Matematiikan ja tilastotieteen laitos
Funktionaalianalyysin perusteet
Harjoitus 4
3.10.2013

1. Let H be a Hilbert space, let $y \in H \setminus \{0\}$ and let $S = Sp\{y\}$. Show that $\{x \in H : (x, y) = 0\}^\perp = S$.

2. Let Y be a closed linear subspace of a Hilbert space H. Show that if $Y \neq H$ then $Y^\perp \neq \{0\}$. Is this always true if Y is not closed? [Hint: consider dense, non-closed linear subspaces.]

3. Let X be a Hilbert space and let $A \subset X$ be non-empty. Show that: (a) $(A^\perp)^\perp = \overline{Sp}A$; (b) $A^{\perp\perp\perp} = A^\perp$ (where $A^{\perp\perp\perp} = ((A^\perp)^\perp)^\perp$.

4. Consider the function $\sin(x)$ on $[0, \pi/2]$. Find the best approximation
$$\min_{b_1, b_2} \int_0^{\pi/2} (\sin(x) - b_1 - b_2 x)^2 \, dx \,.$$
Find the best approximation
$$\min_{b_1, b_2, b_3} \int_0^{\pi/2} (\sin(x) - b_1 - b_2 x - b_3 x^2)^2 \, dx \,.$$

5. To fit a plane $z = Ax + By + C$ through the N points (x_j, y_j, z_j), $j = 1, \ldots, N$, the expression
$$S = \sum_{j=1}^{N} (Ax_j + By_j + C - z_j)^2$$
is minimized by setting up the *normal equations*
$$\frac{\partial S}{\partial A} = 0, \quad \frac{\partial S}{\partial B} = 0, \quad \frac{\partial S}{\partial C} = 0 \,.$$
This yields three linear equations in A, B, C. Write these equations (but do not solve them).