbolic functions*, (Manuscript).