**Matematiikan ja tilastotieteen laitos**
**Funktionaalianalyysin perusteet**
**Harjoitus 5**
**10.10.2013**

1. If $T : C_{\mathbb{R}}[0,1] \to \mathbb{R}$ is the linear transformation defined by

$$T(f) = \int_0^1 f(x)\,dx\,,$$

show that $T$ is continuous.

2. Let $h \in L^\infty[0,1]$.
(a) If $f$ is in $L^2[0,1]$, show that $fh \in L^2[0,1]$.
(b) Let $T : L^2[0,1] \to L^2[0,1]$ be the linear transformation defined by $T(f) = hf$. Show that $T$ is continuous.

3. Let $\mathcal{H}$ be a complex Hilbert space and let $y \in \mathcal{H}$. Show that the linear transformation $f : \mathcal{H} \to \mathbb{C}$ defined by $f(x) = (x, y)$ is continuous.

4. (a) If $(x_1, x_2, x_3, x_4, \dots) \in \ell^2$, show that $(0, 4x_1, x_2, 4x_3, x_4, \dots) \in \ell^2$.
(b) Let $T : \ell^2 \to \ell^2$ be the linear transformation defined by

$$T(x_1, x_2, x_3, x_4, \dots) = (0, 4x_1, x_2, 4x_3, x_4, \dots).$$

Show that $T$ is continuous.

5. A person runs the same track for five consecutive days and is timed as follows

| | | | | | |
|---|---|---|---|---|---|
| day (x) | 1 | 2 | 3 | 4 | 5 |
| time (y) | 15.50 | 15.10 | 15.00 | 14.50 | 14.00 |

make a least square fit to the above data using a function $a + b/x + c/x^2$ .

6. Obtain an approximation in the sense of least squares in the form of a polynomial of degree 2 to the function $1/(1 + x^2)$ on the interval $[-1, 1]$ .