WORKSHOP
ON MODERN TRENDS
IN CLASSICAL ANALYSIS AND
APPLICATIONS

The 4th Chinese-Finnish Seminar
CHIFIN2015

September 18, 2015
University of Turku, Finland
http://users.utu.fi/vuorinen/CHIFIN2015/

Earlier workshops of this series were organized at the University of Turku in 2012, 2013, and at the Aalto University on September 9, 2014.

Programme

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Friday, September 18, 2015

Site of the event:
Turku University Botanical Garden
Ruissalon puistotie 215
Turku
tel. 3339060

9:50	Matti Vuorinen	Opening remarks
10:05-10:50	Mike Porter	Characteristic function for multi-parameter Sturm-Liouville spectral problems
10:50-11:00		**Coffee**
11:00-11:25	Xuxin Yang	Multiplicity of Periodic Solutions for Third-Order Nonlinear Differential Equations
11:30-11:55	Xiaohui Zhang	On isometries of conformally invariant metrics
12:00-13:00		**Lunch**
13:00-13:25	Gendi Wang	The visual angle metric and quasiregular maps
13:30-13:55	Riku Klén	Local convexity properties of the triangular ratio metric
14:00-14:25	Parisa Hariri	Inequalities and bilipschitz conditions for triangular ratio metric
14:30-14:55	Antti Rasila	Minimal surfaces and harmonic mappings
15:00-15:25	Peter Hästö	Generalized Orlicz spaces

Inequalities and bilipschitz conditions for triangular ratio metric

Parisa Hariri

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Abstract. Let $G \subsetneq \mathbb{R}^n$ be a domain and let d_1 and d_2 be two metrics on G. We compare the geometries defined by the two metrics to each other for several metrics such as the distance ratio metric, the triangular ratio metric and the visual angle metric. Finally we apply our results to study Lipschitz maps with respect to metrics.

References

[HVZ] P. Hariri, M. Vuorinen, and X. Zhang: Inequalities and bilipschitz conditions for triangular ratio metric. arXiv: 1411.2747 [math.MG] 16pp.

Generalized Orlicz spaces

Peter Hästö

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Abstract. In this talk I present recent advances in the theory of harmonic analysis in generalized Orlicz spaces, including the boundedness of the maximal operator and the Riesz potential operator.

Local convexity properties of the triangular ratio metric

Riku Klén

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Abstract. We study local convexity properties of the triangular ratio metric balls. In particular, we show that these balls are starlike in starlike domains and convex whenever the radius is less than or equal to $1/2$. We also study inclusion properties of the visual angle metric balls and related hyperbolic type metric balls.

References

[1] R. KLÉN, P. HARIRI, M. VUORINEN: *Local convexity properties of the triangular ratio metric.* Manuscript in preparation, 2015, 13 pages.

[2] R. KLÉN, H. LINDÉN, M. VUORINEN, G. WANG: *The visual angle metric and Möbius transformations.* Comput. Methods Funct. Theory October 2014, Volume 14, Issue 2–3, pp 577–608, DOI 10.1007/s40315-014-0075-x, arxiv.org/abs/1208.2871math.MG

Characteristic function for multi-parameter Sturm-Liouville spectral problems

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Abstract.

The acronym SPPS refers to a recent method [KP] for solving and studying the Sturm-Liouville equation $(py')'(x) + q(x) = \lambda\, r(x)y(x)$ by means of an easily calculated pair of "Spectral Parameter Power Series" of the form $\sum_{k=0}^{\infty} c_k(x)\lambda^k$ which are linearly independent solutions to the differential equation. First we will summarize briefly how this has been applied to a wide variety of initial-value, boundary-value and spectral problems in the last five years. The coefficients $c_k(x)$ are functions on the interval of definition of the equation.

Then we generalize [P] the representation for a general multi-spectral-parameter equation $(py')'(x) + q(x) = (\lambda_1 r_1(x) + \cdots + \lambda_d r_d(x))y(x)$, valid for $\lambda_1, \ldots, \lambda_d$ real or complex. Similarly to the 1-parameter SPPS situation, the coefficients are obtained recursively using indefinite integrals involving the coefficients of lower degree.

We will give some numerical examples using these formulas to solve some boundary value problems for $d = 2$, and also in an application to transmission and reflectance in optics.

References

[KP] V. V. KRAVCHENKO, R. M. PORTER: *Spectral parameter power series for Sturm-Liouville problems*, Math. Meth. Appl. Sci. **33**, 459–468 (2010)

[P]   R. M. PORTER, On Sturm-Liouville Equations with Several Spectral Parameters, arXiv:1504.02003[math.ca].

Minimal surfaces and harmonic mappings

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Abstract. Minimal surfaces are defined as surfaces with zero mean curvature. A minimal surface therefore can be understood as a parametrized surface that satisfies Lagrange's equation, also called the minimal surface equation. Minimal surfaces are, however, intimately connected to the harmonic mappings of the complex plane through the Enneper-Weierstrass parameterization that gives a correspondence between harmonic mappings and minimal surfaces. In this talk we discuss minimal surfaces and their applications in the theory of harmonic mappings. This presentation is based on joint research with S. Ponnusamy and T. Quach [PQR].

References

[PQR] S. PONNUSAMY, T. QUACH AND A. RASILA: Harmonic shears of slit and polygonal mappings. *Applied Mathematics and Computation* **233** (2014) 588–598.

The visual angle metric and quasiregular maps

Gendi Wang

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Abstract. The distortion of distances between points under maps is studied. We first prove a Schwarz-type lemma for quasiregular maps of the unit disk involving the visual angle metric. Then we investigate conversely the quasi-conformality of a bilipschitz map with respect to the visual angle metric on convex domains. For the unit ball or half space, we prove that a bilipschitz map with respect to the visual angle metric is also bilipschitz with respect to the hyperbolic metric. We also obtain various inequalities relating the visual angle metric to other metrics such as the distance ratio metric and the quasihyperbolic metric.

References

[VW] M. VUORINEN AND G.-D. WANG: The visual angle metric and quasiregular maps. arXiv:1505.00607 [math.CA].

Multiplicity of Periodic Solutions for Third-Order Nonlinear Differential Equations

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Abstract. Questions on the existence and the multiplicity of periodic solutions are important topics in qualitative analysis of differential equations. Much work related to periodic solutions for second order differential equations has been done by using various theorems and methods of nonlinear functional analysis. In this presentation, we investigate existence of periodic solutions of the following differential equation:

$$u'''(t) = g(u(t)) - f(t, u(t)),$$

where $g(u) : \mathbb{R} \to \mathbb{R}$, $f(t, u)$ is ω-periodic in t and $\omega > 0$. The method of proof relies on Schauder's fixed point theorem applied in a novel way, where the original equation is transformed into second-order integrodifferential equation through a linear integral operator. Finally, examples are presented to illustrate applications of the main results. This presentation is based on joint research with Weibing Wang and Dingyang Lv [XWL].

References

[XWL] Xuxin Yang, Weibing Wang and Dingyang Lv: *Mathematical Problems in Engineering* **2015** (2015), Article ID 735758, 7 pages
http://dx.doi.org/10.1155/2015/735758

On isometries of conformally invariant metrics

Xiaohui Zhang

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Abstract. This talk is based on the joint paper [KVZ]. We prove that isometries in a conformally invariant metric of a general domain are quasiconformal. In the particular case of the punctured space, we prove that isometries in this metric are Möbius, thus resolving a conjecture of Ferrand, Martin and Vuorinen [FMV, p. 200] in this particular case.

References

[FMV] J. Ferrand, G.J. Martin, and M. Vuorinen: Lipschitz conditions in conformally invariant metrics, J. Analyse Math. 56 (1991), 187–210.

[KVZ] R. Klén, M. Vuorinen, and X.-H. Zhang: *On isometries of conformally invariant metrics.* J. Geom. Anal. 2015 (to appear). DOI 10.1007/s12220-015-9577-7, arXiv:1411.4381 math.CV.